New stem taper functions for 12 Saskatchewan timber species

J. Gál and I.E. Bella

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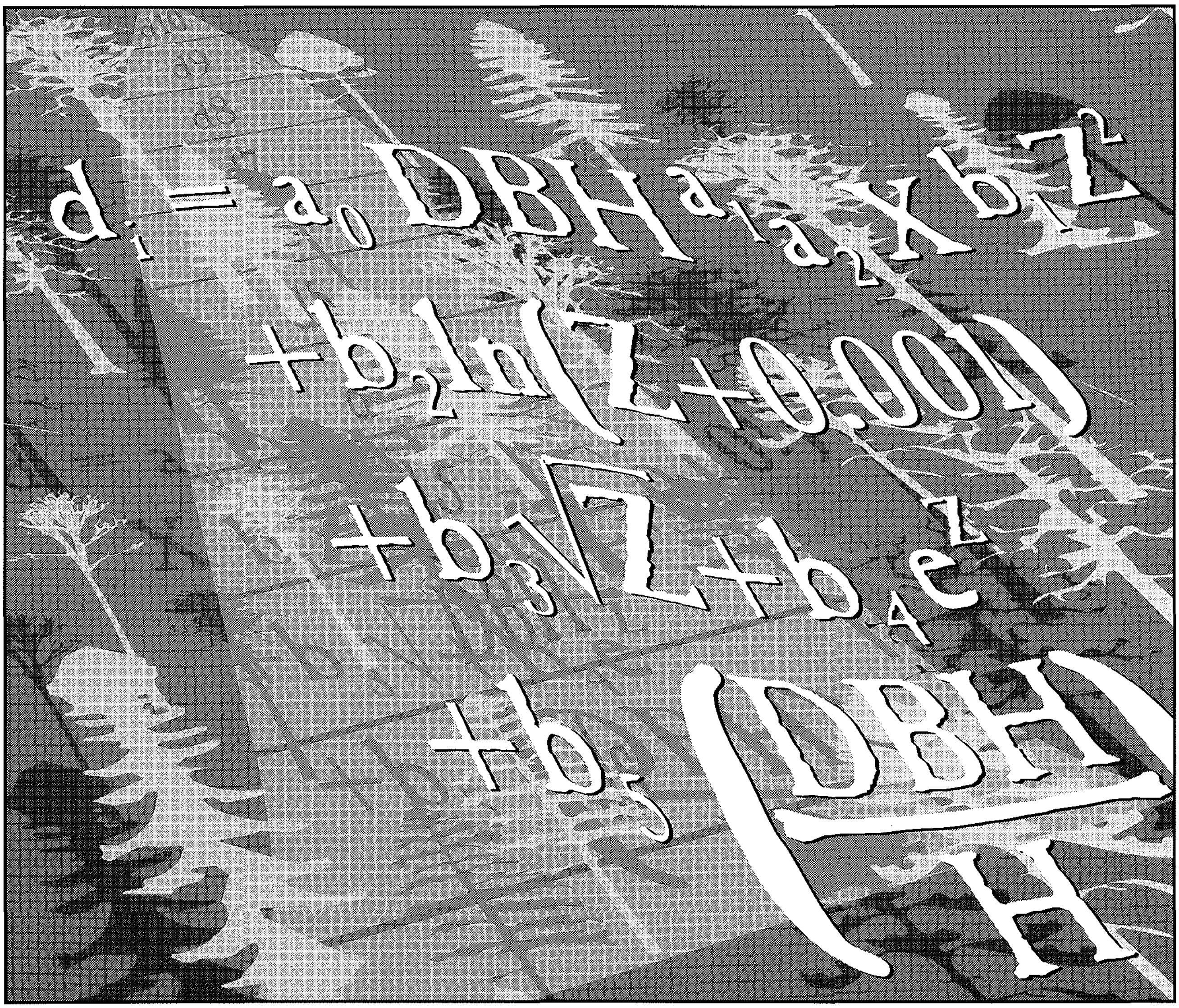

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NEW STEM TAPER FUNCTIONS FOR 12 SASKATCHEWAN TIMBER SPECIES

J. Gál and I.E. Bella

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ABSTRACT

Three taper functions were fitted to tree section data of 12 Saskatchewan timber species to provide a means for estimating individual tree volume in the provincial timber inventory in Saskatchewan, Canada. The fitted functions performances were evaluated in terms of bias, standard error of estimate of volume, and log assortment estimation. The three functions differed in quality of results.

RÉSUMÉ

Trois formules de défilement ont été fixées aux données de sections transversales de 12 espèces d'arbres économiques du Saskatchewan àfin de permettre une estimation du volume des arbres de l'inventaire forestier de la province. Les formules ont été évaluées en fonction des biais, de l'erreur standard de l'estimation du volume et de l'estimation de classification des billots. Les trois formules diffèrent quant à la qualité des résultats obtenus.

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NOTE

The exclusion of certain manufactured products does not necessarily imply disapproval nor does the mention of other products necessarily imply endorsement by Natural Resources Canada.

INTRODUCTION

A timber inventory is expected to provide accurate estimates of timber resources of an area, both in total stem volume and merchantable volume for any desired utilization standard. Taper functions describe stem profiles of trees and are used to estimate total stem volume and merchantable volume to specified utilization standards. They are also used to estimate volumes of certain-sized logs in each stem.

The Saskatchewan Provincial Timber Inventory system (SPTI) in Saskatchewan, Canada, uses a combined-variable volume equation for calculating tree volumes, and taper functions for calculating log volumes. Professor A. Kozak fitted these functions using results published earlier (Demaerschalk and Kozak 1977). In the last few years, officials responsible for this inventory have become concerned that the system might underestimate the volumes of small-sized trees (David Lindenas, Saskatchewan Environment and Resource Management, Prince Albert, Saskatchewan, July 1993. Personal communication.). To address this concern, additional data were collected for small trees and refitting of the old taper function was initiated.

The objective of this current study was to pool and screen all available data; choose new taper functions of potential usefulness for the 12 timber species of Saskatchewan; fit these functions to the data, and; through analysis of their fit, select the function or functions that provide the best fit and most-accurate volume estimates over the range of size classes present. The 12 species used were: white spruce (*Picea glauca* [Moench] Voss), black spruce (*Picea mariana* [Mill.] B.S.P.), jack pine (*Pinus banksiana* Lamb.), lodgepole pine (*Pinus contorta* var. *latifolia* Engelm.), balsam fir (*Abies balsamea* [L.] Mill.), tamarack larch (*Larix laricina* [Du Roi] K. Koch), trembling aspen (*Populus tremuloides* Michx.), black poplar (*Populus balsamifera* L.), white birch (*Betula papyrifera* Marsh.), green ash (*Fraxinus pennsylvanica* var. *lanceolata* [Borkh.] Sarg.), Manitoba maple (*Acer negundo* L.), and white elm (*Ulmus americana* L.).

DATA

Tree section data (from a total of 11 032 trees) were provided on a data tape from Saskatchewan Environment and Resource Management. About 70% of the data were collected prior to 1979; for those trees, location was not recorded. The lack of location information prevented an evaluation of whether significant differences in tree shape occur among regions. In Alberta, Huang (1994) found significant differences in tree forms among ecoregions, and developed an ecoregion-based tree-volume estimating system that improves the accuracy of tree-volume estimates.

When using taper functions for inventory purposes, it is important to ensure that the sample of sectioned trees represents the potential range of heights and diameters of the given species population, and that the frequencies in size classes are similar to the frequencies in the inventory population. Marek Zieba of Saskatchewan Environment and Resource Management (pers. com.) found that the frequencies in the tree section data for the seven main commercial timber species, (white spruce, black spruce, jack pine, balsam fir, trembling aspen, black poplar, and white birch) were approximately proportional to their frequencies by volume in the inventory data.

As the first step in data screening, stem profiles were plotted and visually examined for all the data sets with d_{rel} as abscisa and h_{rel} as ordinate. In this paper, d_{rel} equals diameter inside bark (d) divided by diameter at breast height outside bark (DBH); h_{rel} equals 1 minus the ratio of height from the base to a specified section (h) divided by total stem height (H). The formulae are therefore expressed as:

$$d_{rel} = d/DBH$$

$$h_{rel} = 1 - (h/H)$$

The resulting plots showed high variation in data quality, with old data generally less variable than the new. Many trees in the trembling aspen data had unreasonably large diameters in the tip section, or incorrect section length at the tip. The plots also indicated other possible discrepancies in the data, and suggested possible screening criteria.

The data files, consisting of a header record and an appropriate number of section records, were checked and trees with inconsistencies were deleted based on the criteria (Table 1):

1. Total height given in the header record differed from the running sum of the section record at the tip.

2. Diameter of the last section (tip) was not zero.

3. The section data, other than the tip, had zero diameter.

4. Increasing diameter with increasing height on the stem. If the increase exceeded 5%, both that section and the one below were deleted. A tree with more than two such discrepancies was deleted.

5. Relative diameter too large: $d_{rel} > (0.4 + 1.6h_{rel})$ when $h_{rel} \geq 0.1$.

6. Relative diameter near the top too large:

$$\frac{d_{rel}}{h_{rel}} > 4 \text{ when } h_{rel} < 0.1.$$

7. *DBH/H* ratio too high: *DBH/H* > 4.

8. Relative diameter too small:

$$\frac{d_{rel}}{h_{rel}} < 0.5 \text{ when } h_{rel} > 0.1.$$

9. *DIB* (diameter at breast height inside bark) computed from section data differed more than 20% from the *dbh* in the header record.

10. Height of the first section was greater than 0.75 m.

11. The difference between *dbh* in the header record and that computed from the section data differed more than 10%. In these cases, the value in the header record was replaced with the computed value while retaining the tree.

Next, the stem profiles were replotted using the screened data to check for further outliers. To estimate the relative height of the inflection in relation to total tree height (called parameter '*p*' in Kozak's [1989] equation), dot style plots were prepared from the stem section data for each species. The '*p*' values were: 0.15 for white spruce, black spruce, balsam fir, white birch, and tamarack larch; 0.17 for lodgepole pine, trembling aspen, and Manitoba maple; and 0.19 for jack pine, black poplar, green ash, and white elm.

Actual stem volumes were calculated from section data using the following procedure:

- The stump section, which contains the butt swell[1], was calculated as a cylinder using the upper diameter. This led to a slight underestimation of the total stem volume.

- The volume of the second section was calculated as a neiloid frustum[2].

- The upper sections were treated as cone frustums[3], which became cones for the tips when the top diameter was zero.

Merchantable height for such volumes was calculated using linear interpolation between sections bracketing the specified merchantable diameter limit.

Log volumes were calculated using a 2.6-m log length and a minimum of 8 cm upper diameter inside bark. Log end diameters were calculated using linear interpolation from the two section diameters bracketing the log end. The individual logs were then assigned to three log classes based on upper-end diameter: small (8 cm $\leq d <$ 14 cm), medium (14 cm $\leq d <$ 20 cm), and large ($d \geq$ 20 cm). Their volumes were added up in each class.

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1 Butt swell refers to the swelling in diameter of larger trees near the base, usually below 1.3 m in height.

2 Neiloid frustum, or conic section, essentially refers to a log shape. Neiloid means that there is a narrowing of diameter in the midsection of the log. This is in contrast to a paraboloid frustum, in which the diameter increases in the midsection of the log.

3 Cone frustum refers to a log shape that decreases in diameter in a straight line taper from the large end to the small end of the log.

Table 1. Data summaries and screening results by species and criterion used

Criterion	White spruce	Black spruce	Jack pine	Balsam fir	Tamarack larch	Lodgepole pine	Trembling aspen	Balsam poplar	White birch	Green ash	Manitoba maple	White elm
HTOT ≠ *h* at tip section	1	–[a]	3	1	–	–	11	–	–	–	–	–
d of last section ≠ 0	–	–	–	–	–	–	–	–	–	–	–	–
Section with 0 diameter	–	–	–	–	–	–	–	–	–	–	–	–
Increasing diameter	–	–	–	–	–	–	2	12	1	–	2	2
Relative *d* too large	–	4	2	–	–	1	18	6	2	–	2	1
Relative *d* too large near top	34	9	38	8	6	15	133	35	12	2	6	4
DBH/HTOT ratio too large	–	–	–	–	–	–	2	1	–	–	1	2
Relative *d* too small	1	1	–	–	–	–	2	13	3	–	–	7
DBH-DIB discrepancy	–	–	1	–	–	–	1	10	–	–	–	1
First section too high	–	2	10	–	–	–	17	6	3	–	2	–
Number of stems in sample	1603	2405	1681	396	379	117	2413	810	605	137	228	258
Number of stems used	1567	2389	1627	387	373	101	2227	727	584	135	215	241
Deleted number	36	16	54	9	6	16	186	83	21	2	13	17
Deleted percent	2.2	0.7	3.2	2.3	1.6	13.7	7.7	10.2	3.5	1.5	5.7	6.6
Corrected *DBH* values	8	2	10	4	5	1	579	110	41	0	2	9

Note: *HTOT* = total tree height; *h* = height; *d* = diameter inside bark; *DBH* = diameter at breast height; and *DIB* = diameter at breast height inside bark.

[a] Dash indicates no trees in this category for the species.

The taper function currently used in the SPTI was developed by Demaerschalk and Kozak (1977) and is referred to as K1 in this study. The parameters of this function had to be recalibrated with the additional data collected since its first calibration.

In addition, a literature review suggested two other recently developed taper functions with potential usefulness. The two functions are:

K2 by Kozak (1988), also called the variable exponent model; and

Hilt presented by Hilt (1980) as a modification of a function proposed by Bruce et al. (1968).

Using screened data for each species, the above taper functions were fitted with appropriate nonlinear algorithms. Although requiring considerably more computing power and more complex analysis, nonlinear methods provide better fit and a more realistic residual analysis than the various linear transformations used for fitting these functions in the past (Huang 1994).

K1 Function

This function was included in the analysis because it is currently used in the SPTI for volume estimation. The function describes the stem form with two functions: one for the butt, and one for the top section. The two functions merge at the inflection point of the stem profile. The parameters are conditioned so that at inflection point, the diameters predicted by either function are equal.

The form of the upper stem function is:

$$d = \left[\left(\frac{h/H}{RH}\right)^{b_1} b_2^{\left(1-\frac{h/H}{RH}\right)}\right] DI \tag{1}$$

where: $RH = 1 - \frac{H}{HI}$

and DI is the diameter inside bark of the stem profile at the inflection point; HI is height of the inflection point from the base; and b_1, b_2 are parameters.

The form of the butt function is:

$$d = \left[b_3 - (b_3 - 1)\left(\frac{1-h/H}{RHI}\right)^{b_4}\right] DI \tag{2}$$

where: $RHI = 1 - RH$

and b_3, b_4 are parameters.

These values also had to be estimated from *DBH* because the functions use diameter inside bark at the inflection point. Demaerschalk and Kozak (1977) suggest using two functions. First, estimate diameter inside bark at breast height, and from this value use another function to estimate diameter inside bark at the inflection point. These functions are:

$$DIB = a_1 + a_2 DBH + a_3 DBH^2 \tag{3}$$

$$DI = a_4 + a_5 DIB + a_6 DIB^2 \tag{4}$$

where: a values are parameters.

The coefficients for the regressions *DBH* over *DIB*, and *DIB* over *DI*, were calculated using multiple regression techniques. In estimating *DI*, the estimated value instead of the real value of *DIB* was used because, in application of the functions for estimating *DI* again, the estimated value of *DIB* will be used. These values were calculated from section data using linear interpolation because the original data contained no *DIB* and *DI* values. The estimated parameters and R^2 (coefficient of determination) statistics for Equations (3) and (4) are in Tables 2 and 3, respectively.

In some instances, coefficients a_1, a_3, a_4, a_6 were nonsignificant; nevertheless, they were also included for consistency.

The coefficients b_1 and b_2 of the K1 function were estimated using a nonlinear parameter estimation procedure. This function describes the tree stem only above the inflection point, so the lower sections were deleted and regressions were fitted on the reduced data set. The fit statistics for Equation (1), therefore, represent only this upper part of the tree (Table 4).

Parameter b_3 can be calculated with the following equation:

Table 2. **Parameter estimates and R^2 statistics for Equation (3)**

Species	Parameter a_1	a_2	a_3	N[a]	R^2
White spruce	−0.393221	0.950448	0.000262	1567	0.9878
Black spruce	−0.172585	0.938378	0.000396	2389	0.9964
Jack pine	−0.551719	0.955305	−0.000078	1627	0.9921
Balsam fir	−0.374458	0.970875	−0.000367	387	0.9987
Tamarack larch	−0.499043	0.977679	−0.000167	373	0.9977
Lodgepole pine	0.301728	0.914611	0.000805	101	0.9978
Trembling aspen	−0.457064	0.966300	−0.000709	2227	0.9911
Black poplar	−0.052826	0.904434	−0.000119	727	0.9877
White birch	−0.924184	1.025530	−0.001684	584	0.9934
Green ash	0.305634	0.856641	0.000935	135	0.9922
Manitoba maple	−0.064417	0.888189	0.000814	215	0.9932
White elm	0.001498	0.861505	0.000743	241	0.9906

[a] N = number of observations.

Table 3. **Parameter estimates and R^2 statistics for Equation (4)**

Species	Parameter a_1	a_2	a_3	N[a]	R^2
White spruce	1.151978	0.894175	−0.001173	1567	0.9878
Black spruce	0.295805	0.973551	−0.002627	2389	0.9904
Jack pine	0.273784	0.927046	−0.001413	1627	0.9733
Balsam fir	0.622374	0.939691	−0.001417	387	0.9928
Tamarack larch	0.336730	0.993247	−0.003947	373	0.9904
Lodgepole pine	−0.177269	0.994172	−0.002008	101	0.9836
Trembling aspen	0.369581	0.945650	−0.002546	2227	0.9758
Black poplar	1.197019	0.839997	−0.001101	727	0.9743
White birch	0.972290	0.890632	−0.000858	584	0.9749
Green ash	−0.177666	1.009932	−0.005035	135	0.9675
Manitoba maple	0.435907	0.934107	−0.001428	215	0.9685
White elm	0.355853	0.894385	−0.001483	241	0.9725

[a] N = number of observations.

Table 4. Parameter estimates and residual sum of squares (RSS) for Equation (1)

Species	Parameter a_1	a_2	N[a]	RSS
White spruce	0.964888	1.384107	11 863	3.46
Black spruce	0.775441	1.248411	10 224	0.68
Jack pine	0.801665	1.280415	7 926	2.32
Balsam fir	0.949868	1.375446	2 644	1.62
Tamarack larch	0.977913	1.272527	1 575	1.12
Lodgepole pine	0.963767	1.690012	1 471	3.35
Trembling aspen	0.991947	1.794499	10 499	3.88
Black poplar	1.101415	1.915244	4 322	7.14
White birch	1.208500	1.990195	3 205	5.58
Green ash	0.742181	1.179841	422	1.94
Manitoba maple	1.208500	1.990195	998	3.48
White elm	1.197396	1.649205	1 034	5.08

[a] N = number of observations.

$$b_3 = \frac{\left(\frac{DIB}{DI}\right) - \left(\frac{1-\frac{H-1.3}{H}}{RHI}\right)^{b_4}}{1-\left(\frac{1-\frac{H-1.3}{H}}{RHI}\right)^{b_4}} \quad (5)$$

The coefficient b_4 is conditioned to make the top and bottom functions continuous at the inflection point. The values of b_4 and, subsequently, b_3 and d, are determined for each tree by an iterative process; therefore no fit statistics are presented for Equation (2).

K2 Function

This function describes the whole stem with a varying exponent of the X term in Equation (6). The exponent includes the relative height from the ground, *DBH*, and *H* as independent variables. The form of the function is:

$$d = a_0 DBH^{a_1} a_2^{DBH} X^{b_1 Z^2 + b_2 \ln(Z+0.001) + b_3\sqrt{Z} + b_4 e^{Z} + b_5\left(\frac{DBH}{H}\right)} \quad (6)$$

where:

$Z = h / H$

$X = \frac{1-\sqrt{h/H}}{1-\sqrt{p}}$

p = relative height of inflection point from the ground

The coefficients for the K2 taper function were also fitted using nonlinear regression procedures. To obtain reasonably good initial parameter estimates, the model first was linearized using logarithmic transformation, and the coefficients of the transformed model were estimated with multiple linear regression techniques. With the initial parameters obtained from the transformed model, the parameters and fit statistics were then recalculated using nonlinear regression procedures (Table 5). These realistic initial parameter values also likely ensured convergence to global rather than local minimum of residual sum of squares.

Hilt Function

The original idea for this function was presented by Bruce et al. (1968), who suggested describing the d^2/DBH^2 ratio by the following function:

Table 5. Parameter estimates and residual sum of squares (RSS) for Equation (6)

Species	Parameter								N[a]	RSS
	a_0	a_1	a_2	b_1	b_2	b_3	b_4	b_5		
White spruce	1.028894	0.942483	0.999984	2.162679	−0.520780	3.553711	−1.922393	0.154525	15 824	3.99
Black spruce	0.933348	0.993014	0.998033	1.634033	−0.377332	2.839266	−1.523830	0.224967	15 158	0.59
Jack pine	0.738222	1.080307	0.994994	0.946525	−0.215280	1.529815	−0.769171	0.167008	12 284	1.73
Balsam fir	0.850978	1.037410	0.996623	1.726301	−0.374214	2.879779	−1.535010	0.205714	3 580	1.49
Tamarack larch	0.745588	1.123195	0.990416	1.119887	−0.295350	1.562785	0.756463	0.151162	2 301	1.16
Lodgepole pine	0.965401	0.970357	1.000435	1.507149	−0.263651	1.450598	−0.942382	0.191585	1 824	2.08
Trembling aspen	0.758908	1.053024	0.996492	−0.148669	0.067875	−1.892333	1.009093	0.076416	15 523	3.85
Black poplar	0.767020	1.017008	0.997743	−0.367639	0.056375	−2.018467	1.238476	−0.037714	6 060	7.26
White birch	0.790913	1.062430	0.995247	−0.194606	0.024172	−1.853877	1.153796	−0.040530	4 396	5.64
Green ash	1.000092	0.936587	1.000235	1.467591	−0.483600	2.919951	−1.370717	−0.039554	734	1.79
Manitoba maple	0.766925	1.059614	0.995625	0.205406	−0.034736	−0.720112	0.529446	0.016027	1 545	2.97
White elm	0.784025	1.006287	0.998168	0.393405	−0.097708	−0.616395	0.407093	0.073362	1 625	5.18

[a] N = number of observations.

$$\frac{d^2}{DBH^2} = a_1 x^{3/2} + \sum_{i=1}^{n}\sum_{j=0}^{1}\sum_{k=0}^{1} a_{ijk}(x^{3/2} - x^{b_i})DBH^j H^k \quad (7)$$

where:

$x = \frac{H-h}{H-1.3}$

b_i = vector of possible exponents

i = subscript for possible exponents

j, k = index variables

Bruce et al. (1968) used *DBH* over bark. To condition the function to go through *DIB*, the term $a_1x^{3/2}$ had to be equal to the average d^2/DBH^2 ratio. The elements of vector *b* were determined by trial and error. Bruce et al. (1968) suggested using a two-element vector with elements 3 and 30.

In his version, Hilt (1980) used inside bark diameters. In this case, the constant of the term $x^{3/2}$ is set to 1, forcing the model to go through *DBH*. A separate function is used to estimate *DIB* from *DBH*.

In this analysis, the parameters of the Hilt functions were fitted with multiple linear regression techniques. First, the b_i values in Equation (7) were chosen to be 3 and 30 as suggested above. Then, substituting these b_i values into the function, the *a* parameters could be estimated with multiple linear regression methods. The function form used was:

$$\frac{d^2}{DIB^2} = a_1x^{3/2} + a_2(x^{3/2} - x^3)H + a_3(x^{3/2} - x^3)DIB \cdot H + a_4(x^{3/2} - x^{30})DIB + a_5(x^{3/2} - x^{30})DIB \cdot H \quad (8)$$

Coefficient a_1 was forced equal to 1. Only with the trembling aspen data did a trial fitting the above function with nonlinear techniques give the same parameters as the linear regression. The parameter estimates and fit statistics are in Table 6.

MODEL PERFORMANCE TEST

Taper functions are used to predict total and merchantable volumes and log volumes. Evaluating their accuracy in use for estimating such volumes is best done in terms of those respective volume estimates. Estimated stem and log volumes were therefore calculated and compared to the actual volumes.

The K1 and K2 functions are not integrable, so stem volumes can only be estimated numerically by

Table 6. Parameter estimates and R^2 statistics for Equation (8)

Species	Parameter a_2	a_3	a_4	a_5	N[a]	R^2
White spruce	0.44894E-02[b]	-0.29500E-03	0.27711E-04	-0.84550E-05	15 824	0.9423
Black spruce	0.45465E-01	-0.16784E-02	0.27150E-03	-0.68984E-04	15 158	0.9806
Jack pine	0.40571E-01	-0.95925E-03	0.62940E-03	-0.12800E-03	12 284	0.9738
Balsam fir	0.14669E-01	-0.52808E-03	0.34490E-06	-0.11650E-06	3 580	0.9671
Tamarack larch	-0.54220E-02	-0.75656E-03	0.80641E-04	-0.22620E-04	2 301	0.9596
Lodgepole pine	0.55494E-01	-0.67898E-03	0.21819E-02	-0.26818E-03	1 824	0.9861
Trembling aspen	0.28511E-01	-0.49355E-03	0.39810E-03	-0.98228E-04	15 523	0.9709
Black poplar	0.56280E-04	0.69961E-04	0.48610E-03	-0.10903E-03	6 060	0.9650
White birch	0.51208E-02	0.64262E-04	0.70180E-03	-0.16898E-03	4 396	0.9529
Green ash	-0.86645E-04	0.67701E-03	0.26261E-02	-0.35176E-03	734	0.9782
Manitoba maple	0.74041E-02	0.12284E-03	0.60650E-03	-0.11801E-03	1 545	0.9662
White elm	-0.12131E-01	0.30834E-03	0.14944E-02	-0.25396E-03	1 625	0.9634

[a] N = number of observations.

[b] $0.4489 \bullet 10^{-2}$.

adding up section volumes. In order to ensure compatibility with the estimation system currently used in Saskatchewan, the subroutines provided by Kozak (for the K1 function in 1982, and for the K2 function in 1993) were used to estimate log volumes for those functions. Volumes for the Hilt function, although integrable, were also calculated with the same algorithms as used in the Kozak subroutines. Log volumes estimated by the functions were also added up by the three log size classes.

To compare the three models' performances, statistics were calculated for each species by *DBH* class and by *DBH, H* class within a species.

Average bias of volume prediction is calculated as follows:

$$Bias\ (\%) = 100\ \frac{\sum_{i=1}^{m}(\hat{v}_i - \bar{v})}{m\bar{v}} \qquad (9)$$

where:

$\hat{v}_i$ = predicted volume
$\bar{v}$ = average observed volume
m = number of observations

It should be noted that the bias calculated here is different from the usual bias calculations because it gives positive values for overestimation and negative values for underestimation.

Percentage standard error of estimate (SEE %) is calculated as follows:

$$SEE\ (\%) = \frac{100}{\bar{v}} \sqrt{\frac{\sum_{i=1}^{m}(\hat{v}_i - v_i)^2}{m-k}} \qquad (10)$$

where:

v_i = observed volume
k = number of estimated parameters in the function

DISCUSSION

A comparison of biases and standard errors of estimate between total and merchantable volumes (Table 7) shows similarity within a species; therefore, only total volumes are discussed here. Biases of volume estimates arising from the use of functions K1 and K2 were also similar, and were under 3% in absolute value. In comparison, the Hilt function underestimated total stem volumes for white spruce, black spruce, and balsam fir. For hardwoods, the K2 function gave the best estimates for trembling aspen, while for black spruce and white birch, K1 and Hilt gave similar estimates in terms of bias, while the K2 function slightly underestimated these volumes. For the minor species, except tamarack larch (lodgepole pine, Manitoba maple, and white elm) the Hilt function gave better estimates than K1 and K2. The K2 function estimated green ash with a large positive bias.

To investigate the effect of tree size on volume estimation, the percentage volume biases were summarized by species and *DBH* and *H* classes for both total and merchantable volumes[4].

Generally the lowest biases, in the –2% to +2% range, were observed in *H/DBH* cells near the mean values of height-diameter relationship of each species, coinciding with the highest tree frequencies. Higher biases of up to 60–70%, near the extremities of the data range, arise from some outliers. These biases are of minor consequence.

To investigate the overall effect of *DBH* on volume estimates regardless of heights, summaries were prepared for each species by diameter class. The results showed that *DBH* had little effect on the volume estimation bias. Tables 8–19 present the results for all species.

In another analysis, the biases of estimated log volumes were compared in three log size classes to actual log volumes in the same classes. The results showed that log volume predictions using the K1 and K2 functions were similar (Table 20), and even outperformed the Hilt function except in cases of large logs for green ash and Manitoba maple, and small logs for Manitoba maple and white elm. The

[4] These summaries are available from the senior author upon request.

prediction biases for log volumes and merchantable volumes were also similar, although there was a considerable variation in bias for different log size classes. This variation might be due to bias in estimating small trees or the top of larger trees.

CONCLUSIONS

Generally, the K1 and K2 functions provided better estimates for the main coniferous timber species than the Hilt function. The latter, however, provided somewhat better results for the broadleaved species. The K2 function outperformed the K1 function in estimating small tree volumes. If one model form is preferred for all species, the K2 function should be the choice for volume estimation in Saskatchewan because of its superior overall performance and mathematical elegance.

The results in this study are based on a sample of tree section data. Further analysis would be required to evaluate the effects, in terms of actual inventory volume estimates, of implementing new taper functions into the SPTI system.

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REFERENCES

Bruce, D.; Curtis, R.O.; Vancoevering, C. 1968. Development of a system of taper and volume tables for red alder. For. Sci. 14:339–350.

Demaerschalk, J.P.; Kozak, A. 1977. The whole-bole system: a conditioned dual equation system for precise prediction of tree profiles. Can. J. For. Res. 7:488–497.

Hilt, D. E. 1980. Taper-based system for estimating stem volumes for upland oak. U.S. Dep. Agric., For. Serv., Northeast. For. Exp. Stn., Broomall, Pennsylvania. Res. Pap. NE-458.

Huang, S. 1994. Ecologically based individual tree volume estimation for major Alberta tree species. Individual tree volume estimation procedures for Alberta: methods of formulation and statistical foundations. Alberta Environ. Prot., Land For. Serv., For. Manage. Div., Edmonton, Alberta. Rep. 1. Publ. T/288.

Kozak, A. 1988. A variable-exponent taper equation. Can. J. For. Res. 18:1363–1368.

Table 7. Summary of percentage bias and percentage standard error of estimate (SEE) of total stem and merchantable volumes for the three selected taper functions

Species	Number of stems	Total stem volume						Merchantable volume					
		Bias (%)			SEE (%)			Bias (%)			SEE (%)		
		K1	K2	Hilt	K1	K2	Hilt	K1	K2	Hilt	K1	K2	Hilt
White spruce	1567	–2.7	–2.9	–5.4	14.3	14.2	16.2	–2.9	–2.9	–4.3	14.8	14.8	16.3
Black spruce	2389	0.3	0.3	–3.0	11.9	11.0	12.8	–0.8	–0.5	–3.6	14.0	13.2	15.1
Jack pine	1627	0.3	0.5	0.1	14.9	12.4	13.3	–0.5	0.2	–0.1	15.9	13.7	14.6
Balsam fir	387	–2.4	–1.2	–4.0	11.3	10.6	11.9	–2.3	–1.3	4.2	11.9	11.3	12.2
Tamarack larch	373	–0.2	–1.1	–8.3	13.4	11.7	18.2	–1.4	–1.3	9.2	14.8	14.0	20.5
Lodgepole pine	101	–2.2	2.2	0.5	12.5	9.6	12.5	–2.1	2.3	0.6	12.6	9.9	12.8
Trembling aspen	2227	2.0	0.7	1.4	21.9	21.4	21.6	1.3	0.8	1.5	22.6	22.5	22.6
Balsam poplar	727	1.2	–2.0	1.5	22.3	23.2	22.0	0.0	–2.1	1.3	23.4	24.1	22.8
White birch	584	0.9	–2.0	1.5	26.2	25.6	25.5	–0.5	–2.0	1.4	27.8	27.4	27.3
Green ash	135	2.5	9.1	1.5	14.6	17.7	13.6	0.9	8.9	1.0	16.0	19.2	15.4
Manitoba maple	215	–0.9	–3.7	–1.2	24.5	22.3	21.8	–2.8	–4.0	–1.7	25.0	25.2	24.8
White elm	241	–2.4	–3.7	–1.2	17.9	18.7	17.7	–3.5	–3.7	–1.5	19.9	20.5	19.3

Table 8. **Summary of percentage bias and percentage standard error of estimate (SEE) of total stem and merchantable volumes using three taper functions by *DBH* classes for white spruce**

DBH	Stem quantity (no.)	Total stem volume						Merchantable volume					
		Bias (%)			SEE (%)			Bias (%)			SEE (%)		
		K1	K2	Hilt	K1	K2	Hilt	K1	K2	Hilt	K1	K2	Hilt
4	10	−100.0	48.0	11.2	–[a]	–	–	–	–	–	–	–	–
6	12	−69.2	14.6	−2.8	–	–	–	–	–	–	–	–	–
8	8	2.8	6.2	3.0	–	–	–	−18.8	−62.9	−50.4	–	–	–
10	81	3.4	3.0	−6.7	15.0	11.0	12.5	−8.5	−1.5	−16.8	23.6	22.5	27.5
12	100	−1.8	1.3	−6.8	12.5	10.6	12.6	−6.7	−0.5	−12.9	18.3	16.4	21.7
14	87	−2.6	0.6	−7.4	12.9	9.9	12.6	−6.5	−0.2	−10.8	15.1	12.8	17.3
16	82	0.2	0.6	−5.5	15.7	9.5	11.9	−3.9	0.0	−7.1	14.3	11.7	15.0
18	79	−1.7	−1.9	−5.4	11.1	8.2	10.1	−4.9	−2.5	−6.0	12.1	10.1	12.3
20	92	−0.7	−0.5	−4.2	12.1	10.2	10.6	−3.2	−0.7	−4.0	11.8	11.5	11.9
22	61	−3.6	−2.1	−5.4	11.9	10.3	12.5	−5.4	−2.7	−5.4	13.6	11.6	13.6
24	60	−5.6	−4.2	−7.1	12.9	10.4	13.4	−6.9	−4.5	−6.6	13.9	11.2	13.9
26	71	−3.6	−2.5	−4.8	12.5	11.1	12.6	−4.6	−2.6	−4.0	13.7	12.3	13.5
28	46	−6.7	−4.8	−7.7	13.0	11.6	13.3	−7.4	−4.8	−6.9	13.9	12.2	13.4
30	56	−3.9	−2.5	−4.7	9.3	8.4	9.7	−4.4	−2.5	−3.6	10.0	8.9	9.7
32	64	−5.5	−4.8	−7.5	16.5	9.7	12.2	−6.5	−4.7	−6.5	13.1	10.2	12.1
34	58	−5.6	−3.3	−6.6	10.0	8.1	10.7	−5.8	−3.4	−5.7	10.3	8.4	10.4
36	84	−4.2	−2.2	−5.4	10.6	9.8	11.2	−4.3	−2.1	−4.4	11.0	10.2	11.0
38	73	−3.6	−1.9	−4.9	10.8	10.2	11.4	−3.5	−1.7	−3.6	11.2	10.6	11.4
40	66	−3.2	−2.5	−4.5	11.1	11.0	11.4	−3.3	−2.4	−3.4	11.4	11.3	11.3
42	62	−5.0	−4.0	−6.7	9.5	9.1	10.4	−5.0	−4.0	−5.6	9.7	9.3	10.0
44	61	−2.5	−2.3	−4.3	9.2	9.7	9.6	−2.3	−1.9	−2.9	9.5	9.9	9.4
46	48	−2.3	−2.7	−4.2	11.1	11.1	11.7	−2.4	−2.8	−3.1	11.5	11.4	11.6
48	47	−1.8	−2.6	−4.1	10.3	10.3	11.0	−1.8	−2.5	−2.9	10.6	10.6	10.9
50	43	−1.9	−2.8	−4.6	11.1	11.2	12.0	−1.9	−2.8	−3.4	11.5	11.5	11.9
52	19	−3.3	−4.2	−6.5	11.5	11.8	12.7	−3.3	−4.2	−5.4	12.0	12.3	12.4
54	17	−3.5	−5.4	−6.6	17.1	17.2	17.4	−3.5	−5.4	−5.5	16.9	17.0	16.6
56	17	0.1	−2.0	−3.6	17.9	17.0	17.2	−0.2	−2.3	−2.7	18.5	17.6	17.6
58	14	−3.9	−6.0	−8.2	14.9	14.7	16.9	−4.0	−6.2	−7.2	15.5	15.2	16.7
60	14	−0.3	−4.6	−4.1	13.0	15.6	12.4	−0.1	−4.4	−2.6	14.0	15.9	12.7
62	12	4.0	1.0	−1.2	–	–	–	4.0	0.9	−0.1	–	–	–
64	6	−1.0	−3.1	−7.5	–	–	–	−1.1	−3.3	−6.7	–	–	–
66	6	−1.3	−4.9	−7.1	–	–	–	−1.4	−5.1	−6.1	–	–	–
68	4	−4.4	−6.8	−11.5	–	–	–	−4.5	−7.1	−10.8	–	–	–
70	0	–	–	–	–	–	–	–	–	–	–	–	–
72	1	4.7	6.1	−9.9	–	–	–	4.7	5.8	−9.6	–	–	–
74	1	9.4	7.7	−3.1	–	–	–	9.0	7.2	−2.7	–	–	–
76	4	2.9	−2.1	−5.9	–	–	–	2.2	−3.0	−5.6	–	–	–
78	0	–	–	–	–	–	–	–	–	–	–	–	–
80	1	4.6	−0.4	−5.9	–	–	–	4.4	−0.7	−5.1	–	–	–

[a] Dash indicates SEE not calculated due to diameter class with number of trees <12.

Table 9. Summary of percentage bias and percentage standard error of estimate (SEE) of total stem and merchantable volumes using three taper functions by *DBH* classes for black spruce

		Total stem volume						Merchantable volume					
	Stem quantity	Bias (%)			SEE (%)			Bias (%)			SEE (%)		
DBH	(no.)	K1	K2	Hilt	K1	K2	Hilt	K1	K2	Hilt	K1	K2	Hilt
4	10	–100.0	51.3	32.6	–[a]	–	–	–	–	–	–	–	–
6	11	–20.1	5.2	0.3	–	–	–	–	–	–	–	–	–
8	16	–8.1	0.5	–4.0	39.5	8.9	10.2	–5.0	–19.7	–13.7	54.5	49.9	47.1
10	525	1.7	2.8	–1.3	10.0	9.1	8.7	–2.8	0.2	–3.6	18.4	18.3	18.4
12	527	–0.5	0.7	–2.9	10.3	7.7	8.4	–4.7	–1.1	–5.2	14.1	12.2	13.7
14	341	0.0	1.5	–1.5	10.3	9.0	9.2	–2.0	0.6	–2.5	12.8	12.1	12.6
16	333	–0.5	0.8	–2.0	8.7	7.3	8.0	–1.9	–0.1	–2.7	10.1	9.1	10.1
18	224	–1.4	–0.3	–3.1	8.1	7.8	8.4	–2.0	–0.9	–3.3	9.3	9.2	9.7
20	148	–0.4	0.1	–2.8	9.1	8.1	9.4	–0.8	–0.3	–2.9	10.1	9.2	10.5
22	96	0.2	–0.9	–3.2	7.8	7.6	8.4	–0.1	–1.2	–3.1	8.5	8.5	9.1
24	56	2.7	1.8	–2.1	8.7	8.2	8.4	2.7	1.8	–1.8	9.5	9.0	9.2
26	46	1.6	–1.3	–4.3	10.9	11.5	11.3	1.6	–1.4	–4.0	11.7	12.2	11.9
28	30	2.2	–1.4	–5.0	7.6	7.9	9.3	2.1	–1.5	–4.8	8.4	8.7	9.9
30	16	1.2	–3.7	–7.5	17.2	15.3	18.7	0.8	–4.0	–7.6	17.9	16.1	19.5
32	4	4.5	–0.8	–6.3	–	–	–	4.5	–0.9	–6.2	–	–	–
34	4	0.9	–2.0	–12.9	–	–	–	1.2	–1.7	–12.9	–	–	–
36	1	14.6	5.8	0.1	–	–	–	15.2	6.3	0.8	–	–	–
38	0	–	–	–	–	–	–	–	–	–	–	–	–
40	0	–	–	–	–	–	–	–	–	–	–	–	–
42	1	15.4	6.9	–10.5	–	–	–	17.3	8.6	–9.6	–	–	–

[a] Dash indicates SEE not calculated due to diameter class with number of trees <12.

Table 10. Summary of percentage bias and percentage standard error of estimate (SEE) of total stem and merchantable volumes using three taper functions by *DBH* classes for jack pine

DBH	Stem quantity (no.)	Total stem volume						Merchantable volume					
		Bias (%)			SEE (%)			Bias (%)			SEE (%)		
		K1	K2	Hilt	K1	K2	Hilt	K1	K2	Hilt	K1	K2	Hilt
4	6	–100.0	1.6	–7.0	–[a]	–	–	–	–	–	–	–	–
6	11	–85.4	3.3	1.2	–	–	–	–	–	–	–	–	–
8	15	–25.4	–2.7	–3.7	94.7	10.6	15.1	–21.5	–73.7	–49.4	109.8	147.7	100.0
10	131	1.4	–0.2	–2.3	17.1	12.1	12.9	–15.2	–16.9	–12.8	33.8	32.8	31.5
12	154	3.9	6.0	3.2	14.8	14.8	12.9	–0.2	6.1	2.9	21.4	23.7	21.4
14	186	4.0	6.3	4.0	14.8	14.7	13.1	1.6	7.3	4.3	17.8	19.8	18.0
16	169	3.0	6.6	4.0	14.5	14.7	13.2	1.7	7.6	4.5	16.2	18.3	16.5
18	145	–1.0	4.2	1.2	12.7	13.1	12.3	–2.1	4.0	0.6	14.8	15.0	14.4
20	148	0.7	4.2	2.4	12.4	12.1	12.0	–0.3	3.9	2.0	13.8	13.5	13.6
22	109	–0.4	1.8	0.4	12.6	10.0	10.6	–1.5	1.5	0.1	12.4	11.0	11.7
24	129	–4.6	–2.7	–3.7	12.5	10.4	11.7	–5.3	–3.1	–4.1	13.2	11.1	12.5
26	128	–3.3	–1.5	2.9	9.2	8.7	8.9	–3.5	–1.7	–3.0	9.8	9.2	9.5
28	112	–1.9	–2.3	–2.3	10.1	9.8	10.0	–1.9	–2.2	–2.1	10.5	10.3	10.5
30	71	1.3	–1.3	0.1	10.5	9.9	10.3	1.1	–1.4	0.2	11.1	10.7	11.1
32	25	3.3	–1.0	1.2	12.4	10.5	11.5	2.7	–1.4	1.0	12.2	10.8	11.8
34	17	4.4	–2.6	1.6	13.8	11.4	11.8	4.1	–2.7	1.8	14.2	12.1	12.6
36	16	6.3	–0.5	2.3	14.8	10.3	11.6	6.5	–0.3	2.7	15.3	11.1	12.4
38	11	14.0	1.0	9.5	–	–	–	13.8	1.4	10.2	–	–	–
40	3	12.9	–0.6	7.3	–	–	–	13.0	–0.1	8.2	–	–	–
42	3	14.9	0.6	8.3	–	–	–	14.5	0.4	8.4	–	–	–
44	2	27.1	14.3	16.3	–	–	–	27.5	14.5	16.4	–	–	–
46	0	–	–	–	–	–	–	–	–	–	–	–	–
48	1	1.4	–13.3	–6.8	–	–	–	0.7	–13.9	–7.4	–	–	–
50	0	–	–	–	–	–	–	–	–	–	–	–	–
52	0	–	–	–	–	–	–	–	–	–	–	–	–
54	1	24.3	1.6	11.0	–	–	–	24.7	1.8	11.3	–	–	–

[a] Dash indicates SEE not calculated due to diameter class with number of trees <12.

Table 11. Summary of percentage bias and percentage standard error of estimate (SEE) of total stem and merchantable volumes using three taper functions by *DBH* classes for balsam fir

	Stem	Total stem volume						Merchantable volume					
	quantity	Bias (%)			SEE (%)			Bias (%)			SEE (%)		
DBH	(no.)	K1	K2	Hilt	K1	K2	Hilt	K1	K2	Hilt	K1	K2	Hilt
4	10	–100.0	36.4	21.5	–[a]	–	–	–	–	–	–	–	–
6	11	–24.7	7.2	3.3	–	–	–	–	–	–	–	–	–
8	11	–4.9	–1.9	–5.1	–	–	–	–	–	–	–	–	–
10	19	–3.2	–2.9	–5.7	12.4	9.3	12.3	–10.2	–11.9	–15.3	24.9	23.0	27.6
12	31	–1.0	3.3	–2.9	12.7	12.5	12.2	–4.3	2.2	–7.8	19.4	18.2	20.3
14	21	–2.3	0.0	–4.2	14.8	13.0	14.2	–4.2	–0.1	–6.1	20.3	18.3	20.1
16	26	–3.8	1.8	–5.0	11.9	9.0	12.1	–4.9	1.7	–6.4	14.9	11.3	15.5
18	23	–0.9	3.7	–2.2	13.0	11.1	12.2	–1.1	4.5	–2.0	13.9	12.3	13.8
20	21	–4.7	–1.0	–5.6	12.7	8.0	12.3	–5.7	–1.7	–6.2	14.0	9.1	13.8
22	18	–7.2	–3.2	–8.1	14.5	9.4	14.0	–7.4	–3.2	–7.9	15.7	10.7	15.1
24	27	–5.3	–1.6	–6.1	10.3	7.3	10.4	–5.3	–1.7	–5.8	11.1	8.3	11.1
26	25	–4.8	–0.4	–5.9	8.0	5.0	8.7	–4.6	–0.3	–5.5	8.4	5.6	8.9
28	29	–5.2	–3.1	–6.4	9.4	7.0	10.0	–5.3	–3.3	–6.0	9.9	7.6	10.2
30	21	–6.9	–4.6	–8.5	11.9	9.8	12.5	–7.1	–5.0	–8.2	12.2	10.2	12.4
32	24	–3.7	–0.7	–5.7	8.6	7.1	9.6	–3.5	–0.8	–5.2	8.6	7.3	9.3
34	20	–2.5	–0.6	–4.9	11.2	10.7	11.7	–2.1	–0.5	–4.1	11.6	11.2	11.6
36	16	1.6	1.3	–1.2	11.6	11.7	10.2	1.9	1.5	–0.2	12.3	12.2	10.6
38	11	0.6	–0.7	–2.5	–	–	–	0.7	–0.8	–1.7	–	–	–
40	8	4.0	1.0	0.3	–	–	–	4.8	1.7	2.0	–	–	–
42	5	–0.5	–2.8	–4.6	–	–	–	–0.4	–2.8	–3.7	–	–	–
44	7	0.6	–5.4	–3.5	–	–	–	–0.1	–6.2	–3.1	–	–	–
46	1	–0.4	–3.0	–6.7	–	–	–	0.4	–2.3	–5.4	–	–	–
48	1	11.4	4.2	5.4	–	–	–	11.0	3.6	6.0	–	–	–
50	0	–	–	–	–	–	–	–	–	–	–	–	–
52	1	7.3	0.6	–0.4	–	–	–	8.5	1.5	1.5	–	–	–

[a] Dash indicates SEE not calculated due to diameter class with number of trees <12.

Table 12. Summary of percentage bias and percentage standard error of estimate (SEE) of total stem and merchantable volumes using three taper functions by *DBH* classes for tamarack larch

	Stem	Total stem volume						Merchantable volume					
	quantity	Bias (%)			SEE (%)			Bias (%)			SEE (%)		
DBH	(no.)	K1	K2	Hilt	K1	K2	Hilt	K1	K2	Hilt	K1	K2	Hilt
2	1	−100.0	36.9	21.1	–[a]	–	–	–	–	–	–	–	–
4	9	−24.6	5.0	0.0	–	–	–	–	–	–	–	–	–
6	11	−25.8	−1.2	−5.0	–	–	–	–	–	–	–	–	–
8	9	9.9	−3.0	−7.3	–	–	–	73.6	−38.0	−47.8	–	–	–
10	59	−4.6	−3.7	−6.3	11.5	10.8	12.2	−10.1	−14.0	−15.2	20.6	23.0	24.5
12	58	0.9	−2.2	−5.3	15.0	9.9	12.4	−4.2	−5.6	−10.7	16.8	14.9	19.5
14	47	1.0	−0.2	−5.4	16.4	10.1	14.6	−2.1	−0.8	−8.0	17.6	13.7	19.9
16	35	0.5	−0.1	−6.8	14.2	10.1	14.4	−2.0	−0.2	−8.5	14.2	12.6	18.3
18	32	0.2	−1.8	−7.7	16.5	9.9	17.3	−2.6	−2.2	−8.7	15.9	12.2	21.0
20	32	−1.4	0.2	−9.3	11.4	6.7	17.5	−1.9	0.6	−9.7	11.8	8.1	19.7
22	21	−3.4	−3.0	−11.2	11.3	10.5	17.1	−4.3	−3.1	−11.5	12.4	12.0	18.1
24	23	−1.5	−1.3	−10.5	10.5	11.8	16.6	−1.7	−0.9	−10.2	11.8	13.0	17.1
26	21	0.2	−0.8	−9.4	10.7	10.1	15.1	−0.1	−0.4	−9.0	11.8	11.4	15.8
28	7	2.2	−0.8	−7.0	–	–	–	1.6	−0.6	−6.4	–	–	–
30	5	2.9	−0.3	−9.1	–	–	–	3.4	1.0	−7.9	–	–	–
32	2	2.4	−5.8	−6.9	–	–	–	0.5	−6.3	−6.5	–	–	–
34	1	24.2	13.4	11.8	–	–	–	23.1	13.7	13.0	–	–	–

[a] Dash indicates SEE not calculated due to diameter class with number of trees <12.

Table 13. **Summary of percentage bias and percentage standard error of estimate (SEE) of total stem and merchantable volumes using three taper functions by *DBH* classes for lodgepole pine**

		Total stem volume						Merchantable volume					
	Stem quantity	Bias (%)			SEE (%)			Bias (%)			SEE (%)		
DBH	(no.)	K1	K2	Hilt	K1	K2	Hilt	K1	K2	Hilt	K1	K2	Hilt
8	1	−9.1	−0.5	−9.8	–[a]	–	–	40.6	−79.0	−43.4	–	–	–
10	1	1.8	12.8	3.1	–	–	–	0.3	28.1	10.4	–	–	–
12	3	−11.0	−2.9	−9.1	–	–	–	−15.9	−0.5	−11.4	–	–	–
14	4	−10.7	−1.3	−7.1	–	–	–	−13.7	−0.7	−8.8	–	–	–
16	6	−0.5	5.6	2.3	–	–	–	−2.3	7.0	2.3	–	–	–
18	2	−4.6	7.6	1.5	–	–	–	−3.3	10.4	3.2	–	–	–
20	5	−3.2	4.6	1.6	–	–	–	−3.6	5.1	1.7	–	–	–
22	8	−11.2	−2.3	−5.9	–	–	–	−11.6	−2.3	−6.1	–	–	–
24	5	−10.5	−4.8	−6.1	–	–	–	−11.1	−5.1	−6.4	–	–	–
26	9	−5.8	2.0	−0.6	–	–	–	−5.8	2.2	−0.5	–	–	–
28	4	−6.6	0.0	−1.8	–	–	–	−6.4	0.2	−1.6	–	–	–
30	10	−4.7	2.4	0.0	–	–	–	−4.1	2.9	0.6	–	–	–
32	10	1.2	4.7	5.1	–	–	–	1.4	5.1	5.7	–	–	–
34	6	−1.2	5.0	2.7	–	–	–	−0.8	5.4	3.1	–	–	–
36	5	0.4	6.0	3.7	–	–	–	0.5	5.9	3.7	–	–	–
38	6	−6.5	−3.3	−4.1	–	–	–	−6.6	−3.5	−4.3	–	–	–
40	3	−0.9	2.4	1.0	–	–	–	−0.9	2.3	0.9	–	–	–
42	3	−2.3	1.6	−1.1	–	–	–	−2.1	1.6	−1.0	–	–	–
44	4	2.7	5.0	3.2	–	–	–	2.6	4.8	3.0	–	–	–
46	2	3.9	3.8	4.1	–	–	–	4.1	3.7	4.0	–	–	–
48	2	−3.3	−0.6	4.6	–	–	–	−2.4	0.2	−4.0	–	–	–
50	0	–	–	–	–	–	–	–	–	–	–	–	–
52	1	−5.2	−2.5	−8.7	–	–	–	−5.6	−3.1	−9.6	–	–	–
54	0	–	–	–	–	–	–	–	–	–	–	–	–
56	1	26.7	14.6	27.4	–	–	–	26.2	13.4	27.0	–	–	–

[a] Dash indicates SEE not calculated due to diameter class with number of trees <12.

Table 14. Summary of percentage bias and percentage standard error of estimate (SEE) of total stem and merchantable volumes using three taper functions by *DBH* classes for trembling aspen

		Total stem volume						Merchantable volume					
	Stem quantity	Bias (%)			SEE (%)			Bias (%)			SEE (%)		
DBH	(no.)	K1	K2	Hilt	K1	K2	Hilt	K1	K2	Hilt	K1	K2	Hilt
4	11	–100.0	–14.8	–3.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
6	10	–70.5	–12.3	–10.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
8	33	–25.3	–7.2	–6.6	59.1	19.0	18.7	–50.1	–71.1	–49.8	106.5	122.7	103.2
10	214	8.0	–1.7	0.7	15.6	12.2	12.2	4.2	–25.9	–6.3	28.4	38.6	28.7
12	194	8.9	1.0	2.5	16.5	11.1	11.7	9.0	–0.1	0.9	21.5	18.7	19.4
14	173	6.8	1.5	2.6	13.6	11.0	11.5	6.1	2.0	1.1	16.5	15.0	15.4
16	179	4.2	0.5	1.2	11.1	10.0	10.2	3.1	0.7	0.2	12.9	12.3	12.5
18	157	2.5	–0.1	0.5	11.4	10.9	11.0	1.5	0.0	–0.1	12.9	12.6	12.7
20	147	1.9	0.0	0.4	11.7	11.3	11.4	1.1	0.2	0.2	12.7	12.5	12.6
22	139	1.6	0.6	0.8	10.7	10.3	10.4	1.0	0.8	0.8	11.3	11.0	11.2
24	125	0.4	–0.4	–0.1	12.3	12.4	12.2	–0.3	–0.3	0.0	13.0	13.1	12.9
26	109	2.8	2.2	2.6	12.1	11.8	11.9	2.3	2.4	2.9	12.5	12.4	12.5
28	116	0.9	0.0	0.7	12.2	12.1	12.1	0.2	0.1	1.0	12.6	12.6	12.6
30	108	0.2	–0.2	0.3	11.5	11.5	11.4	–0.5	–0.1	0.5	11.9	11.9	11.8
32	105	–0.5	–1.2	–0.4	13.5	13.3	13.3	–1.3	–1.1	–0.2	13.9	13.8	13.8
34	83	0.2	–0.4	0.4	14.7	14.8	14.5	–0.5	–0.3	0.7	15.3	15.4	15.2
36	58	1.2	0.3	1.4	15.3	15.6	15.0	0.7	0.7	1.9	15.8	16.2	15.6
38	64	1.9	1.6	2.1	16.7	17.0	16.4	1.4	1.9	2.5	17.3	17.7	17.1
40	44	2.0	1.1	2.0	15.1	14.8	14.7	1.3	1.3	2.4	15.4	15.3	15.1
42	33	4.5	3.5	4.4	20.0	19.3	19.3	3.9	4.0	5.0	20.3	19.9	19.8
44	26	4.2	2.6	4.1	19.4	18.5	18.4	3.4	3.0	4.5	19.7	19.1	19.0
46	23	–0.5	–1.9	–0.9	20.5	19.8	19.5	–1.4	–1.7	–0.6	20.9	20.2	19.7
48	15	–7.3	–8.4	–7.8	17.9	18.3	16.1	–8.2	–8.4	–7.8	19.3	18.7	16.5
50	18	–2.7	–4.6	–3.4	17.8	17.4	16.6	–3.6	–4.3	–3.1	17.9	17.2	16.4
52	12	8.4	7.1	7.1	0.0	0.0	0.0	7.5	7.4	7.4	0.0	0.0	0.0
54	15	13.2	11.0	11.5	29.8	26.2	25.1	12.0	11.3	11.7	29.2	27.0	25.6
56	4	–1.7	–3.8	–3.4	0.0	0.0	0.0	–2.3	–3.1	–2.8	0.0	0.0	0.0
58	2	9.6	8.2	6.8	0.0	0.0	0.0	8.2	8.2	6.4	0.0	0.0	0.0
60	6	8.6	5.3	6.1	0.0	0.0	0.0	7.4	5.8	6.3	0.0	0.0	0.0
62	3	25.7	21.5	22.8	0.0	0.0	0.0	24.8	22.6	23.6	0.0	0.0	0.0
64	0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
66	0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
68	0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
70	1	10.7	10.8	1.2	0.0	0.0	0.0	10.1	11.5	1.2	0.0	0.0	0.0

Table 15. Summary of percentage bias and percentage standard error of estimate (SEE) of total stem and merchantable volumes using three taper functions by *DBH* classes for black poplar

	Stem	Total stem volume						Merchantable volume					
	quantity	Bias (%)			SEE (%)			Bias (%)			SEE (%)		
DBH	(no.)	K1	K2	Hilt	K1	K2	Hilt	K1	K2	Hilt	K1	K2	Hilt
4	11	−100.0	−10.3	7.6	–[a]	–	–	–	–	–	–	–	–
6	7	−86.1	−17.1	−3.7	–	–	–	–	–	–	–	–	–
8	7	3.2	−2.5	9.6	–	–	–	−15.1	−51.0	−5.2	–	–	–
10	16	1.7	−14.1	−7.6	19.0	26.2	20.8	6.4	−38.7	−15.6	37.7	63.9	42.4
12	21	18.0	−5.1	5.4	28.6	11.5	12.8	22.4	−11.6	6.6	34.6	23.2	21.7
14	21	9.1	−4.4	0.8	15.0	10.9	10.3	9.7	−7.2	−1.9	19.9	17.8	16.4
16	27	10.0	−1.3	2.5	19.3	14.7	14.3	0.0	−1.9	1.1	22.8	18.8	17.8
18	30	8.2	−0.9	1.6	14.6	10.8	10.8	7.5	−1.3	0.6	15.7	12.8	12.6
20	39	3.5	−4.9	−2.6	11.7	11.5	10.7	1.6	−5.7	−3.8	11.9	13.2	12.5
22	32	6.9	0.9	1.9	14.6	11.7	12.3	6.3	0.8	1.7	15.1	13.0	13.5
24	38	−0.5	−5.3	−4.5	16.9	17.6	17.3	−1.6	5.9	5.0	19.0	19.7	19.4
26	36	2.1	−2.2	−1.4	11.3	11.3	10.4	1.1	−2.5	−1.6	12.0	12.1	11.2
28	38	5.9	1.5	3.0	14.9	13.1	13.4	5.0	1.6	3.3	15.4	14.2	14.6
30	42	1.4	−1.8	−1.3	12.5	12.6	12.1	0.5	−2.0	−1.2	13.3	13.4	12.9
32	52	−0.9	−3.8	−3.0	12.2	12.6	12.4	−1.9	−4.0	−3.0	12.9	13.3	13.0
34	41	−0.2	−3.1	−1.7	15.4	15.7	15.0	−1.2	−3.2	−1.6	16.8	16.9	16.1
36	24	1.2	−1.4	0.1	13.1	12.9	12.1	0.3	−1.4	0.3	13.5	13.3	12.6
38	21	1.1	−0.9	−0.2	13.9	13.4	13.0	0.8	−0.4	0.5	14.2	13.6	13.4
40	17	0.0	−2.9	0.0	15.2	14.9	13.8	−0.8	−2.4	0.7	14.8	14.4	13.3
42	16	5.2	2.9	5.0	18.6	16.5	16.5	4.3	3.0	5.4	19.4	17.9	17.8
44	16	−4.2	−6.4	−4.2	17.7	17.7	16.2	−5.4	−6.4	−4.1	19.0	18.4	16.9
46	13	2.3	0.0	2.5	17.9	16.1	15.4	1.2	0.2	2.7	18.2	16.6	15.8
48	14	0.3	−2.0	1.0	13.3	12.7	11.7	−0.7	−1.8	1.2	14.5	13.6	12.6
50	15	3.7	1.5	4.4	17.9	15.9	16.7	3.0	2.0	4.9	17.7	16.1	17.0
52	13	7.2	5.0	8.0	23.6	20.3	20.0	6.4	5.4	8.3	23.3	20.9	20.6
54	12	4.6	2.6	5.3	–	–	–	3.8	2.9	5.7	–	–	–
56	16	−2.0	−4.1	−1.2	25.1	24.3	22.2	−3.2	−4.1	−1.3	25.8	24.6	22.5
58	11	−2.5	−5.0	−1.0	–	–	–	−3.9	−5.1	−1.2	–	–	–
60	12	0.4	−2.3	1.9	–	–	–	−0.9	−2.2	1.8	–	–	–
62	12	5.5	2.9	6.8	–	–	–	4.2	2.9	6.6	–	–	–
64	8	−2.8	−5.5	−1.2	–	–	–	−4.2	−5.6	−1.5	–	–	–
66	11	9.8	6.5	11.8	–	–	–	8.4	6.8	11.7	–	–	–
68	6	1.6	−1.9	3.8	–	–	–	−0.5	−2.3	3.0	–	–	–
70	3	7.7	4.4	8.3	–	–	–	6.0	4.2	7.7	–	–	–
72	8	−9.3	−13.4	−6.0	–	–	–	−11.9	−14.0	−7.2	–	–	–
74	4	−8.9	−12.4	−6.7	–	–	–	−10.9	−12.8	−7.6	–	–	–
76	5	13.8	9.1	17.0	–	–	–	12.0	9.3	16.6	–	–	–
78	2	−4.6	−9.7	0.3	–	–	–	−6.4	−9.1	0.2	–	–	–
80	0	–	–	–	–	–	–	–	–	–	–	–	–

Table 15. Concluded

DBH	Stem quantity (no.)	Total stem volume						Merchantable volume					
		Bias (%)			SEE (%)			Bias (%)			SEE (%)		
		K1	K2	Hilt	K1	K2	Hilt	K1	K2	Hilt	K1	K2	Hilt
82	5	8.7	4.5	9.5	–	–	–	7.7	4.9	9.3	–	–	–
84	1	–4.6	–8.2	–5.0	–	–	–	–5.5	–8.0	–5.2	–	–	–
86	1	4.2	–1.1	7.9	–	–	–	2.2	–1.2	7.1	–	–	–
88	0	–	–	–	–	–	–	–	–	–	–	–	–
90	0	–	–	–	–	–	–	–	–	–	–	–	–
92	2	–7.5	–13.0	–4.1	–	–	–	–9.7	–13.2	–5.1	–	–	–
94	0	–	–	–	–	–	–	–	–	–	–	–	–
96	1	–20.4	–25.3	–17.9	–	–	–	–22.3	–25.7	–19.1	–	–	–

[a] Dash indicates SEE not calculated due to diameter class with number of trees <12.

Table 16. Summary of percentage bias and percentage standard error of estimate (SEE) of total stem and merchantable volumes using three taper functions by *DBH* classes for white birch

	Stem	Total stem volume						Merchantable volume					
	quantity	Bias (%)			SEE (%)			Bias (%)			SEE (%)		
DBH	(no.)	K1	K2	Hilt	K1	K2	Hilt	K1	K2	Hilt	K1	K2	Hilt
4	10	−77.6	−15.2	−20.5	–[a]	–	–	–	–	–	–	–	–
6	10	−0.2	−22.5	−20.2	–	–	–	–	–	–	–	–	–
8	9	20.9	−10.8	−3.7	–	–	–	8.4	−54.0	−35.7	–	–	–
10	59	2.1	−5.6	−0.3	14.9	14.0	13.7	2.0	−21.0	−9.4	29.0	35.4	30.5
12	63	9.9	−2.7	2.8	24.8	15.8	17.7	7.0	−5.7	−1.2	28.2	25.5	26.9
14	36	7.3	−2.0	1.7	16.4	11.3	11.3	4.3	−2.9	−0.8	15.9	15.5	15.3
16	49	0.8	−4.7	−1.8	13.3	11.5	11.3	−1.6	−5.6	−3.7	14.2	14.7	14.7
18	33	4.5	−3.4	0.1	18.2	15.5	14.1	0.3	−4.2	−1.3	17.5	18.6	17.2
20	34	3.4	−0.9	2.5	16.7	16.3	15.7	2.3	−0.3	2.9	19.1	18.9	18.1
22	27	4.2	1.6	3.5	14.1	12.4	12.5	3.8	2.2	4.2	14.0	13.2	13.3
24	24	3.5	0.4	2.8	20.5	18.4	17.4	1.9	0.4	2.9	20.4	19.6	18.7
26	24	3.9	2.3	4.5	18.1	17.4	17.2	3.3	2.6	5.0	19.8	19.1	18.9
28	22	3.8	−2.1	0.4	30.7	18.9	18.3	−0.2	−2.3	0.4	21.2	20.1	19.4
30	32	1.9	0.7	2.8	21.8	21.9	20.7	1.1	0.8	3.0	23.9	23.8	22.6
32	31	0.2	−0.9	1.2	22.4	22.7	21.0	−0.4	−0.6	1.7	24.1	23.9	22.3
34	31	−5.7	−7.3	−4.1	15.6	16.1	14.2	−6.9	−7.3	4.2	16.9	16.8	14.9
36	21	0.6	−1.2	2.2	20.2	20.7	19.0	−0.7	−1.2	2.2	21.7	21.4	19.8
38	16	3.5	2.5	5.1	23.0	21.6	21.0	3.0	2.8	5.5	24.0	22.6	22.1
40	11	2.8	1.0	4.7	–	–	–	1.8	1.3	4.8	–	–	–
42	42	−7.7	−9.7	−5.3	17.2	18.2	12.1	−9.2	−9.9	−5.7	19.3	18.5	12.7
44	6	4.3	1.5	5.8	–	–	–	3.8	2.7	6.7	–	–	–
46	6	−6.2	−8.3	−3.9	–	–	–	−6.7	−7.6	−3.6	–	–	–
48	6	3.4	0.1	5.2	–	–	–	1.5	0.2	4.7	–	–	–
50	2	10.4	8.1	11.9	–	–	–	10.0	8.7	12.1	–	–	–
52	2	−16.4	−20.1	−13.1	–	–	–	−19.2	−20.7	−14.6	–	–	–
54	2	−17.7	−21.7	−14.4	–	–	–	21.4	−23.0	−16.9	–	–	–
56	2	−6.8	−10.0	−4.6	–	–	–	−7.3	−9.2	−4.4	–	–	–
58	1	24.2	18.1	29.4	–	–	–	21.5	18.5	28.3	–	–	–
60	1	17.0	9.3	22.9	–	–	–	13.7	10.2	21.8	–	–	–
62	0	–	–	–	–	–	–	–	–	–	–	–	–
64	0	–	–	–	–	–	–	–	–	–	–	–	–
66	0	–	–	–	–	–	–	–	–	–	–	–	–
68	0	–	–	–	–	–	–	–	–	–	–	–	–
70	1	63.3	53.6	65.4	–	–	–	63.1	55.9	66.0	–	–	–

[a] Dash indicates SEE not calculated due to diameter class with number of trees <12.

Table 17. Summary of percentage bias and percentage standard error of estimate (SEE) of total stem and merchantable volumes using three taper functions by *DBH* classes for green ash

DBH	Stem quantity (no.)	Total stem volume						Merchantable volume					
		Bias (%)			SEE (%)			Bias (%)			SEE (%)		
		K1	K2	Hilt	K1	K2	Hilt	K1	K2	Hilt	K1	K2	Hilt
8	1	28.4	11.3	5.0	–[a]	–	–	26.1	–16.8	–19.7	–	–	–
10	8	15.8	15.8	5.8	–	–	–	–0.9	8.1	–5.7	–	–	–
12	10	6.7	9.9	0.0	–	–	–	2.6	9.5	–4.2	–	–	–
14	14	8.7	12.5	2.6	18.6	22.3	12.2	7.9	14.2	1.5	24.6	29.4	18.5
16	22	4.3	9.5	–0.4	10.5	15.0	8.4	2.9	9.0	–2.2	12.8	17.0	11.4
18	17	4.4	9.6	0.6	15.8	19.1	13.3	3.2	9.3	–0.4	19.1	21.6	16.9
20	19	7.4	9.2	0.6	22.3	17.7	14.4	3.8	8.7	–0.2	16.5	18.5	15.9
22	10	4.4	11.4	2.3	–	–	–	4.1	11.5	2.3	–	–	–
24	10	3.2	9.1	2.3	–	–	–	2.7	9.9	2.8	–	–	–
26	7	–3.9	3.3	–3.9	–	–	–	–4.9	2.9	–4.2	–	–	–
28	6	1.0	9.4	2.6	–	–	–	–0.3	8.8	2.1	–	–	–
30	4	4.8	14.1	8.1	–	–	–	3.9	14.3	8.3	–	–	–
32	2	–1.8	9.2	1.1	–	–	–	–1.4	9.7	2.1	–	–	–
34	3	–0.6	9.3	5.0	–	–	–	–1.8	9.4	5.1	–	–	–
36	1	–6.0	6.1	–0.8	–	–	–	7.4	4.7	–1.6	–	–	–
38	1	–9.4	1.3	–1.7	–	–	–	1.2	0.6	2.4	–	–	–

[a] Dash indicates SEE not calculated due to diameter class with number of trees <12.

Table 18. Summary of percentage bias and percentage standard error of estimate (SEE) of total stem and merchantable volumes using three taper functions by *DBH* classes for Manitoba maple

DBH	Stem quantity (no.)	Total stem volume						Merchantable volume					
		Bias (%)			SEE (%)			Bias (%)			SEE (%)		
		K1	K2	Hilt	K1	K2	Hilt	K1	K2	Hilt	K1	K2	Hilt
4	4	−100.0	0.8	12.2	–[a]	–	–	–	–	–	–	–	–
6	5	−28.8	−12.3	−4.9	–	–	–	–	–	–	–	–	–
8	6	−40.9	−4.1	0.3	–	–	–	−33.3	−48.5	−27.4	–	–	–
10	16	9.9	0.4	2.2	25.6	18.4	16.2	5.0	−11.2	−7.1	42.7	43.3	37.3
12	19	4.1	−0.3	0.2	21.1	15.2	16.0	1.9	−1.9	−4.8	24.4	22.0	23.4
14	31	4.9	2.2	1.8	19.5	15.8	16.5	4.6	4.2	0.7	23.9	22.3	22.7
16	24	−0.8	−1.4	−2.2	21.5	18.5	20.1	−2.2	−1.4	−4.0	25.8	23.3	25.2
18	21	−6.8	−5.9	−7.6	17.0	14.0	16.1	−8.6	−6.8	−9.4	19.3	16.3	19.1
20	19	−4.9	−5.3	−5.7	18.9	17.1	17.4	−7.1	−6.1	−7.0	21.4	19.3	19.8
22	17	−2.0	−3.6	−2.5	24.8	23.2	22.4	−4.5	−4.3	−3.4	28.4	26.6	25.6
24	12	−7.5	−7.6	−7.2	–	–	–	−8.6	−7.8	−7.3	–	–	–
26	9	1.0	−0.9	1.9	–	–	–	−0.4	−1.0	2.0	–	–	–
28	5	−4.8	−4.7	−3.8	–	–	–	−5.9	−5.3	−4.1	–	–	–
30	9	3.4	−7.0	−2.2	–	–	–	−2.2	−6.4	−1.4	–	–	–
32	8	8.8	2.8	9.5	–	–	–	6.5	3.1	10.0	–	–	–
34	5	0.0	−2.6	2.3	–	–	–	−0.3	−2.2	3.1	–	–	–
36	2	−7.6	−11.6	−4.9	–	–	–	−8.8	−11.9	−4.9	–	–	–
38	3	−0.9	−6.7	2.7	–	–	–	−1.6	−6.3	3.4	–	–	–

[a] Dash indicates SEE not calculated due to diameter class with number of trees <12.

Table 19. Summary of percentage bias and percentage standard error of estimate (SEE) of total stem and merchantable volumes using three taper functions by *DBH* classes for white elm

DBH	Stem quantity (no.)	Total stem volume						Merchantable volume					
		Bias (%)			SEE (%)			Bias (%)			SEE (%)		
		K1	K2	Hilt	K1	K2	Hilt	K1	K2	Hilt	K1	K2	Hilt
4	5	−100.0	−12.6	9.8	–[a]	–	–	–	–	–	–	–	–
6	3	−100.0	−0.2	13.8	–	–	–	–	–	–	–	–	–
8	5	19.1	3.2	8.8	–	–	–	61.1	−48.3	−20.2	–	–	–
10	12	17.5	−0.9	6.4	–	–	–	7.5	−22.9	4.9	–	–	–
12	12	17.6	8.2	12.5	–	–	–	18.4	9.8	17.0	–	–	–
14	17	5.5	3.1	4.5	14.8	14.5	12.4	4.5	5.8	4.7	23.3	25.0	20.1
16	19	−0.8	−2.6	−1.9	13.6	13.5	11.9	−4.4	−2.3	−3.8	18.4	18.2	16.7
18	15	−4.5	−2.7	−4.6	14.5	12.6	13.3	−5.9	−1.9	−5.8	17.5	14.9	16.3
20	14	−8.6	−7.0	−8.1	19.4	16.7	16.3	−10.8	−7.5	−9.8	22.7	18.1	18.7
22	21	−3.1	−3.3	−2.1	16.3	16.4	14.8	−5.3	−3.4	−2.9	19.3	18.6	17.0
24	17	−4.0	−3.2	−3.2	15.5	15.0	13.8	−6.1	−3.8	−4.3	19.0	17.6	16.4
26	14	−1.5	−1.0	−1.1	17.4	19.3	13.7	−2.9	−0.9	−1.4	20.6	21.9	15.6
28	19	−7.4	−7.9	−6.4	21.1	21.0	19.0	−9.1	−8.2	−6.8	23.5	22.6	20.6
30	16	−5.8	−4.2	−5.4	21.5	21.5	18.1	−6.7	−4.6	−5.9	23.1	22.5	18.7
32	8	−1.7	−2.6	−0.2	–	–	–	−2.7	−2.4	−0.1	–	–	–
34	11	−5.1	−5.6	−3.7	–	–	–	−5.8	−5.4	−3.5	–	–	–
36	6	−4.9	−6.6	−2.7	–	–	–	−5.8	−6.6	−2.7	–	–	–
38	6	−4.4	−6.2	−2.3	–	–	–	−6.0	−7.0	−3.1	–	–	–
40	5	1.8	0.3	2.9	–	–	–	1.7	0.8	3.4	–	–	–
42	3	−5.4	−7.1	−3.6	–	–	–	−7.4	−8.5	−5.1	–	–	–
44	4	−1.7	−2.0	−2.3	–	–	–	−0.6	−0.8	−1.2	–	–	–
46	1	−8.8	−21.5	−0.7	–	–	–	−12.0	−21.8	−1.1	–	–	–
48	3	9.8	5.6	12.8	–	–	–	11.6	7.9	15.2	–	–	–
50	0	–	–	–	–	–	–	–	–	–	–	–	–
52	0	–	–	–	–	–	–	–	–	–	–	–	–
54	1	−7.1	−11.5	−5.1	–	–	–	−7.0	−11.0	−4.7	–	–	–
56	1	4.5	−3.0	9.7	–	–	–	2.4	−4.2	8.2	–	–	–
58	2	11.5	6.2	12.2	–	–	–	10.7	5.7	11.3	–	–	–
60	0	–	–	–	–	–	–	–	–	–	–	–	–
62	1	−2.1	−8.5	−0.1	–	–	–	−1.9	−7.9	0.2	–	–	–

[a] Dash indicates SEE not calculated due to diameter class with number of trees <12.

Table 20. Log volume biases in four log size classes estimated by the three functions for each species

Log size class and taper function	White spruce	Black spruce	Jack pine	Balsam fir	Tamarack larch	Lodgepole pine	Trembling aspen	Balsam poplar	White birch	Green ash	Manitoba maple	White elm
Small logs (8 cm ≤ d < 14 cm)												
K1	1.8	−0.7	−1.6	−2.8	−2.4	1.9	0.7	−1.4	4.0	5.1	1.6	1.3
K2	3.0	2.1	2.5	1.9	−1.5	−6.1	0.6	−5.6	0.4	6.2	2.5	1.2
Hilt	6.4	−1.2	0.1	2.8	−3.3	−7.2	1.3	2.5	6.6	1.8	6.7	8.6
Medium logs (14 cm ≤ d = < 20 cm)												
K1	−1.3	−0.7	0.9	−1.5	2.0	−0.9	2.0	9.5	−1.2	9.6	0.3	−3.5
K2	0.6	−1.7	4.7	3.0	5.0	0.8	1.6	7.4	−2.7	4.5	0.5	0.6
Hilt	0.4	−4.3	2.2	2.9	−11.8	0.5	1.3	8.4	4.1	−3.2	−2.1	−2.8
Large logs (d ≥ 20 cm)												
K1	−3.6	−2.1	−0.5	−2.7	−6.3	−2.8	1.2	−1.2	−1.1	−13.7	−8.3	−4.7
K2	−3.8	−6.7	−4.4	−3.4	−13.7	3.5	0.5	−3.1	−2.3	21.2	−11.6	−6.9
Hilt	−5.9	−11.5	−1.5	−7.8	−16.9	1.0	1.8	0.4	−0.2	8.8	−7.1	−4.2
Top-end logs												
K1	−1.4	−0.2	−4.6	3.5	−0.3	16.1	−3.3	−8.1	−4.6	−7.6	−4.8	−11.0
K2	−3.3	−0.6	−9.6	−1.4	1.0	5.9	−2.0	−5.0	−2.8	−6.3	−9.5	−11.7
Hilt	−5.5	−1.6	−6.8	−4.8	−1.5	21.5	−5.9	−9.8	−4.8	−12.7	−3.8	1.1