FOREST MANAGEMENT NOTE

PREDICTING DENSITY-RELATED LODGEPOLE PINE HEIGHT GROWTH IN ALBERTA FOR THINNING APPLICATIONS

For most tree species, height growth of dominant and codominant trees is unaffected by stand density over a fairly wide range. Lodgepole pine (*Pinus contorta* Dougl. var. *latifolia* Engelm.) is an exception because even moderate density reduces its height growth and associated timber production, thus diminishing the value of traditional site index (SI) as a productivity measure for this species.

The approach used in this note recognizes that reduction of density-related lodgepole pine height growth is a dynamic process that changes during the life of a stand; it may not occur in a stand at the juvenile stage but become a very strong factor in the young and intermediate-age stand, and then lessen in the old stand. Models using cumulative functions and/or the traditional fixed-age SI models (e.g., Johnstone 1976) are unsuitable for describing this phenomenon. Cieszewski and Bella (1993) developed a new height-growth model based on annual height increments as a function of density. This model can represent changing conditions caused by thinning or natural mortality, and at the same time it could be used with minimum input requirements for thinning prescriptions across a range of stand densities.

This note describes the application of the new density-related height-growth model for lodgepole pine (Cieszewski and Bella 1993) that allows stand density changes to be examined and results predicted in terms of height growth, and provides the basis for spacing and thinning prescriptions.

DENSITY–HEIGHT-GROWTH MODEL

The new density–height-growth model (for details, see Cieszewski and Bella 1993) is based on Czarnowski's (1961) crowding measure or index (CI) of squared top height (TH; average height in metres of the 100 largest trees per hectare) and number of trees per hectare:

[1] $CI = TH^2 \times NT \times 10^{-4}$

where 10^{-4} converts hectares into square metres.

Developed from permanent sample plot (PSP) data collected by the Alberta Forestry Service and Forestry Canada (Cieszewski and Bella 1993), this new model is an extension of the variable-age–site-index (VASI) model developed earlier (Cieszewski and Bella 1989).

The starting point in developing the density–height-growth model was a basic difference equation that describes height growth in annual increments. With such an equation, a height curve can be constructed by simply cumulating annual height increments. Thus the model predicts next year height from current year height, breast height age, and stand crowding (Cieszewski and Bella 1993) and it has the following form:

$$[2] \quad H_{t+1}(h_t,t,CI) = \frac{h_t + \delta + \sqrt{(h_t - \delta)^2 + \zeta h_t/t^\iota}}{2 + \zeta/(t+1)^\iota \left[h_t - \delta + \sqrt{(h_t - \delta)^2 + \zeta h_t/t^\iota}\right]^{-1}}$$

where $H_{t+1}(h_t,t,CI)$ is height growth in the year $t+1$ minus breast height (i.e., 1.3 m); h_t is an observed top height minus 1.3 m at breast height age t; CI is crowding index; $\delta = 20f_2(CI)/t_{SI}^{1+f_1(CI)}$; $\zeta = 80f_2(CI)$; $\iota = 1 + f_1(CI)$; $f_1(CI) = 0.37389 - 0.004254 \times CI$; and $f_2(CI) = 92.960 - 0.35966 \times CI$ (see Cieszewski and Bella 1993).

After making appropriate substitutions and redefining H_{t+1} and h_x as total heights from ground level, the equation reads as follows:

$$[3] \quad H_{t+1}(h_t,t,CI) = \frac{Root + \dfrac{1859.2 - 7.1932 \times CI}{t_{SI}^{1.37389 - 0.004254 \times CI}}}{2 + \dfrac{7436.8 - 28.7728 \times CI}{(t+1)^{1.37389 - 0.004254 \times CI}}\left(Root - \dfrac{1859.2 - 7.1932 \times CI}{t_{SI}^{1.3789 - 0.004254 \times CI}}\right)^{-1}} + 1.3$$

where $Root = \sqrt{\left(h_t - 1.3 - \dfrac{1859.2 - 7.1932 \times CI}{t_{SI}^{1.37389 - 0.004254 \times CI}}\right)^2 + \left[\dfrac{(7436.8 - 28.7728 \times CI)(h_t - 1.3)}{t^{1.37389 - 0.004254 \times CI}}\right]^{-1}} + h_t - 1.3$

Model Predictions

When applied to open stand conditions (i.e., no density effect), eq. [3] gives similar predictions to the height-growth SI curves for lodgepole pine (Cieszewski and Bella 1991) based on stem analysis data (Fig. 1a). As density increases, however, height growth is reduced, and this reduction at extreme densities may exceed 50% (Fig. 1b). The model indicates greatest height-growth reduction in young high-density stands. In older stands, changes in density result in a more modest height-growth reduction. The model also predicts a small height-growth reduction in excessively open stands.

Application of Model to Thinning Evaluation

Three main steps are required to evaluate the effects of thinning: prediction of height growth at current density from current age to rotation; prediction of potential height growth at postthinning densities in yearly iterations for the same period; and evaluation and comparison of the two results.

Height Growth at Current Density

When predicting height growth at current density, CI is calculated first. The resulting CI is used with current height and age to predict any future height growth from eq. [3]. For untreated older stands, CI is assumed to be constant ($NT_1 \times TH_1^2 = NT_2 \times TH_2^2 = ... = NT_n \times TH_n^2$); with such stands, therefore, the same value of CI can be used for a specific time or for the whole growth period. In the latter case, eq. [2] would be used as a cumulative function.

Potential Height Growth After Thinning

Prediction of potential height growth after thinning requires repetitions of three additional steps:

1) calculation of CI with postthinning NT_t and TH_t;

2) prediction of height (TH_{t+1}) for following year from eq. [3]; and

3) use of this height to calculate a new CI from NT_t and TH_{t+1}.

Steps 1 to 3 should be repeated until the end of the desired period, ensuring that the CI used in eq. [2] does not exceed the value before thinning.
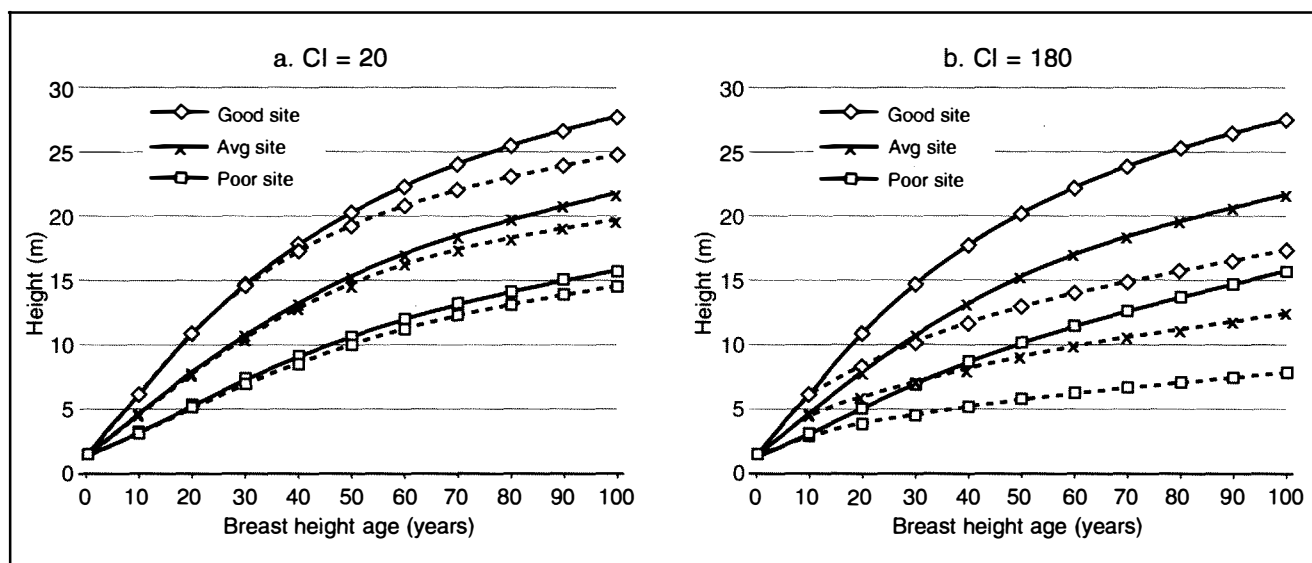
Figure 1. Height-growth curves for lodgepole pine, generated by the new density–height-growth model (broken lines) and the variable-age–site-index model (solid lines; Cieszewski and Bella 1991) for good, average, and poor sites: a) *CI* = 20; **b)** *CI* = 180.

Comparisons

The evaluation of thinning response can be based on the comparison of the above results with respect to local objectives and management criteria.

Examples of Computer Applications

The above procedure can be facilitated by a computer program. For example, the Lotus 1-2-3 program shown in Figure 2 is the basis for the spreadsheet in Table 1.

This program can be used for creating more complex evaluation tables for thinning as required by the user. An example of a further development of this program on the LOTUS 1-2-3 spreadsheet is shown in Table 2, in which the possible outcome of a thinning scenario is illustrated and compared to that of a no-thinning scenario. By examining several such scenarios it is possible to determine the density required to achieve near-maximum height growth. If the LOTUS 1-2-3 or any other spreadsheet is not available, the FORTRAN program shown in

Figure 3 (PROGRAM HtDens) can be used for the same purpose on any computer.[1] This interactive program predicts future top height from present height, breast height age, and number of trees per hectare, and it simulates height-growth response to thinning.

SUMMARY

The approach presented here accounts only for crowding-related mortality. The user should also allow for site-specific competition-independent mortality, based on local information and experience.

The spreadsheets in Tables 1 and 2 can be used by any field forester with a personal computer to estimate potential outcomes of different thinning or spacing scenarios in young stands growing under a range of conditions. This approach can also be used by growth and yield scientists or by silvicultural researchers for modeling lodgepole pine stand development.

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[1] PROGRAM HtDens is available on Internet, Vax@NoFC, and on diskette from C. Cieszewski, Forestry Canada, Northwest Region, Northern Forestry Centre, 5320 – 122 Street, Edmonton, Alberta T6H 3S5.

```
A1: "Age
B1: "TH1
C1: "PresNT
D1: "CI
E1: "alpha
F1: "beta
G1: "gama
H1: "R
A2: 10
B2: (F0) 6
C2: (F0) 50000
D2: (F1) @MIN(C2/10000*B2^2,183)
E2: "<== These are the initial conditions
A3 to H3: \=
A4: (F0) "Thin to=>
C4: (F0) 2000
D4: (F1) +D6
A5: "Height growth as a function of age and density:
A6: +A2
B6: (F2) +B2
C6: (F0) +C4
D6: (F1) +C6/10000*B6^2
E6: 0.004543*D6-1.33121384
F6: 7317.4681216-38.3082064*D6
G6: +F6*50^E6/4
H6: +B6-1.3+@SQRT((B6-1.3-G6)^2+F6*(B6-1.3)*A6^E6)

A7: +A6+1
B7: (F2) (H6+G6)/(2+F6*A7^E6/(H6-G6))+1.3
C7: (F0) @IF(D7D$2,C6,$D$2*10000/B7^2)
D7: (F0) @MIN($D$2,C6/10000*B7^2)
E7: 0.004543*D7-1.33121384
F7: 7317.4681216-38.3082064*D7
G7: +F7*50^E7/4
H7: +B7-1.3+@SQRT((B7-1.3-G7)^2+F7*(B7-1.3)*A7^E7)
  .
  .
  .
```

Figure 2. Basic LOTUS 1-2-3 program.

Table 1. LOTUS 1-2-3 spreadsheet from the basic program

```
-----A----B------C------D------E------F------G-------H--
 1  Age    TH  PresNT  CI   alpha  beta   gamma    R
 2  10      6  50000 180.0  <==These are the initial conditions
 3  =============================================================
 4  Thin to=>   2000   7.2
 5  Height growth as a function of age and density:
 6  10  6.00    2000    7.2 -1.298 7041.6 10.952 45.973
 7  11  6.51    2000      8 -1.292 6993.1 11.124 46.135
 8  12  7.00    2000     10 -1.286 6941.8 11.308 46.315
 9  13  7.49    2000     11 -1.280 6887.8 11.505 46.511
10  14  7.96    2000     13 -1.273 6831.7 11.712 46.721
11  15  8.43    2000     14 -1.266 6773.6 11.930 46.945
12  16  8.88    2000     16 -1.259 6713.8 12.157 47.181
13  17  9.31    2000     17 -1.252 6652.7 12.393 47.428
14  18  9.74    2000     19 -1.244 6590.3 12.637 47.684
15  19 10.16    2000     21 -1.237 6527.0 12.888 47.950
16  20 10.56    2000     22 -1.229 6463.0 13.147 48.223
 .
 .
 .
```

ACKNOWLEDGMENTS

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I.E. Bella
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Table 2. Example of LOTUS 1-2-3 spreadsheet use: a comparison of two lodgepole pine stand densities on good sites

```
Good site, no thinning:

-----A----B------C------D------E------F------G------H-----I------J
 1  Age   TH1  PresNT    CI  alpha   beta   gamma     R    TH2 VASI
 2   10   7.6   30980 180.0 <== These are the initial conditions
 3  ================================================================
 4  Thin  to=>  30980 180.0
 5    1                                                          1.8
 6    3               |    VASI SI 24.00 |                       3.1
 7    6               |    HTDE SI 12.21 |                       4.6
 8    8               ====================                       6.2
 9   10   7.62  30980 180.0 -0.513 421.99 14.153 35.974  7.86   7.6
10   20   9.45  20170   180 -0.513 421.99 14.153 35.974  9.59  13.4
11   30  10.63  15929   180 -0.513 421.99 14.153 35.974 10.73  17.9
12   40  11.51  13586   180 -0.513 421.99 14.153 35.974 11.59  21.3
13   50  12.21  12073   180 -0.513 421.99 14.153 35.974 12.27  24.0
14   60  12.79  11003   180 -0.513 421.99 14.153 35.974 12.84  26.1
15   70  13.28  10200   180 -0.513 421.99 14.153 35.974 13.33  27.9
16   80  13.71   9571   180 -0.513 421.99 14.153 35.974 13.75  29.3
17   90  14.09   9064   180 -0.513 421.99 14.153 35.974 14.13  30.4
18  100  14.43   8643   180 -0.513 421.99 14.153 35.974 14.46  31.4


Good site, thinning to 1200 trees/ha:

-----A----B------C------D------E------F------G------H-----I------J
 1  Age   TH1  PresNT    CI  alpha   beta   gamma     R    TH2 VASI
 2   10   7.6   30980 180.0 <== These are the initial conditions
 3  ================================================================
 4  Thin  to=>   1200   7.0
 5    1                                                          1.8
 6    3               |    VASI SI 24.00 |                .      3.1
 7    6               |    HTDE SI 23.07 |                       4.6
 8    8               ====================                       6.2
 9   10   7.62   1200   7.0 -1.299 7050.3 10.921 53.836  8.28   7.6
10   20  13.35   1200    21 -1.234 6498.0 13.005 56.131 13.83  13.4
11   30  17.50   1200    37 -1.164 5909.4 15.541 58.931 17.85  17.9
12   40  20.62   1200    51 -1.099 5363.5 18.170 61.692 20.89  21.3
13   50  23.07   1200    64 -1.040 4869.8 20.745 64.294 23.29  24.0
14   60  25.09   1200    76 -0.987 4423.2 23.180 66.718 25.27  26.1
15   70  26.79   1200    86 -0.939 4017.3 25.415 68.957 26.95  27.9
16   80  28.26   1200    96 -0.895 3646.6 27.399 70.997 28.39  29.3
17   90  29.54   1200   105 -0.855 3307.0 29.088 72.818 29.66  30.4
18  100  30.66   1200   113 -0.818 2995.5 30.444 74.400 30.77  31.4
```

```
      IMPLICIT NONE
      DOUBLE PRECISION Est_Ht, hx, pred, t, xl, switch, bh_age,
     &    CI, coef_a, coef_b, gen_al, gen_a2, gen_bl, gen_b2, calc_cf,
     &    no_trees, thin_trs, orig_CI
      CHARACTER*1 cont, model, new_mdl
      INTEGER i, count
      parameter (gen_al=0.33121384, gen_a2=-0.004543, gen_bl=91.468352,
     &    gen_b2=-0.478853)

      WRITE (*,'(T25,34A/,2(T25,A/),T25,34A//,A,2(A/))') ('-',i=1,34),
     &  '   LP Height Growth Estimator','     using Ht-Density Model',
     &  ('-',i=1,34)

1     switch = 0
      WRITE (*,'(/A,A)') ' Select (g)rowth model predictions,',
     &      ' or (t)hinning simulation (g/t)?'
      READ (*,'(A1)') model

10    IF ((model.EQ.'t').OR.(model.EQ.'T')) THEN
          count = 0
          switch = 1
      ENDIF
      WRITE(*,'(/A,A)') ' Enter a known BH AGE and HT, eg., 50 15.0'
      READ (*,*) xl, hx

      WRITE(*,'(/A)') ' Enter number of trees per ha at this age:'
      READ (*,*) no_trees
      CI = hx**2 * no_trees / 10000

      IF (switch.EQ.1) THEN
          WRITE(*,'(/A)') ' Enter number of trees per ha after thinning:'
          READ (*,*) thin_trs
          orig_CI = CI
          CI = hx**2 * thin_trs / 10000
      ENDIF

      coef_a = gen_al + gen_a2 * CI
      coef_b = gen_bl + gen_b2 * CI

* check for a realistic calculated SI (should be 5 < ht < 30 m at bh age 50)

      pred = Est_Ht(50.0, coef_a, coef_b, xl, hx, switch)
      IF (pred.LT.5.0) THEN
          WRITE (*,*) '      * WARNING: Trees unusually short-future',
     &              ' ht predictions may be unreliable.'
      ENDIF
      IF (pred.GT.30.0) THEN
          WRITE (*,*) '      * WARNING: Trees unusually tall-future',
     &              ' ht predictions may be unreliable.'
      ENDIF

      IF (switch.EQ.1) THEN
          WRITE(*,'(/A,F8.1,A)') ' After thinning to',
     &              thin_trs, ' trees per ha . . .'

*  do thinning simulation, calculating ht every year and using it as new hx

13        IF (xl.LE.100) THEN
              t = xl + 1
              pred = Est_Ht(t, coef_a, coef_b, xl, hx, switch)
              count = count + 1
              IF (count.EQ.10) THEN
```

Figure 3. Application of a FORTRAN program: PROGRAM HtDens (by C. Cieszewski and M. Vant Erve).

```
              count = 0
              WRITE(*,15) 'At bh age',t,', ht will be',pred,' m.'
            ENDIF
            hx = pred
            x1 = t
            CI = hx**2 * thin_trs / 10000
            IF (CI.GT.orig_CI) CI = orig_CI
            coef_a = gen_a1 + gen_a2 * CI
            coef_b = gen_b1 + gen_b2 * CI
            GOTO 13
          ENDIF
        ELSE

*  predict heights every 10 years

          t = x1 + 10.0
14        IF (t.LE.100) THEN
            pred = Est_Ht(t, coef_a, coef_b, x1, hx, switch)
            WRITE(*,15) 'At bh age',t,', ht will be',pred,' m.'
15          FORMAT (1X, A, F6.1, A, F8.4, A)
            t = t + 10.0
            GOTO 14
          ENDIF
        ENDIF

        WRITE (*,'(/A)') ' Again (y/n)?'
        READ (*,'(A1)') cont
        IF ((cont.NE.'N').AND.(cont.NE.'n')) THEN
          WRITE (*,*) 'Choose new model (y/n)?'
          READ (*,'(A1)') new_mdl
          IF ((new_mdl.EQ.'Y').OR.(new_mdl.EQ.'y')) GOTO 1
          GOTO 10
        ENDIF
        WRITE (*,'(/A)') ' Good-bye.'
        END

        DOUBLE PRECISION FUNCTION Est_Ht(t,coef_a,coef_b,x1,hx,switch)

        IMPLICIT NONE

        DOUBLE PRECISION t,coef_a,coef_b,x1,hx,switch,z,j,d,hxRoot
        z = 80 * coef_b
        j = - 1 - coef_a
        d = 20 * coef_b * 5d1**j

        hxRoot=(hx-1.3) + DSQRT(((hx-1.3)-d)**2 + z*(hx-1.3)*x1**j)
        Est_Ht = ( hxRoot + d) / ( 2 + z*t**j/(hxRoot-d) ) + 1.3

        RETURN
        END
```

Figure 3. Concluded.