MODELING GROWTH AND YIELD OF ASI

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ABSTRACT

Using experimental plot data from numerous studies across the aspen growing regions of North America we developed a system of nonlinear equations describing annual changes of quadratic mean diameter and mortality in aspen stands. These relationships are the starting point for developing a growth and yield prediction system for this species. The system is based in Alberta. It allows for easy implementation of stand treatments such as thinning and fertilization, and is robust with respect to extrapolation beyond the data range.

INTRODUCTION

Less than a decade ago aspen was considered a weed tree in the vast boreal forests of western Canada. The recent dramatic rise in aspen harvest changed that and also increased the need for information related to the management of this species. Improved growth and yield prediction ranks high among tools and information urgently needed.

The main objective of the current study is to develop a comprehensive system of growth and yield prediction that would be applicable in western Canada for all major commercial tree species and cover types, including aspen. The system should be compatible with available forest inventory information and used for updating the inventory and calculating annual allowable cut. It should also be useful to evaluate growth and yield under a range of management scenarios and stand conditions. The model structure should be robust enough to allow its development from limited data and also allow reasonable extrapolation beyond.

Yield models currently used in the region do not meet the above objectives because they are based on cumulative functions with fixed yield trajectories and provide no allowance or adjustment for variations in stand density. Among the theories used as a basis of stand modeling, the self-thinning rule (Westoby 1984) seems to be the most suitable, being relatively simple and without major drawbacks. This theory states that in fully stocked stands mean plant size is a negative linear function of stand density, when both variables are in logarithms (Fig. 1). In other words, in such stands, density and radial growth are self-limiting. In open stands, the theory has to be complemented by a growth-survival model, derived from representative data, to simulate appropriate stand dynamics.

The self-thinning theory, or rule, has been applied for modeling growth and yield of various tree species growing in stands around the world, including lodgepole pine in Alberta[1]. In the latter case, the rule describes the relation between stand total volume, mean quadratic diameter, stand average height, and the number of trees per hectare.

Ciewzewski, C.J.; Bella, I.E.; Perala, D.A. 1991. Modeling growth and yield of aspen in western Canada. *In* S. Navratil and P.B. Chapman, editors. Aspen management for the 21st century. Proceedings of a symposium held November 20-21, 1990, Edmonton, Alberta. For. Can., Northwest Reg., North. For. Cent. and Poplar Counc. Can., Edmonton, Alberta.

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[1] Cieszewski, C.J.; Bella, I.E. LPSIM—Lodgepole Pine Simulator for Alberta (in preparation).

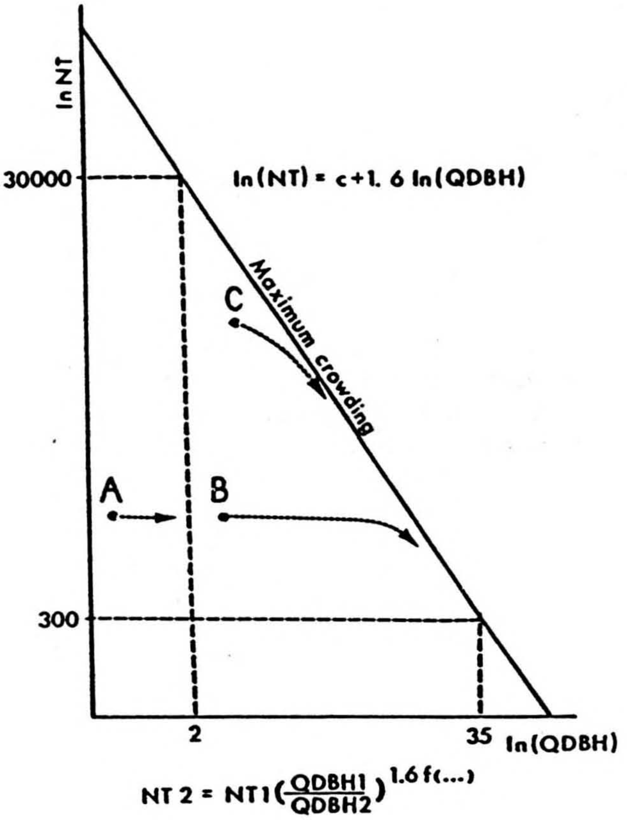

Figure 1. The self-thinning rule diagram.

In this paper we report on the development of a system of nonlinear equations describing annual changes of quadratic mean diameter and mortality in aspen stands to be used as a starting point for developing a growth and yield prediction system for this species. These equations are process-based advancements of empirical equations developed earlier from the same data base[2].

DATA AND METHODS

The experimental sample plot measurement data used in this analysis were obtained from numerous studies across the aspen growing region of North America and Europe (Table 1). Data consisted of 274 observation pairs, the bulk of which originated from the Lake States and prairie provinces (Table 2).

In line with the self-thinning rule, stand density, i.e., crowding, affects stand dynamics by reducing the trees' radial growth and causing mortality. One of the main challenges in formulating the basic model was to select a suitable density measure. This measure had to be expressed in such a way that it would be compatible with the self-thinning rule and would combine tree size and number of trees per hectare into a stand characteristic capable of quantifying stand density. We tried the following three measures of stand density based on the log-log relation between quadratic mean diameter and number of trees per hectare:

a) where the measure of density is the intercept, on the number of trees (Y) axis, of a line parallel to the limiting trajectory and passing through the analyzed stand density and diameter (Fig. 2a);
b) where the measure of density is the absolute vertical distance between the limiting trajectory and the point defined by the stand. That is, the difference between the logarithm of the maximum number of trees for the given diameter, defined by the limiting trajectory, and the logarithm of the actual number of trees in the stand (Fig. 2b); and
c) where the measure of density is the relative vertical distance, i.e., the ratio, between the maximum density, defined by the limiting trajectory for the actual stand diameter, and the actual number of trees in the stand (Fig. 2c).

In developing the lodgepole pine model we found that the best measure of the stand density was the third one, the relative vertical distance between the trajectory defined by the stand and the maximum trajectory. However, during the present analysis of the aspen data we found that for this species a better measure was the intercept value of trajectory for the stand in

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[2] Perala, D.A.; Cieszewski, C.J. Generic growth and yield equations for trembling aspen (*Populus tremuloides* Michx.) based on the self-thinning rule (in review).

Table 1. Data sources

Author	Stand location	Cases	Variable and range Site index (m @ 50 yr)	Age (year)	Dbh (cm)
Day 1958	Lower Michigan	3	18-24	10-25	3-11
Hubbard 1972	North central	6	27	7-24	3-14
Noreen 1986	Minnesota	6	24	4-20	1-10
Perala 1974	Minnesota	7	21	1-10	1-4
Perala 1978	Minnesota	48	23-25	13-53	4-31
Perala (on file)	Minnesota	19	24	15-39	5-17
Perala and Laidly 1989	Minnesota	24	25-31	5-21	3-13
Schlaegel 1971, 1972	Minnesota	62	18-24	10-62	5-35
Schlaegel and Ringold 1971	Minnesota	8	26	37-47	17-24
Pike 1953	Manitoba	3	17-18	35-55	8-16
Steneker 1974	Manitoba	52	16-21	11-44	3-23
Steneker 1969	Saskatchewan	21	19-21	14-30	4-15
Elfving 1986	Sweden	4	34	9-32	7-25
Vuokila 1977	Finland	11	22-23	11-48	3-25

Table 2. Summary statistics of data used for fitting diameter-density relationship for aspen using 274 observation pairs

Variable[a]	Mean	Standard deviation	Minimum	Maximum
Age 1	21.1	10.3	2	44
Age 2	32.2	14.5	3	62
QDBH1	9.53	5.251	0.55	22.21
QDBH2	15.03	7.504	0.88	34.75
Density 1	3348.5	7275.3	333	46085
Density 2	2173.9	3951.1	187	33853

[a] 1 = first measurement; 2 = second measurement.

question, i.e., the first density measure (a), which was then chosen for use.

MORTALITY MODEL DEVELOPMENT

As for lodgepole pine yield modeling, we started the mortality submodel development for aspen with a formulation of an equation describing annual mortality. We approached this using annual mortality or increments to achieve greater flexibility in modeling stand growth and to facilitate the consideration of different stand and environmental conditions. The required nonlinear relation was derived from the self-thinning rule ($\ln NT = c + 1.6 \times \ln QD$) and it defines mortality trajectories on the log-log diagram (Fig. 1). The relationship indicates high mortality near the maximum crowding line, and lesser or no mortality as stands approach open growing conditions. For example, in Figure 1, stands A and B represent open growing conditions with no mortality; stand C, near the maximum crowding line, represents high density and high mortality. The quantification of mortality from open to dense conditions is accomplished by a nonlinear function of the diameter ratio exponent (Fig. 3).

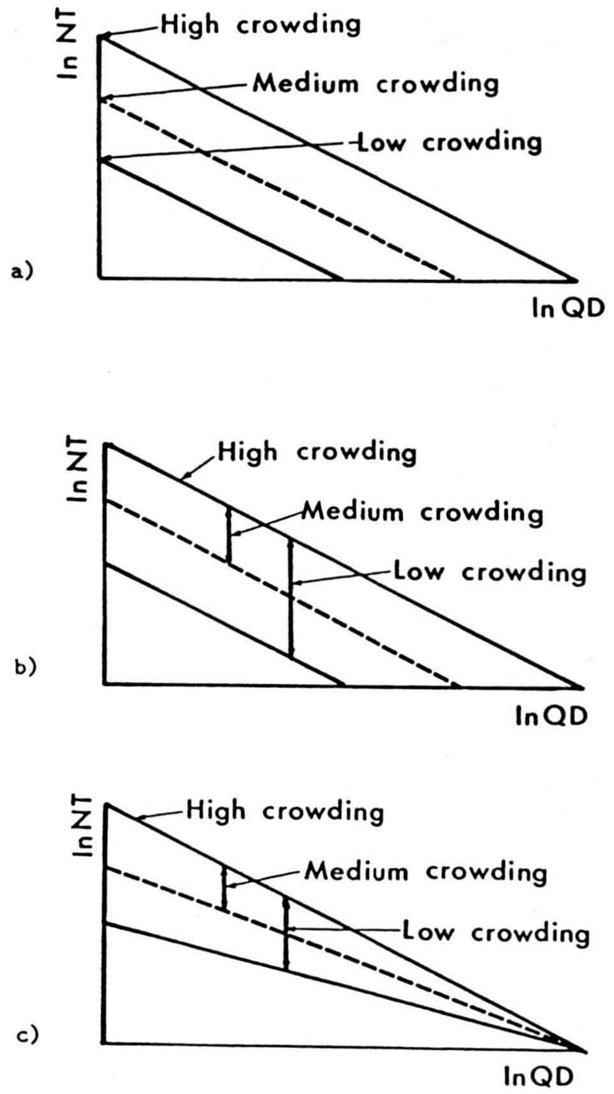

Figure 2. Three measures of density shown in terms of the self-thinning rule: a) intercept; b) absolute vertical distance from the limiting trajectory; c) relative vertical distance from the limiting trajectory.

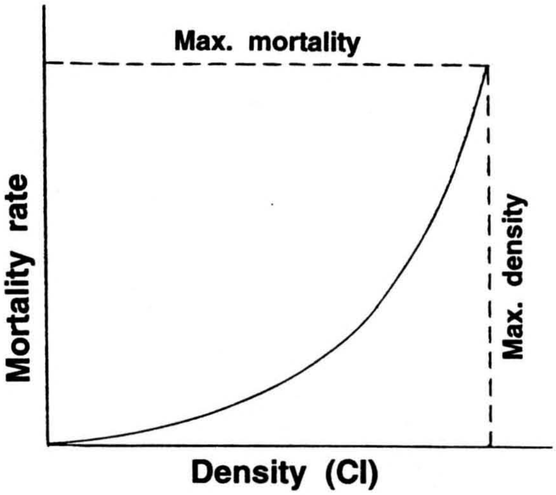

Figure 3. A function defining mortality rate, i.e., the slope on the log-log density diagram.

For our analysis, we had to express density changes on an annual basis. Our data of number of trees over age indicated an inverse exponential relationship (Fig. 4). In order to obtain annual mortality values, we did a logarithmic transformation on the data, which thus allowed a linear interpolation (Fig. 5).

In developing the mortality model we assumed that trees die from two basic causes: 1) crowding effects; and 2) other causes, which can be considered as constant, and related chiefly to insect, disease, mammal, and abiotic factors.

The crowding dependent, or variable mortality was represented by an asymptotic model adhering to the self-thinning rule; the constant mortality was modeled as an intercept and density independent. This means that a minimum mortality will occur even in open stands with no competition. After formulating the basic model structure we tested various hypotheses on the influence of age, site, and elevation on the two types of mortality.

DIAMETER GROWTH MODEL DEVELOPMENT

We started out with the relationship between quadratic mean diameter and a number of trees per hectare developed for lodgepole pine. Because our approach here is based on annual increment data, it was desirable to do the same for the diameter model. Therefore, the data had to be expressed and interpolated accordingly. We approximated mean annual quadratic diameter increments using linear interpolation, which was justified by the consistently linear trend of the mean quadratic diameter data (Fig. 6). For aspen, as for lodgepole pine, we used the differential form of the Von Bertalanffy growth model to describe annual increment of quadratic mean diameter (QD). This model essentially states that the increments are the differences between anabolic (photosynthesis) and catabolic (respiration) processes. The first is expressed as $n \times QD^m$ and the second as $k \times QD$; where n, m, and k are coefficients. In developing the model, our task was to meaningfully express these coefficients as functions of stand density and test their sensitivity to other variables.

CONCLUSIONS

A basic model structure was developed based on the self-thinning rule to describe aspen mortality and annual radial growth. The aspen growth and survival data used conformed well to the self-thinning rule[3] and demonstrated the rule's usefulness in growth and yield modeling of this species. As the model structure shows, only a portion of the mortality in the data is explained by the self-thinning rule and is independent of both density and radial growth. To complete a yield forecasting model for aspen stands, we now have the component mortality and diameter increment models; the height growth model that was developed in a separate study[3]; and we still need to develop a way to project the stand table over time. Then, with this information, stand yield can be estimated simply by using suitable standard volume tables.

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[3] Cieszewski, C.J.; Bella, I.E. 1991. Height growth-site index equations for the major tree species in Alberta (in preparation).

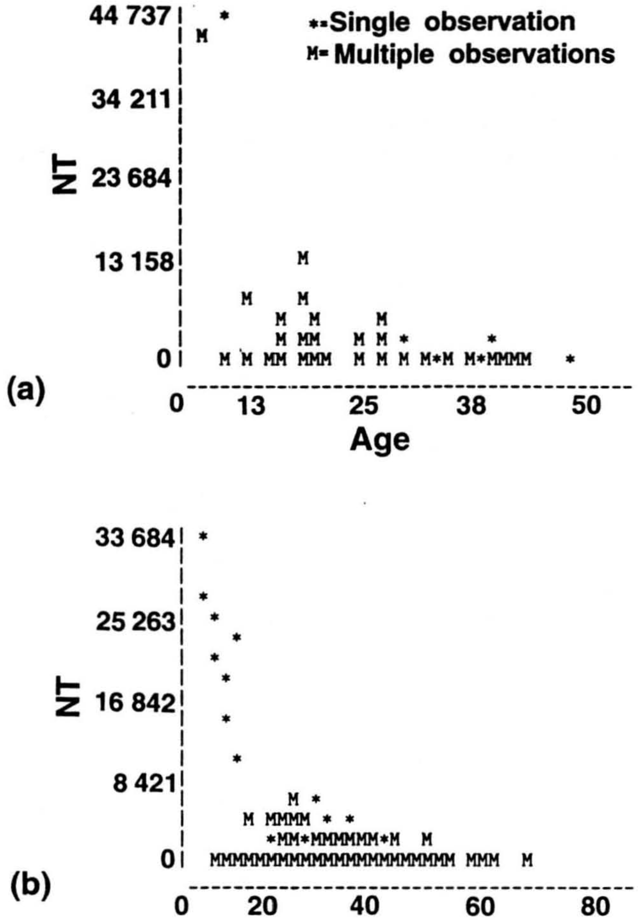

Figure 4. Number of trees per hectare over age (274 observations): (a) first measurement, (b) second measurement.

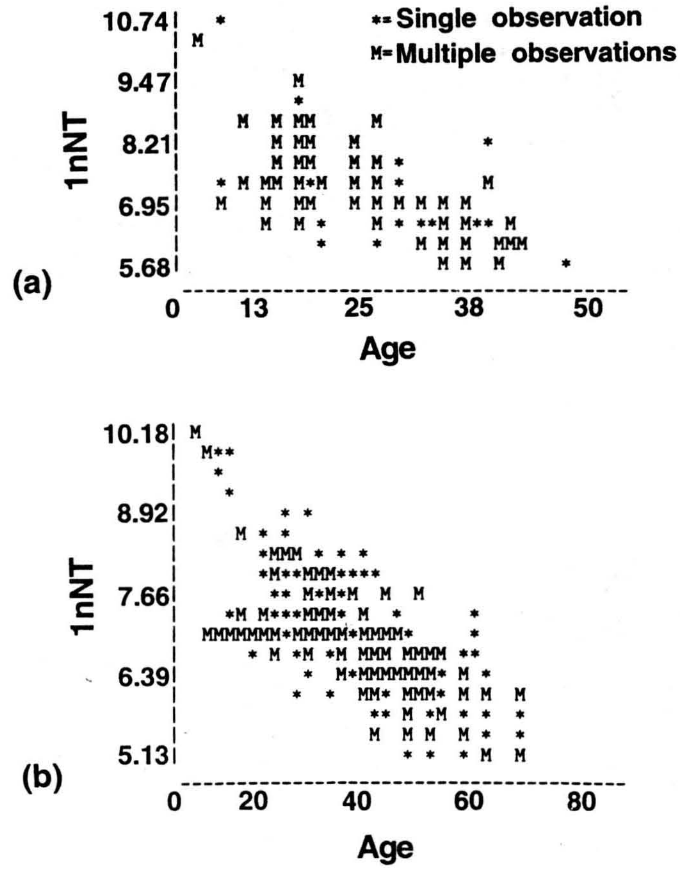

Figure 5. Logarithmically transformed number of trees per hectare over age (274 observations): (a) first measurement, (b) second measurement.

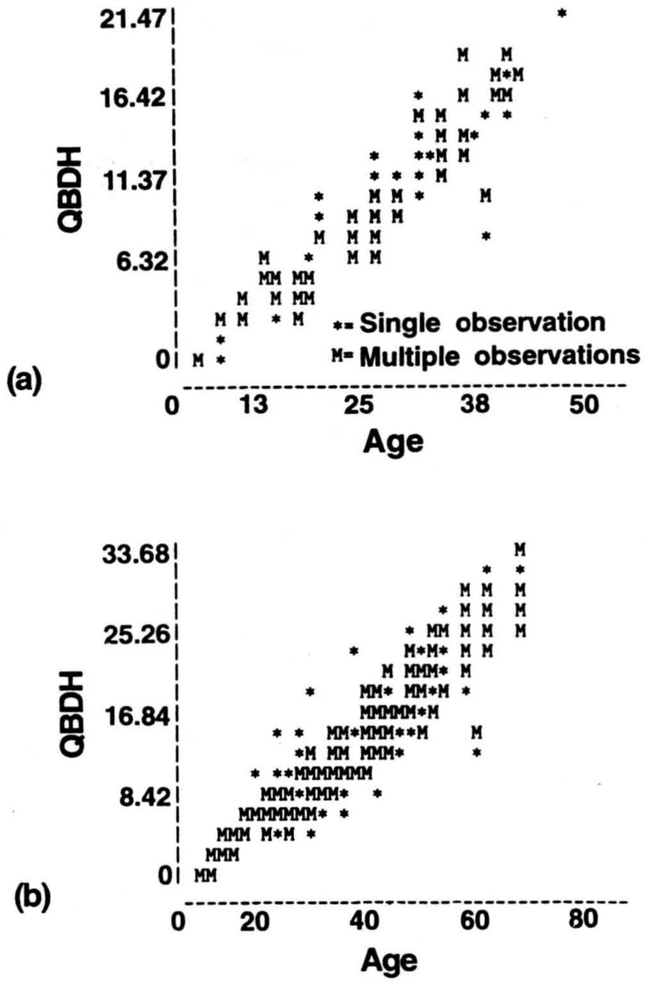

Figure 6. Quadratic mean diameter over age (274 observations): (a) first measurement, (b) second measurement.

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