# FOREST MANAGEMENT NOTE 

Note 51

## POLYMORPHIC HEIGHT AND SITE INDEX CURVES FOR THE MAJOR TREE SPECIES IN ALBERTA

In 1989 Cieszewski and Bella developed a new, two-coefficient variable-age-site index (VASI) height model for lodgepole pine in Alberta. This model 1) provided compatible site index (SI) and height-growth estimates, and 2) predicted height growth at any age, without prior knowledge of SI, as a function of any other height and corresponding age.

The VASI model had fewer coefficients than the previously developed models used for the major tree species in Alberta, and also could predict height with greater accuracy and precision (Cieszewski and Bella 1989).

## Variable-Age Site Index Equation

The biologically based, nonlinear height-growth model(Cieszewski and Bella 1989) was derived from a process of formulating and testing a biological hypothesis on the polymorphism of lodgepole pine growth, and presented as:
[1] $H\left(t, h_{x}, x\right)=$

$$
\frac{h_{x}+\delta+\sqrt{\left(h_{x}-\delta\right)^{2}+\zeta h_{x} x^{1}}}{2+\zeta t^{1} /\left(h_{x}-\delta+\sqrt{\left.\left(h_{x}-\delta\right)^{2}+\delta h_{x} x^{1}\right)}\right.}
$$

where $\delta=20$ ange $_{\mathrm{SI}}^{-1 \cdot \alpha}, \zeta=80 \beta, x^{1}=x^{-1-\alpha}, t^{1}=t^{-1 \cdot \alpha}$, and age $_{\text {SI }}=50$.

After making appropriate substitutions, the equation reads:
[2] $H\left(t, h_{x}, x\right)=$

$$
\frac{h_{x}+20 \beta 50^{-1 \cdot \alpha}+\sqrt{\left(h_{x}-20 \beta 50^{-1-\alpha}\right)^{2}+80 \beta h_{x} x^{-1-\alpha}}}{2+\frac{80 \beta t^{-1-\alpha}}{h_{x}-20 \beta 50^{-1 \cdot \alpha}+\sqrt{\left(h_{x}-20 \beta 50^{-1-\alpha}\right)^{2}+80 \beta h_{x} x^{-1-\alpha}}}}
$$

Equations 1 and 2 generate biologically sound, nonlinear and polymorphic growth curves as a

Table 1. Data screening criteria and summaries by species

${ }^{\mathrm{a}} \mathrm{bh}=$ breast height.
${ }^{\mathrm{b}}$ Not applicable.
${ }^{\mathrm{c}} \mathrm{SI}=$ site index.

Table 2. Summary statistics of the data used for fitting height-growth SI $^{a}$ curves

| Species | Number of trees | $\mathrm{bh}^{\text {b }}$ Age |  |  |  | Height above bh (m) |  |  |  | SI above bh (m) |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Avg | $\mathrm{SD}^{\text {c }}$ | Min. | Max. | Avg | SD | Min. | Max. | Avg | SD | Min. | Max. |
| Lodgepole pine | 1163 | 98.9 | 32.83 | 50.0 | 260.0 | 19.22 | 4.73 | 7.70 | 35.50 | 13.83 | 3.54 | 4.40 | 23.88 |
| White spruce | 698 | 104.6 | 31.61 | 50.0 | 250.0 | 23.90 | 5.24 | 8.69 | 40.92 | 14.22 | 3.43 | 5.01 | 24.45 |
| Black spruce | 282 | 94.8 | 28.35 | 50.0 | 190.0 | 13.96 | 3.70 | 6.87 | 26.50 | 8.99 | 2.36 | 3.98 | 15.87 |
| Trembling aspen | 276 | 70.0 | 19.95 | 50.0 | 140.0 | 20.66 | 3.26 | 9.04 | 28.25 | 17.47 | 2.83 | 8.28 | 24.93 |

${ }^{\text {a }} \mathrm{SI}=$ site index.
${ }^{\mathrm{b}} \mathrm{bh}=$ breast height.
${ }^{\mathrm{c}} \mathrm{SD}=$ standard deviation.
function of prediction age and height at any other age. When age and height at this age are used to produce a reference point on a height curve (instead of using the standard fixed-age SI), the problem of incompatibility between height and SI predictions is precluded. The reference points of each new curve are, by definition, part of a height curve: this means that height at any age can be used to create a set of base-age invariant height curves, because any point on a height curve will unequivocally define the entire growth curve.

## Equation Coefficients

The traditional fixed-age SI was not used in this study, and the term height was given the following two meanings:

1) a computed tree height $\left[H\left(t, h_{x}, x\right)\right]$ for a prediction age $(t)$; and
2) a known height ( $h_{\mathrm{x}}$ ) at any other age ( $x \neq t$ ), i.e., this height ( $h_{\mathrm{x}}$ ) was used in conjunction with this age ( $\mathbf{x}$ ) as a reference point, instead of the SI.
Estimable model coefficients were denoted as $\alpha$ and $\beta$. The coefficient for all species were determined by nonlinear regressions, using stem analysis data.

## Four Major Trees Species

Because of the excellent performance of the VASI model (eq. 1) with lodgepole pine data, it was decided to establish accurate and precise heightgrowth curves for all the major tree species in Alberta. Therefore, following the incorporation of individual data for the four species in eq. 2, this paper presents height-growth SI curves for lodgepole pine (Pinus contorta var. latifolia Engelm.), white spruce [Picea glauca (Moench) Voss], black spruce [Picea mariana (Mill.) B.S.P.], and trembling aspen (Populus tremuloides Michx.).

## DATA SOURCES AND COLLECTION

In this study, stem analysis data from dominant and codominant trees were provided by the Alberta Forest Service (AFS), Weldwood of Canada Limited, Hinton Division, and Forestry Canada. The sample covered each species' commercial range in Alberta.

The total height of each tree was measured, and sections for ring (age) counts were obtained at stump, at breast height (bh) of 1.3 m , and at fixed intervals (of from 1.0 m to 2.5 m ) above bh.

## Data Screening Criteria

Initial data screening for all species was based on the following criteria:

1) non-decreasing heights and ages;
2) a minimum of three valid measurements above bh per tree; and
3) a minimum tree bh age of 50 years.

These three constraints screened out of the study up to $65 \%$ of the trees (Table 1). Another factor in the initial screening was that aspen data showed much greater variation than conifer data; therefore, a more rigorous screening criterion had to be devised for aspen. Only trees that reached bh by age five were retained in the study (Table 1), because this species has very fast early growth.

In the next step, individual tree data were plotted and screened for suppressed early growth or top damage (Table 1).

Plots of all individual trees were screened for: 1) a minimum SI (which was calculated from average SI minus three standard deviations); 2) a maximum age at which the tree had to reach the first section measurement; and 3 ) an early growth suppression (Table 1).

The decadal values of height growth, (i.e., heights at age $10,20,30$, etc. years), were interpolated from the data that passed all of the selection criteria (Table 2). These values were then used in all further analysis.

Table 3 presents distribution of screened data for the new height-growth SI curves by source and species.

Table 3. Screened data sources by species

| Source | Lodgepole <br> pine | White <br> spruce | Black <br> spruce | Trembling <br> aspen |
| :--- | :---: | :---: | :---: | :---: |
| AFS | 1110 | 698 | 282 | 202 |
| For. Can. | $-^{\text {a }}$ | - | - | 74 |
| Weldwood | 53 | - | - | - |

[^0]
## DATA ANALYSIS

Equation 2 was chosen for height-growth SI modelling because it has flexibility similar to Monserud's (1984) modified logistic equation currently used by AFS for these same species (Alberta Department of Energy and Natural Resources 1985, 1988), but it solves the problem of compatibility of height versus SI prediction and is simple to use.

Fitting eq. 2 was done using least-square nonlinear regressions performed on the decadal height values. Predicted height was computed as a function of bh prediction age, and another height and its age. The other height and age were always the preceding decadal values of height and its age. Only nonoverlapping decadal periods were fitted, following the procedure described by Borders et al. (1984) for SI-free models derived through the algebraic difference approach. All regression analyses were initially performed using customized SIMPLEX software, and they were then rerun using SAS to obtain additional statistics.

Table 4 presents regression coefficients, their standard errors (SE), and $t$-statistics, as well as standard errors of height predictions (SE of Ht ),
residual variation coefficients

$$
\left(\mathrm{RVC}=\frac{\mathrm{SE}}{\text { Mean } \mathrm{Ht}}\right)
$$

and $R$-squared for each species.

## Height-Growth Curve Comparisons

New height-growth curves were generated for lodgepole pine, white spruce, black spruce, and trembling aspen (Fig. 1) using eq. 2. To provide comparisons, Figure 1 also includes point estimates, in 10-year steps, of height-growth curves used by the AFS at this time (Alberta Department of Energy and Natural Resources 1985).

## Interactive Computer Program

The interactive program shown as Figure 2 is an example of the model's application in the computation of lodgepole pine SI height growth. The same program can be used for other species by changing coefficients $a$ and $b$ in the PARAMETER statement.

Table 4. Regression statistics of the new height-growth $\mathrm{SI}^{\text {a }}$ model

| Species | Coefficient | Estimate | $\mathrm{SE}^{\mathrm{b}}$ | $t$-statistic | $\mathrm{SE} \mathrm{of} \mathrm{Ht}^{\mathrm{c}}$ | $\mathrm{RVC}^{\mathrm{d}}$ <br> $(\%)$ | $R^{2}$ |
| :--- | :---: | ---: | :---: | :---: | :---: | :---: | :---: |
| Lodgepole pine | $\alpha$ | 0.20372424 | 0.0058002482 | 35.123 | 0.84 | 6.3 | 0.970 |
|  | $\beta$ | 97.37473618 | 1.2735295123 | 76.461 |  |  |  |
| White spruce | $\alpha$ | 0.3235139 | 0.0069955077 | 46.246 | 1.21 | 8.1 | 0.965 |
|  | $\beta$ | 260.9162652 | 4.0621902401 | 64.230 |  |  |  |
| Black spruce | $\alpha$ | 0.1992266 | 0.0121355265 | 16.417 | 0.66 | 7.4 | 0.969 |
|  | $\beta$ | 114.8730018 | 2.5131359871 | 45.709 |  |  |  |
| Trembling aspen | $\alpha$ | 0.2644606 | 0.0138721799 | 19.064 | 0.69 | 4.9 | 0.977 |
|  | $\beta$ | 117.3695371 | 3.2205179969 | 36.444 |  |  |  |

[^1]

Figure 1. New height-growth curves generated by the VASI height model (solid lines) and height curves currently used by the AFS (symbols) ${ }^{\text {a }}$ for minimum, average, and maximum SIs of the data available for the major tree species in Alberta.

[^2]```
        DOUBLE PRECISION t,a,b,d,z,j,hx,x1,hxRoot,Ht(15)
        CHARACTER*1 Y
        PARAMETER ( }\textrm{a}=0.20372424DO,\textrm{b}=97.37473618DO
        WRITE(*,'(//,A,A)')' THIS PROGRAM COMPUTES HEIGHTS ',
        & 'OF LODGEPOLE PINE IN ALBERTA AS'
        WRITE(*,'(A,A,/)') ' A FUNCTION OF THE PREDICTION AGE ',
        & 'AND ANY OTHER AGE AND HEIGHT.'
        GOTO 10
    5 WRITE(*,'(//,A)') ' Please REenter your input...'
10 WRITE(*,'(/,$,A)') ' Please enter AGE OF PREDICTION:
    READ(*,*) t
    WRITE(*,'($,A)') ' Enter a KNOWN HEIGHT value: '
    READ(*,*) hx
    WRITE(*,'($,A)') ' Enter the KNOWN HEIGHT''S AGE:
    READ(*,*) x1
    z = 80*b
    j = -1-a
    d = 20*b*5d1**j
    hxRoot = hx-1.3DO + DSQRT( (hx-1.3D0-d)**2 + z*(hx-1.3DO)*x1**j )
    pred = ( hxRoot + d ) / ( 2 + z*t**j/(hxRoot-d) ) + 1.3D0
    DO 20 I = 1, 15
    Ht(I) = ( hxRoot + d ) / ( 2 + z*(I*10)**j/(hxRoot-d) ) + 1.3D0
    IF ( Ht(5) .GT. 25.3DO ) THEN
        WRITE(*,'(//,A)') ' Sorry, pine does not grow that high!'
        hxRoot = 24DO + DSQRT( (24DO-d)**2 + z*24D0*5D1**j)
        pred = ( hxRoot + d ) / ( 2 + z*x1**j/(hxRoot-d)) + 1.3D0
        WRITE(*,'(/,A,F5.2)') ' MAXIMUM HEIGHT FOR SPECIFIED AGE IS: ',
    &
                pred
            GOTO 5
    ELSE IF ( Ht(5) .LT. 5DO ) THEN
        WRITE(*,'(//,A)') ' Sorry, this tree is too short!'
        hxRoot = 3.7DO + DSQRT( (3.7DO-d)**2 + z*3.7DO*5D1**j )
        pred = ( hxRoot + d ) / ( 2 + z*x1**j/(hxRoot-d) ) + 1.3D0
        WRITE(*,'(/,A,F5.2,A,/)')' AT THE SPECIFIED AGE PINE IS MIN.',
& pred, 'm TALL!'
        GOTO 5
    END IF
    WRITE(*,'(//,A,I4,1X,A,F5.2,//)') 'Height for age:',INT(t),
& ' is: ', pred
    WRITE(*,'(A,F5.2,//)') 'SI of the subject tree is: ', Ht(5)
    WRITE(*,*), DECADAL HEIGHTS OF THE SUBJECT TREE'
    WRITE(*,'(A,15(I4,1X))') 'Age', (I, I= 10, 150,10)
    WRITE(*,'(A,15(F4.1,1X))') 'Ht. ', (Ht(I),I=1,15)
    WRITE(*,'(//,A,//)') 'Another height? (Y/n)'
    READ(*,'(A1)') Y
    IF ( Y .NE. 'N' .AND. Y .NE. 'n' ) GOTO 10
    WRITE(*,'(A,A1,A)') 'The request was: ', Y, '... Good bye!'
    STOP
    END
```

Figure 2. Application of the height-growth SI model in an interactive computer program (e.g., lodgepole pine).

## SUMMARY

The major points of this study are as follows:

1) These new curves are quite similar in shape to the curves currently used by the AFS, with slight differences showing up in the extremes of site index and age.
2) For each species the equation has only two coefficients, so it is basically simpler than other models currently in use.
3) The main advantage of the equation is that it can predict height directly from a known height at any age for each species, without prior estimation of site index.

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[^0]:    ${ }^{\mathrm{a}}$ Not applicable.

[^1]:    ${ }^{\text {a }} \mathrm{SI}=$ site index.
    ${ }^{\mathrm{b}} \mathrm{SE}=$ standard error.
    ${ }^{\mathrm{c}} \mathrm{SE}$ of $\mathrm{Ht}=$ standard error of height prediction.
    ${ }^{\mathrm{d}} \mathrm{RVC}=$ residual variation coefficient.

[^2]:    ${ }^{a}$ Source: Alberta Department of Energy and Natural Resources. 1985. Alberta phase 3 forest inventory: yield tables for unmanaged stands. Rep. 60.

