LINEAR PROGRAMMING TECHNIQUES FOR INITIAL ATTACK RESOURCE DEPLOYMENT[1]

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ABSTRACT: Linear programming techniques are used to optimally place fire suppression resources in a presuppression role. Travel times required to reach individual cells within a forest region (attack times) are calculated using fire behavior models and attack time objective sizes or perimeters. A linear programming formulation is then presented so that the number of resources required to meet all attack times are minimized and optimally placed among available bases to meet the attack times objectives of each cell. A second formulation is presented which maximizes coverage of cells with a limited number of resources.
KEYWORDS: Fire management, presuppression planning, operations research.

INTRODUCTION

The prepositioning of fire suppression resources to meet the daily fire danger situation in a region is a fundamental problem for forest protection officers. To help officers arrive at this decision, a number of preparedness planning systems have been designed to determine the required manning levels using fire weather models to estimate the relative fire hazard in the forest (Gray and Janz 1983; Lanoville and Mawdsley 1990; De Groot 1991; Hirsch 1991). In more recent years, computer-aided decision support systems have modelled potential fire behavior with available forest inventory, terrain, and weather, thus allowing a more direct method to assess coverage efficiency (Lee and Anderson 1990). Operations research (OR) and linear programming (LP) have direct applications in arriving at optimal solutions to these analysis techniques.

Operations research is a relatively new science developed during the Second World War. Following its initial military uses, OR was successfully applied to solve various economic and managerial problems. Generally, OR problems take the form of finding the optimal combination of economic factors such as maximizing profits while minimizing costs. Linear programming (LP) is a mathematical technique used to solve a certain class of OR

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problems that lend themselves to a linear combination of variables. By defining the problem as a matrix, linear algebra techniques such as the simplex method are used to solve the problem.

METHODOLOGY

Coverage assessment is an analysis technique used to evaluate the coverage achieved by initial attack resources, such as air tankers and helicopter carried ground crews, over a forest region (Lee and Anderson 1990). Though this paper does not attempt to define spatial coverage requirements, a brief summary of the technique used by Lee and Anderson is presented, by way of example.

For coverage assessment, the forest region is first rasterized into an array of cells. Typically, these take the form of squares with a scale in the order of 1-10 kilometres a side. Then the coverage for each cell is calculated as follows. Fuel, topography, and weather for the cell are retrieved from databases and used as inputs for fire growth models. From initial attack policy, the models calculate the time it would take a fire in the cell to reach a critical size or perimeter. This time is referred to as the attack time and the size or perimeter as the attack time objective. Finally, the coverage is defined as the number of resources that can reach the cell within the attack time, given resource get-away times, travel speeds and distances from manned bases to the cell.

Determining attack times is an essential step in the coverage assessment analysis. In the above example, attack time calculations are based on attack time objectives such as a critical size or perimeter. A simpler approach would be to set the attack time to a constant, thus requiring all fires be attacked within a predetermined time period, say 30 minutes. Another possible alternative would be to base the attack times on a fire containment model.

LP problems can be formulated to provide an optimal deployment of initial attack resources. These problems are an integer programming class of LP problems where variables must assume nonnegative integer valued solutions.

The problems take the following general form. A number of cells require a degree of coverage by fire suppression resources. To achieve this coverage, a number of fire suppression resources can be based at predefined initial attack bases. Initial attack bases are numbered 1 to n, with a subscript i, while cells are numbered 1 to m with a subscript j.

The variables that enter the problems are:

b_i = the resource allocation

a_{ij} = a binary coefficient (0 or 1)

R_j = the required coverage for a cell

c_j = the provided coverage for a cell

The resource allocation, b_i, is the number of resources deployed at the ith base; if two crews were deployed at base 12, then b_{12} would equal two. The binary coefficient a_{ij} could be better described as a logical variable set to one if it is true, zero if it is false. The condition it is based on is whether a resource from base b_i can provide coverage for cell j. If atleast one resource can reach the cell, a_{ij} equals one, otherwise it is set to zero. The required coverage variable, R_j, indicates the number of resources policy may require to cover a cell. For example, policy may dictate that fires with a head fire intensity of over 2000 kW/m should be attacked by two resources. On the other hand, the provided coverage, c_j, is the coverage actually provided from presently positioned resources, that is the number of resources that can reach a potential fire in the cell before it grows beyond its attack time objective. The provided coverage c_j has an upper limit equal to the required coverage R_j; any coverage provided over that required is ignored in the calculations.

The general problem can be described with two unique LP optimization formulations. The first is to minimize the number of resources required to achieve total coverage over a region. The second formulation is to maximize coverage over a region under the restraint of a predetermined number of resources.

Minimal Resources Problem

The minimal resource problem can be described as minimizing the number of suppression resources required to provide full coverage (where possible) of all the cells within a forest region. This problem falls into the coverage class of integer programming problems (Schrage 1986).

Formulation

For j cells, the resource problem is:

minimize

$$(1) \quad \sum_{i=1}^{n} b_i$$

subject to

$$(2) \quad \sum_{i=1}^{n} a_{ij} b_i \geq R_j$$

Equation 1 defines the objective of the problem: to minimize the total number of deployed resource, b_i. Equation 2 sets the constraint that, for every cell, a number of resources equal to the required coverage, R_j, must be able to reach the cell within the attack time objective.

Maximum Coverage Problem

A variation of the minimal resource problem is to maximize coverage of the forest region while the number of resources is limited. This problem can best be classified as a distribution of effort (Wagner 1969).

Formulation

The maximum coverage problem uses much of the same reasoning as the minimum resource problem but it is considerably more involved. If we define the resource limit as N, the problem becomes:

maximize

$$(3) \quad \sum_{i=1}^{n} c_j - \sum_{j=1}^{m} b_i$$

subject to

$$(4) \quad \sum_{i=1}^{n} a_{ij} b_i - c_j \geq 0$$

and

$$(5) \quad c_j \leq R_j$$

and

$$(6) \quad \sum_{i=1}^{n} b_i \leq N$$

Equation 3 states that the objective of the problem is to maximize coverage, c_j. In addition, the objective function minimizes the number of deployed resources, b_i. The intent is to introduce b_i as a variable that must be solved for, otherwise the allocation of resources is unknown. As well, by minimizing the number of resources, the possibility of deploying more resources than are required to reach the optimal deployment described by the minimal resource problem is eliminated.

Admittedly, the units for coverage and resource allocations are not the same and this is a serious concern. Yet, in normal conditions, the sum of the coverage greatly outweighs the number of activated bases and the difference in units can be ignored. The proper method would be to equate the variables to costs, which is discussed later.

Equations 4, 5, and 6 set the constraints. Equation 4 states that the provided coverage for a cell subtracted from the number of resources with travel times less than the cell's attack time must be nonnegative. In other words, the provided coverage cannot exceed the number of resources capable of covering the cell. Equation 5 further limits the provided coverage, c_j, to the required coverage, R_j, thus the combination of equations 4 and 5 will set the provided coverage, which is being maximized, to the lesser of $\Sigma a_{ij} b_i$ or R_j. Equation 6 sets the limit of the number of deployed resources which cannot exceed the constant N.

EXAMPLE

To illustrate the formulation, we define a hypothetical forest composed of 25 square cells as shown in figure 1. Each cell contains an initial attack base. The fuel, weather, and topography in the forest is homogeneous producing fire behavior characteristics that are constant throughout the forest.

The attack times for each cell are the same and are equivalent to the travel time required for a resource to travel from one cell to an adjacent cell. In other words, a positioned resource can cover the cell it occupies and the four adjacent cells, i.e., a resource positioned in cell 13 is capable of covering cell 13 plus the four adjacent cells: 8, 12, 14, and 18.

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

Figure 1. A hypothetical forest of 25 cells, each cell containing one initial attack resource. A resource in cell 13 is capable of covering cells 8, 12, 13, 14, and 18, indicated with shading.

If one resource is required to cover each cell ($R_j = 1$), the minimal resource problem would take the following form.

minimize

$$b_1 + b_2 + b_3 + \cdots + b_{25} \qquad (7)$$

subject to

$$\begin{aligned} b_1 + b_2 + b_6 &\geq 1 \\ b_1 + b_2 + b_3 + b_7 &\geq 1 \\ b_2 + b_3 + b_4 + b_8 &\geq 1 \\ &\vdots \\ b_{20} + b_{24} + b_{25} &\geq 1 \end{aligned} \qquad (8)$$

The solution to the problem indicates that bases b_2, b_9, b_{10}, b_{11}, b_{18}, b_{22}, and b_{25} should be activated. Actually, a number of solutions can be derived with seven activated bases but the minimum number remains as seven.

Though the solution to the minimal resource problem does satisfy the needs of covering all the cells with a minimum number of resources, it may not be the optimal solution desired. In this example, seven resources are required to provide total coverage of the forest region. Of the twenty-five cells, four are being covered by more than one resource (multiple coverage). Also five of the resources have their coverage extending outside the forest region. Though these may be desirable effects, it may be an indication of an overcommittment of resources.

If it is decided that the coverage provided by the minimal resource problem is too much, or if there is a limited number of available resources (and that this is less than seven), the maximum coverage problem for the same situation can be set up for the available number of resources.

maximize

$$c_1 + c_2 + c_3 + \cdots + c_{25} - b_1 - b_2 - b_3 - \cdots - b_{25} \tag{9}$$

subject to

$$\begin{aligned} b_1 + b_2 + b_6 \quad - c_1 &\geq 0 \\ b_1 + b_2 + b_3 + b_7 - c_2 &\geq 0 \\ b_2 + b_3 + b_4 + b_8 - c_3 &\geq 0 \\ &\vdots \\ b_{20} + b_{24} + b_{25} - c_{25} &\geq 0 \end{aligned} \tag{10}$$

and

$$\begin{aligned} c_1 &\leq 1 \\ c_2 &\leq 1 \\ c_3 &\leq 1 \\ &\vdots \\ c_{25} &\leq 1 \end{aligned} \tag{11}$$

and

$$b_1 + b_2 + b_3 + \ldots + b_{25} \leq N \tag{12}$$

where N is the number of resources. The solutions to this problem depend on the number of resources chosen. They are shown in Table 1.

Table 1 summarizes the solutions of the maximum coverage problem for a given number of resources. Coverage levels achieved, shown as percentage, are classified as multiple, single, and none, indicating the area covered by more than one, one or no resources respectively. The number of resources that extend their coverage outside the forest region are also shown in Table 1. The final column lists the activated bases.

From Table 1, adequate levels of coverage can be achieved with fewer than seven resources. It is not the authors' position to dictate what choice of resources is optimal. That choice would depend on agency policy and the level of risk that fires will occur.

Table 1. Summary of maximum coverage established for a chosen number of resources.

Number of resources chosen	Coverage achieved (%)			Resources with extended coverage	Bases activated
	Multiple	Single	None		
7	16	84	0	5	b_2, b_9, b_{10}, b_{11}, b_{18}, b_{22}, b_{25}
6	8	84	8	4	b_3, b_6, b_{10}, b_{13}, b_{17}, b_{24}
5	0	84	16	4	b_4, b_6, b_{15}, b_{17}, b_{24}
4	0	72	28	2	b_6, b_9, b_{17}, b_{24}
3	0	56	44	1	b_4, b_7, b_{19}
2	0	40	60	0	b_8, b_{17}
1	0	20	80	0	b_8

DISCUSSION

Solving these problems for a real situation requires considerable computing resources. Forest inventory databases, weather interpolation schemes, and fire behavior models all must be integrated to formulate the attack time requirements for each rasterized cell. Computerized fire management systems, such as IFMIS, the Intelligent Fire Management Information System, (Lee 1990) provide good platforms in which to do this. In addition, software must be incorporated to solve the LP problems. The LINDO software (Schrage 1986) was used to test the concepts presented in this paper.

There are limitations to the approach presented in this paper. Cells in which no resources can meet the attack time are excluded from the analysis. Future investigations will look for a formulation that ensures at least one initial attack base meets the initial attack objective time for each cell. Also, the calculations assume a constant travel speed and get-away time for all resources. To include a variety of aircraft with different cruising speeds requires a more complex problem with an insurmountable number of calculations.

Perhaps the most serious shortcoming of this approach is that this is an answer for a static world; however, in reality, it must be applied to a dynamic world. Often initial attack resources are mobilized and sent on patrols over potentially troublesome areas. When resources are dispatched to fires, holes are also created in the coverage network. This may be serious in the case of multiple fire starts.

While the current formulation could incorporate a cost of activating each man-up base, this has not been attempted. To factor this in, weight the number of deployed resources, b_i, with a cost. This could be further developed to show economics of the impact of fire. This can be done by weighting the required coverage variable, R_j, with the value of the timber.

CONCLUSIONS

Using linear programming techniques to optimally place fire suppression resources is a practical option available to computerized fire management systems. It has been shown that linear programming can be used to solve two problems: that of minimizing the number of resources required to provide total coverage of a forest region, and to maximize coverage of a forest region by a limited number of resources. Undoubtedly, there are more applications that can be designed.

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