Interim Equations and Tables For the

YIELD OF FULLY STOCKED SPRUCE-POPLAR STANDS IN THE MIXEDWOOD FOREST SECTION OF ALBERTA

W.D. JOHNSTONE

INFORMATION REPORT NOR-X-175

NOVEMBER 1977

NORTHERN FOREST RESEARCH CENTRE
CANADIAN FORESTRY SERVICE
FISHERIES AND ENVIRONMENT CANADA
5320 - 122 STREET
EDMONTON, ALBERTA, CANADA
T6H 3S5

Johnstone, W.D. 1977. Interim equations and tables for the yield of fully stocked spruce-poplar stands in the Mixedwood Forest Section of Alberta. Fish, Environ. Can., Can. For. Serv., North. For. Res. Cent. Edmonton, Alberta. Inf. Rep. NOR-X-175

ABSTRACT

Least-square equations were derived to predict the yield of natural, fully stocked spruce-poplar stands of the Mixedwood Forest Section in Alberta from age and site productivity. The equations give reliable yield estimates, particularly for the softwood stand-component, over a wide range of age and site conditions. Yield estimates, in metric units, are tabulated for stands ranging in age from 20 to 140 years at 0.3 m growing on sites ranging from 12.5 to 25.0 m at 70 years stump age. The importance of considering both site quality and degree of utilization in managing mixedwood stands is discussed.

RESUME

L'auteur a dérivé des équations des moindres carrés pour prévoir le rendement des peuplements naturels entièrement remplis d'Epinette-Peuplier de la section Forêts mixtes de l'Alberta, du point de vue âge et productivité. Les équations fournissent des évaluations de rendement fiables, particulièrement chez les composantes des peuplements re résineux, couvrant une vaste gamme d'âges et de conditions de stations. Les estimations de rendement en unités métriques sont disposées en forme de tables pour les peuplements âgés de 20 à 140 ans à 0.3 m, croissant sur des stations variant entre 12.5 et 25.0 m, à l'âge d'exploitation 70 ans. L'auteur nous renseigne sur l'importance de considérer à la fois la qualité de la station et le degré d'utilisation dans l'aménagement des peuplements mixtes.

CONTENTS

INTRODUCTION

White spruce (*Picea glauca* (Moench) Voss) is the most prominent and commercially important conifer in Alberta. In 1972, this species made up about 45% of the gross merchantable volume of primary coniferous growing stock on productive crown forest land in Alberta and constituted 74.1% of Alberta's total sawmill production (Teskey and Smyth 1975). A large proportion of the province's white spruce inventory is located in the Mixedwood Forest Section (B18a) of the Boreal Forest Region (Rowe 1972) where the spruce-poplar is the dominant forest type. Stands in the spruce-poplar type are composed principally of white spruce interspersed with aspen (*Populus tremuloides* Michx.), but commonly contain some balsam poplar (*Populus balsamifera* L.) and white birch (*Betula papyrifera* Marsh.). These stands are most often of fire origin and therefore are usually even-aged.

With the continued expansion of Alberta's forest economy, there is an increasing need for reliable yield information upon which rational forest management planning can be based. The only yield information presently available for Alberta's spruce-poplar stands is that provided by MacLeod and Blyth (1955). Although this information has been useful in the past, it is available only in a tabular form that seriously limits its use in today's computerized inventory and planning systems.

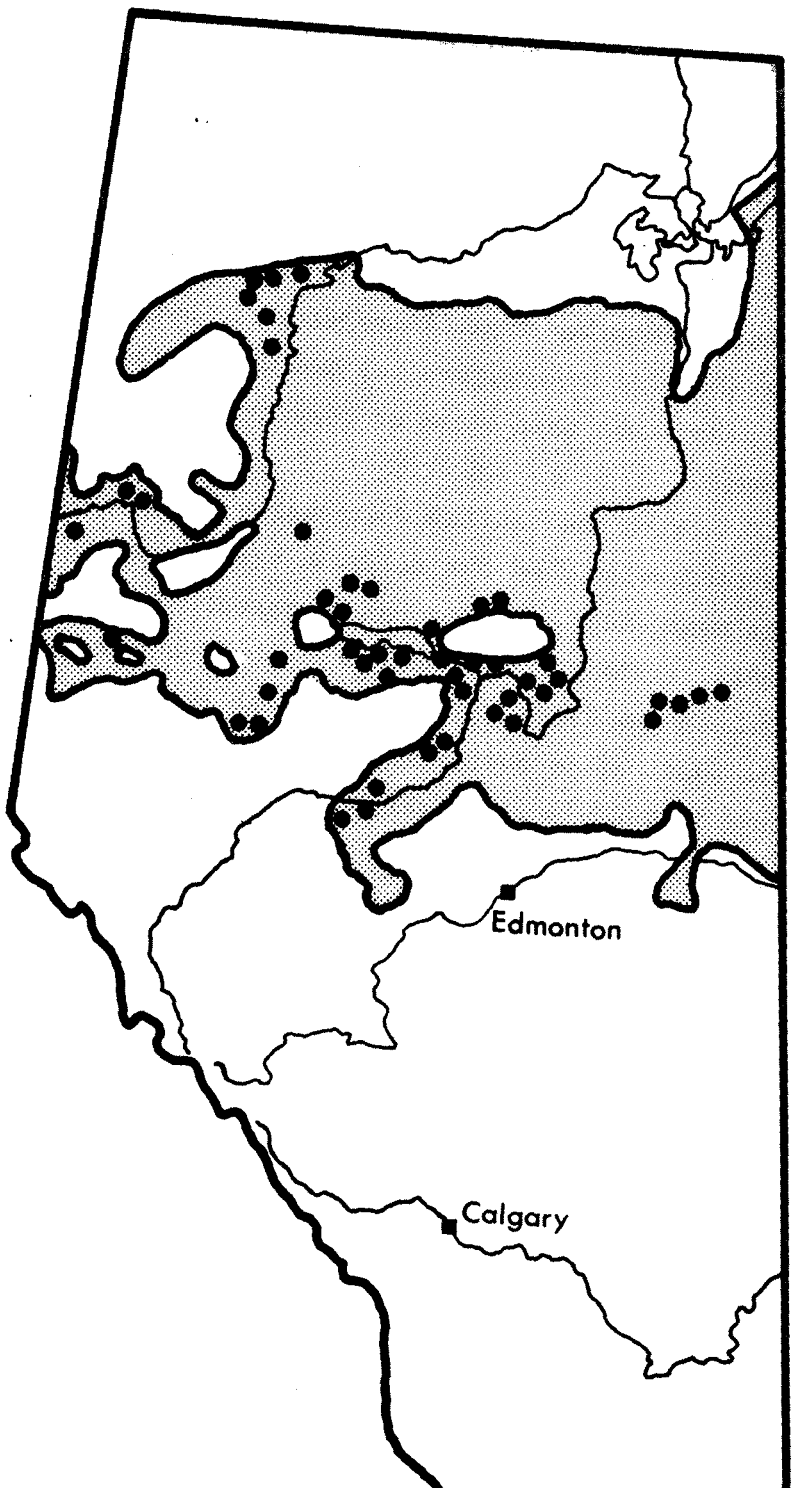
The purpose of this report is to present equations and tables in metric units for predicting the yield, to different merchantability limits, of fully stocked spruce-poplar stands in the Mixedwood Forest Section of Alberta's Boreal Forest Region. Although emphasis is placed upon the softwood component of these stands, an estimate of hardwood yield is also provided.

METHODS AND MATERIALS

SOURCES OF DATA

The basic data used in this study were obtained from 167 Canadian Forestry Service (CFS) plots plus 75 Alberta Forest Service (AFS) plots. All of the plots were established in spruce-poplar stands located within the Mixedwood Forest Section of Alberta's boreal forest (Fig. 1). The

Figure 1. Distribution of sample plots in the B18a (Mixedwood) Section of the Boreal Forest Region of Alberta



Each dot represents one or more sample plots used in this study.

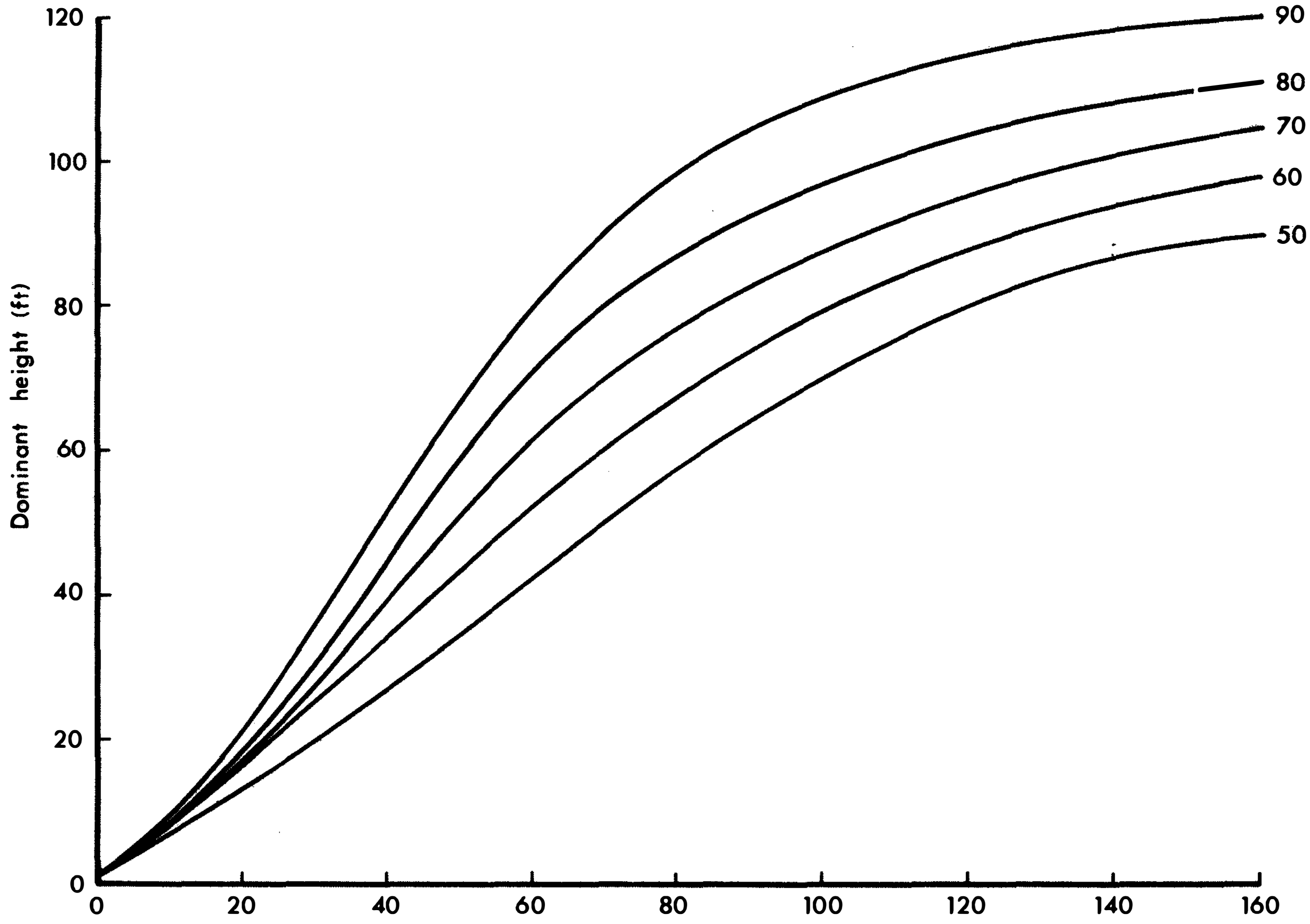
CFS plots, which varied in size (0.01 to 0.24 ha) and contained at least 100 trees in each, were located in fully stocked even-aged stands whose stump age varied from 16 to 146 years. The AFS plots varied in size from 0.08 to 0.20 ha and were located in "C" density (fully stocked) stands that varied from 59 to 123 years in stump age. All plots contained both white spruce and aspen, and some also contained a small component of balsam poplar, white birch, balsam fir (*Abies balsamea* (L.) Mill.), and/or black spruce (*Picea mariana* (Mill.) B.S.P.). None of the plots contained lodgepole pine (*Pinus contorta* Dougl. var. *latifolia* Engelm.) or tamarack (*Larix laricina* (Du Roi) K. Koch).

A dbhob-tally by species was available for all plots. When possible, separate height-diameter curves were used for each species in each plot. Dominant height (h_{dom}) measurements of the spruce were available for all plots; a spruce site index (S) was assigned to each plot using AFS site index curves (Fig. 2). For the softwood component (white spruce, black spruce, and balsam fir) of each plot, the number of trees (N), basal area (G), arithmetic-mean diameter ($\bar{d}$), quadratic-mean diameter (dg: diameter of mean basal area), arithmetic-mean height ($\bar{h}$), Lorey's height (h_L: average height weighted by basal area), and average age (T) of the spruce at stump height (0.3 m) were determined. Individual tree volumes were calculated using Honer's (1967) equations (Appendix 1), based upon the following size and merchantability standards:

1. Total volume ($v_{1.5}$)--trees $\geq$ 1.5 cm dbhob and including stump and top volumes
2. Small roundwood volume ($v_{11.7}$)--trees $\geq$ 11.7 cm dbhob from a 0.3-m stump to a 7.6-cm dib top
3. Large roundwood volume ($v_{14.2}$)--trees $\geq$ 14.2 cm dbhob from a 0.3-m stump to a 10.2-cm dib top
4. Sawlog volume ($v_{21.8}$)--trees $\geq$ 21.8 cm dbhob from a 0.3-m stump to a 15.2-cm dib top

For the hardwood component (aspen, poplar, and birch), total volume, using the equations in Appendix 1, and basal area were determined. The means and variations of the various stand characteristics analyzed are presented in Table 1.

Figure 2. White spruce site index curves (index age 70 yr)



Source: A.F.S. (unpublished data)

Table 1. Summary of the stand characteristics of 242 sample plots of spruce-poplar stands in the Mixedwood Forest Section in Alberta's Boreal Forest Region

Stand characteristic		Mean	Standard deviation	Minimum value	Maximum value
Softwood component:					
S	(m @ 70 yr)	20.6	3.5	12.0	30.0
T	(yr)	81.5	30.9	16.0	146.0
h_{dom}	(m)	21.5	7.4	3.0	34.0
$\bar{h}$	(m)	14.7	5.8	3.1	26.6
h_L	(m)	18.6	6.8	3.1	30.1
$\bar{d}$	(cm)	15.8	7.4	2.5	35.9
dg	(cm)	17.4	8.0	2.5	37.9
N	(/ha)	1 524.5	1 245.1	163.0	6 919.0
G	(m^2/ha)	24.2	10.3	0.2	49.5
$V_{1.5}$	(m^3/ha)	205.2	113.8	0.3	510.3
$V_{11.7}$	(m^3/ha)	177.8	113.0	0.0	475.0
$V_{14.2}$	(m^3/ha)	163.3	112.2	0.0	460.8
$V_{21.8}$	(m^3/ha)	116.1	101.5	0.0	408.5
Hardwood component:					
$V_{1.5}$	(m^3/ha)	149.9	81.8	0.0	413.1
G	(m^2/ha)	17.1	8.4	0.0	41.3
Entire stand:					
$V_{1.5}$	(m^3/ha)	355.2	124.5	48.5	609.0

ANALYSIS

In a preliminary analysis, all of the yield characteristics for the softwood component were used as dependent variables, and site, age, and various transformations and interactions thereof were used as independent variables. This analysis indicated that many of the dependent variables could not be estimated with a high degree of certainty using only age and site. Further analysis indicated that the estimation of arithmetic-average and quadratic-mean diameters and basal area could be greatly improved by using Lorey's height as an independent variable in combination with site index and age. It was also found that all softwood volumes could be reliably estimated using softwood basal area and Lorey's height as independent variables. Because of the difficulty in reliably predicting number of stems, and in order to ensure agreement in the interrelationship between number of stems, quadratic-mean diameter, and basal area, it was decided to calculate number of stems from estimated basal area and estimated quadratic-mean diameter. Both hardwood basal area and volume correlated very poorly with all of the softwood stand characteristics, so regression equations for the hardwood portions of stands were not developed. However, because a fairly reliable equation could be developed to predict the total volume ($V_{1.5}$) of the entire stand (softwoods plus hardwoods), an estimate of hardwood total volume was obtained by subtracting estimated softwood total volume from the estimated total volume of the entire stand.

The following site index equation was derived by a separate analysis based upon the data presented in the AFS site index curves (Fig. 2):

$$h_{dom} = 1.6466X + 0.4189X^2 - 0.0417X^3 - 0.0066X^4 + 0.00076X^5 + (1.0 + 0.03788X - 0.03074X^2 + 0.00215X^3 + 0.00044X^4 - 0.000047X^5) \cdot S$$

Where: $X = (T-70)/10$

A detailed discussion of the derivation of this equation is presented in Appendix 2.

RESULTS

YIELD EQUATIONS AND TABLES

Least-square regression equations and their associated statistics for the various yield characteristics are presented in Table 2. Judged on the basis of their coefficients of determination (R^2) and standard errors of estimate ($s_{y \cdot x}$), these equations appear to provide highly reliable yield estimates, with the exception of basal area. In addition to developing yield tables, the stand-volume equations will provide quick, highly reliable estimates when used in conjunction with horizontal point sampling (p.p.s.), because estimates of both basal area (number of softwood trees tallied x the basal area factor) and Lorey's height (arithmetic-mean height of the tallied softwood trees) are readily available. In addition, because the same regression model was used, the stand-volume equations are additive, and consequently equations can be calculated to predict volume differences between merchantability limits merely by subtracting the corresponding regression coefficients (Johnstone 1976).

Yield estimates for fully stocked spruce-poplar stands from 20 to 140 years old and for a range of site productivities are presented in tabular form in Appendixes 3 to 8. These tables were developed using the yield equations in Table 2 and the site index equation derived in Appendix 2. For each table, dominant height, arithmetic-mean height, and Lorey's height were estimated for each age and site; then, arithmetic-mean diameter, quadratic-mean diameter, and basal area were estimated for each age, site, and corresponding estimated Lorey's height; next, number of stems was calculated from estimated basal area and quadratic-mean diameter. Finally, stand volumes were calculated from estimated Lorey's height and estimated basal area. As noted previously, the total hardwood volume was calculated by subtracting total softwood volume from the total stand volume.

Because predicted values of Lorey's height and/or basal area were used as independent variables in developing the yield table estimates, the standard errors of estimate of the base equations (Table 2) are not directly applicable to the tabular values. In order to obtain an indication of the precision of the estimates when predicted values are used,

Table 2. Yield equations for fully stocked spruce-aspen stands in the Mixedwood Forest Section of the Boreal Forest Region of Alberta

Softwood component:

$\bar{h}$ (m) = $42.598 - 1.055T - 127.538 \text{ Log}_{10}T + 32.705T^{0.5} + 0.004(S \cdot T)$

$n = 242$ $\qquad R^2 = 0.866$ $\qquad s_{y \cdot x} = 2.14$ m

h_L (m) = $94.559 + 0.290T - 73.413 \text{ Log}_{10}T - 59.260 \text{ Log}_{10}S$

$+ 5.717 (S^{0.5} \cdot T^{0.5}) - 0.194 (S^{0.5} \cdot T) - 0.230 (S \cdot T^{0.5})$

$n = 242$ $\qquad R^2 = 0.954$ $\qquad s_{y \cdot x} = 1.47$ m

$\bar{d}$ (cm) = $0.560 - 0.155T^{0.75} + 8.293 \text{ Log}_{10}h_L + 0.040h_L{}^2 - 0.017(S \cdot h_L)$

$n = 242$ $\qquad R^2 = 0.897$ $\qquad s_{y \cdot x} = 2.40$ cm

dg (cm) = $0.481 - 0.132T^{0.75} + 8.254 \text{ Log}_{10}h_L + 0.041h_L{}^2 - 0.015(S \cdot h_L)$

$n = 242$ $\qquad R^2 = 0.927$ $\qquad s_{y \cdot x} = 2.18$ cm

G (m^2/ha) = $-20.005 + 0.006(T \cdot S) + 36.886 \text{ Log}_{10}h_L - 0.029(S \cdot h_L)$

$n = 242$ $\qquad R^2 = 0.612$ $\qquad s_{y \cdot x} = 6.46$ m^2/ha

N (/ha) = $G/(0.00007854 \text{ dg}^2)$

$V_{1.5}$(m^3/ha) = $-6.297 + 1.951G + 0.282h_L + 0.317(G \cdot h_L)$

$n = 242$ $\qquad R^2 = 0.999$ $\qquad s_{y \cdot x} = 2.61$ m^3/ha

$V_{11.7}$(m^3/ha) = $-10.117 - 1.219G + 0.632h_L + 0.410(G \cdot h_L)$

$n = 242$ $\qquad R^2 = 0.993$ $\qquad s_{y \cdot x} = 9.67$ m^3/ha

$V_{14.2}$(m^3/ha) = $-2.218 - 3.440G + 0.375h_L + 0.482(G \cdot h_L)$

$n = 242$ $\qquad R^2 = 0.989$ $\qquad s_{y \cdot x} = 11.96$ m^3/ha

$V_{21.8}$(m^3/ha) = $27.317 - 8.441G - 0.781h_L + 0.613(G \cdot h_L)$

$n = 242$ $\qquad R^2 = 0.958$ $\qquad s_{y \cdot x} = 20.92$ m^3/ha

Entire stand (softwood plus hardwood components):

$V_{1.5}$(m^3/ha) = $66.021 + 3.213G + 11.819h_L - 0.017(G \cdot h_L)$

$n = 242$ $\qquad R^2 = 0.676$ $\qquad s_{y \cdot x} = 71.37$ m^3/ha two statistics, the aggregate difference (A.D.) and the mean absolute deviation (M.A.D.), were determined for each yield characteristic of the 242 plot observations using the following formulas:

$$\text{A.D.} = \frac{\Sigma\hat{Y} - \Sigma Y}{\Sigma\hat{Y}} \times 100\%$$

$$\text{M.A.D.} = \frac{\Sigma(|\hat{Y} - Y|)}{n}$$

Where: $\hat{Y}$ = estimated value of dependent variable

Y = observed value of dependent variable

n = number of observations

Aggregate differences and mean absolute deviations for the various yield characteristics are presented in Table 3. The small magnitude of the A.D.'s indicates that the estimates are essentially free from bias; however, the M.A.D.'s indicate that some loss of precision has resulted because predicted Lorey's height and/or predicted basal area were used as independent variables.

INCREMENT AND ROTATION

The results in Table 4 clearly demonstrate the direct effect of site productivity on mean annual increment (m.a.i.) and its inverse effect upon the age at which the m.a.i.'s culminate. Although basal area m.a.i. and the age at which it culminates are obviously affected by site productivity (Table 4), basal area appears to reach a uniform level of about 33.9 m^2/ha at 140 years on a range of sites (Appendixes 5 to 8). This may account for the low correlation ($r = 0.07$) observed between basal area and site index.

Rotation age is often selected to coincide with the age at which mean annual volume increment culminates, because at this age the stand will yield the maximum possible volume per acre per year. The results presented show that the more completely these mixedwood stands are utilized for small-sized material, the shorter will be the length of the rotation. On the other hand, if the stands are utilized for sawlog material, a long rotation (in excess of 145 years) should be anticipated, and only the better sites should be managed for this purpose.

Table 3. Aggregate differences (A.D.) and mean absolute deviations (M.A.D.) for 242 sample plots of spruce-poplar stands when predictions of Lorey's height and/or predicted basal area are used as independent variables

Yield characteristic	A.D. (%)	M.A.D. (±)
Softwood component:		
h_{dom}	-0.12	0.40 m
$\bar{h}$	0.00	1.76 m
h_L	0.00	1.14 m
$\bar{d}$	0.45	2.34 cm
dg	0.43	2.31 cm
N	2.24	509.57/ha
G	-0.26	5.12 m^2/ha
$V_{1.5}$	-0.11	43.79 m^3/ha
$V_{11.7}$	-0.04	39.06 m^3/ha
$V_{14.2}$	0.03	37.08 m^3/ha
$V_{21.8}$	0.28	32.64 m^3/ha
Hardwood component:		
$V_{1.5}$	0.03	61.50 m^3/ha
Entire stand:		
$V_{1.5}$	-0.05	53.94 m^3/ha

Table 4. Maximum mean annual increments[1] and the age at which they occur for the various stand characteristics of fully stocked spruce-aspen stands in the Mixedwood Forest Section of the Boreal Forest Region in Alberta[2]

Site Index	Softwood Component									
	G		$V_{1.5}$		$V_{11.7}$		$V_{14.2}$		$V_{21.8}$	
	m.a.i. (m^2/ha)	Age (yr)	m.a.i. (m^3/ha)	Age (yr)	m.a.i. (m^3/ha)	Age (yr)	m.a.i. (m^3/ha)	Age (yr)	m.a.i. (m^3/ha)	Age (yr)
12.5	.288	66	(2.02)	(155)	(1.88)	(192)	(1.82)	(208)	(-)	(>220)
15.0	.325	56	2.25	132	(2.09)	(165)	(2.03)	(179)	(1.74)	(202)
17.5	.357	49	2.46	117	(2.28)	(147)	(2.22)	(160)	(1.92)	(180)
20.0	.382	44	2.63	105	2.45	135	2.40	(147)	(2.08)	(165)
22.5	.403	40	2.75	97	2.59	126	2.53	137	(2.21)	(154)
25.0	.418	37	2.84	90	2.68	119	2.64	130	(2.32)	(146)

[1]M.a.i.'s determined by dividing estimated basal area and volumes by age

[2](Bracketed values are beyond range of tables (Appendixes 3-8))

Culmination of m.a.i. of the hardwood and the hardwood plus softwood components will occur much earlier than for the softwood component alone because the hardwoods are somewhat older than the softwoods, and the hardwoods--particularly aspen and poplar-exhibit a much faster juvenile growth rate than the softwoods.

DISCUSSION

The equations and tables presented provide yield estimates for fully stocked spruce-poplar stands in the Mixedwood Forest Section of Alberta. Emphasis is placed on the spruce component because of its current economic importance in the province. If used in conjunction with overstocked or understocked stands, an adjustment for stocking should be made, preferably based on volume.

The results demonstrate the importance of considering both site quality and degree of utilization in selecting rotation ages and making wood supply projections for mixedwood stands. The higher the degree of utilization of smaller trees, as well as stumps and tops, the shorter the length of rotation. Generally, the rotation ages observed agree with those suggested for spruce-aspen stands in Alberta by MacLeod and Blyth (1955) and in Saskatchewan by Kirby (1962). The results also suggest that only those stands growing on high-quality sites should be managed for sawlogs.

The equations and tables presented in this report are intended for interim use only. The inherent weaknesses in normal yield tables are widely recognized, and the range of the basic data was by no means all-encompassing. As the province's softwood allowable annual cut becomes more fully committed, the hardwood component of the mixedwood stands will no doubt become more important, as will the need for more sophisticated information on hardwood yields.

ACKNOWLEDGMENTS

The author is indebted to the Alberta Forest Service for making its sample-plot data available, and to Messrs. Stan Lux and Bill Chow for their assistance in the analysis of the data.

REFERENCES

Honer, T.G. 1967. Standard volume tables and merchantable conversion factors for the commercial tree species of central and eastern Canada. Can. Dep. For. Rural Dev., For. Br. Inf. Rep. FMR-X-5. 21 pp. plus appendixes.

Johnstone, W.D. 1976. Variable-density yield tables for natural stands of lodgepole pine in Alberta. Can. Dep. Fish. Environ., Can. For. Serv., For. Tech. Rep. 20. 110 pp.

Kabzems, A. 1971. The growth and yield of well stocked white spruce in the Mixedwood Section in Saskatchewan. Sask. Dep. Nat. Resour., For. Br. Tech. Bull. 5. 75 pp.

Kirby, C.L. 1962. The growth and yield of white spruce-aspen stands in Saskatchewan. Sask. Dep. Nat. Resour., For. Br. Tech. Bull. 4. 58 pp.

MacLeod, W.K. and A.W. Blyth. 1955. Yield of even-aged fully stocked spruce-poplar stands in Northern Alberta. Can. Dep. North. Aff. Natl. Resour., For. Br., For. Res. Div. Tech. Note 18. 33 pp.

Nordin, V.J. 1956. Heart rot in relation to the management of spruce in Alberta. For. Chron. 32:79-85.

Rowe, J.S. 1972. Forest regions of Canada. Can. Dep. Environ., For. Serv. Publ. 1300. 172 pp.

Teskey, A.G. and J.H. Smyth. 1975. The economic importance of sawmilling and other primary wood-using industries in Alberta, 1972. Can. Dep. Environ., Can. For. Serv., North. For. Res. Cent. Inf. Rep. NOR-X-145. 206 pp.

Thomas, G.P., D.E. Ethridge, and G.D. Paul. 1960. Fungi and decay in aspen and balsam poplar in the Boreal Forest Region, Alberta. Can. J. Bot. 38:459-466.

APPENDIX 1. INDIVIDUAL TREE GROSS-VOLUME EQUATIONS (Honer 1967).

In the following equations d = dbhob (in.) and h = total tree height (ft).

1. White spruce[1]:

$$v_{1.5}\ (m^3) = \left[\frac{d^2}{1.440 + (342.175/h)}\right] \cdot 0.0283168$$

$$v_{11.7}\ (m^3) = v_{1.5} \cdot (0.9611 - 0.2456X - 0.6801X^2)$$

$$\text{Where: } X = \left(\frac{3.0}{d}\right)^2 \cdot \left(1.0 + \frac{1.0}{h}\right)$$

$$v_{14.2}\ (m^3) = v_{1.5} \cdot (0.9611 - 0.2456X - 0.6801X^2$$

$$\text{Where: } X = \left(\frac{4.0}{d}\right)^2 \cdot \left(1.0 + \frac{1.0}{h}\right)$$

$$v_{21.8}\ (m^3) = v_{1.5} \cdot (0.9611 - 0.2456X - 0.6801X^2)$$

$$\text{Where: } X = \left(\frac{6.0}{d}\right)^2 \cdot \left(1.0 + \frac{1.0}{h}\right)$$

2. Black spruce:

$$v_{1.5}\ (m^3) = \left[\frac{d^2}{1.588 + (333.364/h)}\right] \cdot 0.0283168$$

$$v_{11.7}\ (m^3) = v_{1.5} \cdot (0.9526 - 0.1027X - 0.8199X^2)$$

$$\text{Where: } X = \left(\frac{3.0}{d}\right)^2 \cdot \left(1.0 + \frac{1.0}{h}\right)$$

$$v_{14.2}\ (m^3) = v_{1.5} \cdot (0.9526 - 0.1027X - 0.8199X^2)$$

$$\text{Where: } X = \left(\frac{4.0}{d}\right)^2 \cdot \left(1.0 + \frac{1.0}{h}\right)$$

$$v_{21.8}\ (m^3) = v_{1.5} \cdot (0.9526 - 0.1027X - 0.8199X^2)$$

$$\text{Where: } X = \left(\frac{6.0}{d}\right)^2 \cdot \left(1.0 + \frac{1.0}{h}\right)$$

3. Balsam fir:

$$v_{1.5}\ (m^3) = \left[\frac{d^2}{2.139 + (301.634/h)}\right] \cdot 0.0283168$$

$$v_{11.7}\ (m^3) = v_{1.5} \cdot (0.9352 - 0.0395X - 0.8147X^2)$$

$$\text{Where: } X = \left(\frac{3.0}{d}\right)^2 \cdot \left(1.0 + \frac{1.0}{h}\right)$$

$$v_{14.2}\ (m^3) = v_{1.5} \cdot (0.9352 - 0.0395X - 0.8147X^2)$$

$$\text{Where: } X = \left(\frac{4.0}{d}\right)^2 \cdot \left(1.0 + \frac{1.0}{h}\right)$$

$$v_{21.8}\ (m^3) = v_{1.5} \cdot (0.9352 - 0.0395X - 0.8147X^2)$$

$$\text{Where: } X = \left(\frac{6.0}{d}\right)^2 \cdot \left(1.0 + \frac{1.0}{h}\right)$$

4. Aspen[2]:

$$v_{1.5}\ (m^3) = \left[\frac{d^2}{-0.312 + (436.683/h)}\right] \cdot 0.0283168$$

5. Balsam poplar[2]:

$$v_{1.5}\ (m^3) = \left[\frac{d^2}{0.420 + (394.644/h)}\right] \cdot 0.0283168$$

6. White birch:

$$v_{1.5}\ (m^3) = \left[\frac{d^2}{2.222 + (300.373/h)}\right] \cdot 0.0283168$$

---

[1]An indication of the allowances to be made to white spruce volumes for decay and cull are given by Nordin (1956) and Kabzems (1971).

[2]Decay losses for aspen and balsam poplar have been reported by Thomas *et al.* (1960).

APPENDIX 2. DERIVATION OF SITE INDEX EQUATION.

In this study the assessment of site productivity was based upon the site index curves shown in Fig. 2, for which there was no equation. It was deemed desirable, therefore, to develop an equation for future use based upon these curves. This equation was developed in the following manner:

1. Dominant height at 5-year age-intervals, from ages 20 to 140 were obtained for site index classes 50 to 90 from Fig. 2.
2. All height and site values were converted from feet to metres using the factor of 0.3048.
3. Using the following general model, a simple linear regression was derived with dominant height (h_{dom}) as the dependent variable and site index (S) as the independent variable for each 5-year age-interval (i):

$$h_{dom_{(i)}} = b_{0_{(i)}} + b_{1_{(i)}} S_{(i)}$$

4. The intercept values (b_0 regression coefficients) determined for each age-interval were then related to age using a least-square polynomial. The polynomial was conditioned to ensure that the intercept was equal to 0 when age was equal to 70 years (index age). A fifth-degree polynomial was required to satisfactorily fit the coefficients (Fig. 3). The derived polynomial is:

$$\hat{b}_0 = 0. + 1.6466X + 0.4189X^2 - 0.0417X^3 - 0.0066X^4 + 0.00076X^5$$

$n = 26 \qquad R^2 = 0.999 \qquad s_{y \cdot x} = 0.183$

Where: $X = (T - 70)/10$

5. Similarly, the slope values (b_1 regression coefficients) from each age-interval were related to age using a fifth-degree polynomial conditioned to equal 1.0 at index age (70 years). The following regression was derived and is shown in Fig. 4:

$$\hat{b}_1 = 1. + 0.03788X - 0.03074X^2 + 0.00215X^3 + 0.00044X^4 - 0.000047X^5$$

$$n = 26 \qquad R^2 = 0.999 \qquad s_{y \cdot x} = 0.0098$$

6. Substituting the $\hat{b}_0$ polynomial for b_0 and $\hat{b}_1$ polynomial for b_1 in the general model (step 3) produces the following anamorphic site index equation:

$$h_{dom} = 1.6466X + 0.4189X^2 - 0.0417X^3 - 0.0066X^4 + 0.00076X^5 + (1.0 + 0.03788X - 0.03074X^2 + 0.00215X^3 + 0.00044X^4 - 0.000047X^5) \cdot S$$

$$n = 128 \qquad \hat{R}^2 = 0.999^a \qquad \hat{s}_{y \cdot x} = 0.29 \text{ m}^b$$

Where: $X = (T - 70)/10$

---

[a] $$\hat{R}^2 = \frac{SS_{total} - \hat{SS}_{residual}}{SS_{total}}$$

$$\hat{s}_{y \cdot x} = \pm \sqrt{\frac{\hat{SS}_{residual}}{n - m - 1}}$$

Where: $\hat{SS}_{residual} = \Sigma(Y - \hat{Y})^2$

SS_{total} = sum of square of Y

Y = observed value of dependent variable

$\hat{Y}$ = estimated value of dependent variable

n = number of observations

m = number of independent variables

Figure 3. The relationship between stump age and the intercepts of the dominant height / site index regressions.

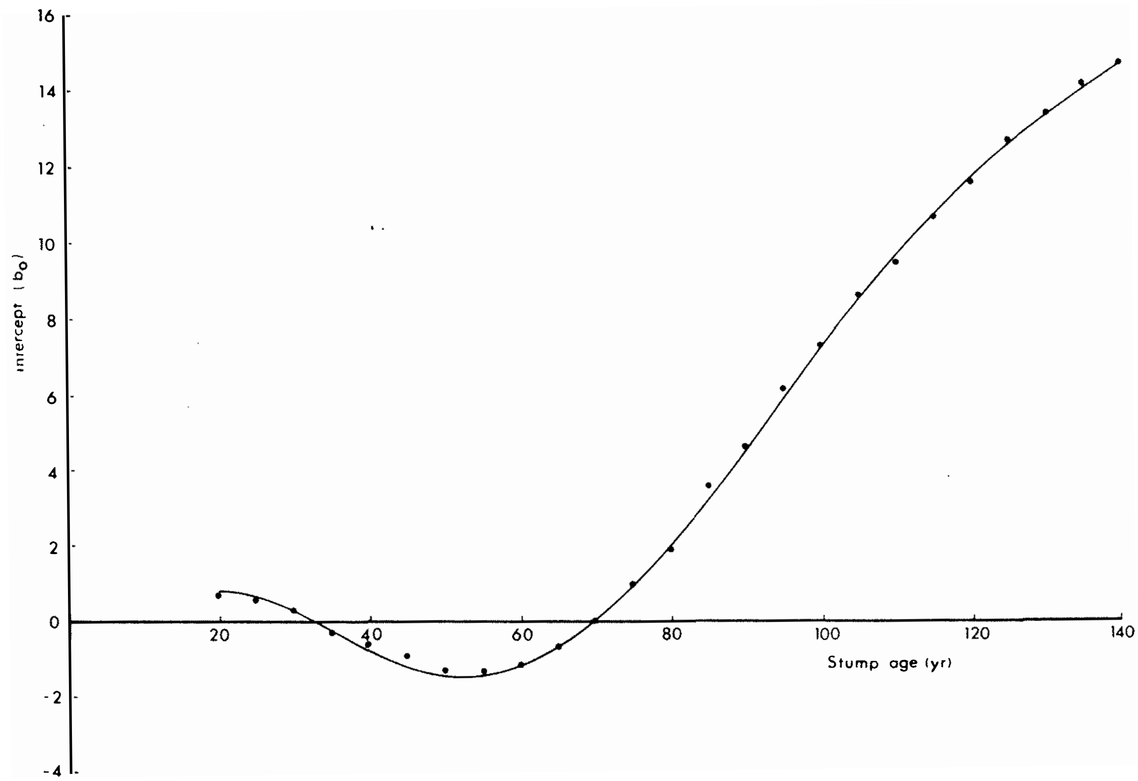
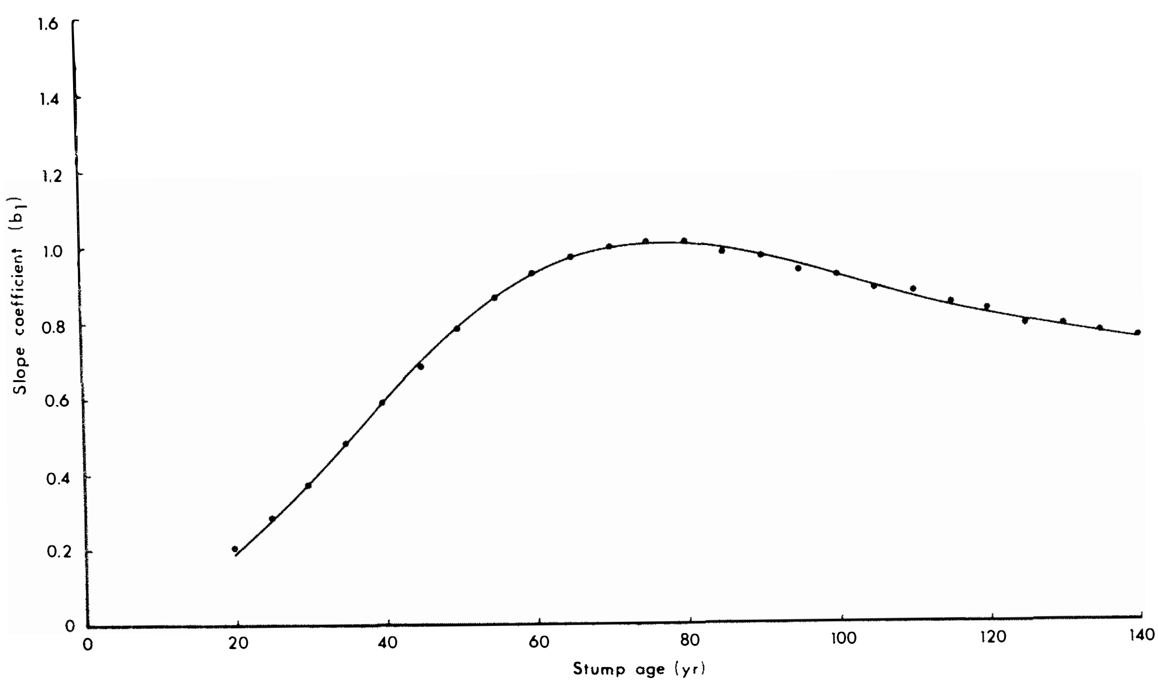


Figure 4. The relationship between stump age and the slope coefficients of the dominant height / site index regressions.



APPENDIX 3. YIELD PER HECTARE OF FULLY STOCKED SPRUCE-ASPEN STANDS

Site index 12.5 (@ stump age 70 years)

	SOFTWOOD COMPONENT, MAINLY SPRUCE											HDWD. COMP., MAINLY ASPEN	ENTIRE STAND
Stump age (yr)	Dom. height (m)	Mean height (m)	Lorey's height (m)	A. mean diam. (cm)	Q. mean diam. (cm)	Basal area (m^2)	No. of stems ≥1.5 cm	Cubic-metre volume (ib) of trees with: dbhob ≥1.5 cm	dbhob ≥11.7 cm	dbhob ≥14.2 cm	dbhob ≥21.8 cm	Volume (ib) ≥1.5 cm dbhob (m^3)	Volume (ib) ≥1.5 cm dbhob (m^3)
20	3.4	2.8	3.7	3.5	3.7	1.0	870	0.0	0.0	0.0	0.0	112.3	112.3
30	5.1	3.2	4.2	3.6	3.9	3.7	3159	7.1	0.0	0.0	0.0	120.3	127.4
40	6.7	4.9	5.8	4.5	4.9	9.0	4675	29.2	3.8	0.0	0.0	132.9	162.0
50	8.5	6.9	7.6	5.7	6.2	13.5	4424	55.0	20.5	3.9	0.0	142.9	197.9
60	10.4	8.8	9.6	7.0	7.7	17.2	3709	82.1	42.4	21.4	0.0	149.4	231.5
70	12.5	10.6	11.4	8.4	9.3	20.1	2997	109.3	67.1	43.9	0.0	152.8	262.1
80	14.6	12.0	13.2	9.9	10.9	22.6	2410	136.3	93.4	69.2	9.7	153.7	290.0
90	16.7	13.2	14.9	11.5	12.7	24.6	1956	162.7	120.2	96.1	33.3	152.8	315.5
100	18.7	14.1	16.5	13.2	14.4	26.4	1612	188.3	147.1	123.5	58.9	150.4	338.7
110	20.5	14.7	18.0	14.8	16.2	28.0	1352	213.1	173.6	151.0	85.6	146.9	360.0
120	21.9	15.1	19.3	16.5	18.0	29.4	1154	237.0	199.5	178.1	112.6	142.4	379.4
130	23.1	15.2	20.6	18.1	19.7	30.7	1003	259.9	224.7	204.6	139.5	137.3	397.2
140	24.1	15.2	21.7	19.6	21.4	31.9	886	282.0	249.0	230.4	166.0	131.5	413.5

APPENDIX 4. YIELD PER HECTARE OF FULLY STOCKED SPRUCE-ASPEN STANDS

Site index 15.0 (@ stump age 70 years)

SOFTWOOD COMPONENT, MAINLY SPRUCE												HDWD. COMP., MAINLY ASPEN	ENTIRE STAND
								Cubic-metre volume (ib) of trees with:					
Stump age (yr)	Dom. height (m)	Mean height (m)	Lorey's height (m)	A. mean diam. (cm)	Q. mean diam. (cm)	Basal area (m^2)	No. of stems ≥1.5 cm	dbhob ≥1.5 cm	dbhob ≥11.7 cm	dbhob ≥14.2 cm	dbhob ≥21.8 cm	Volume (ib) ≥1.5 cm dbhob (m^3)	Volume (ib) ≥1.5 cm dbhob (m^3)
20	3.9	3.0	3.7	3.4	3.7	1.2	1143	0.0	0.0	0.0	0.0	113.8	113.8
30	6.0	3.5	5.0	4.1	4.4	6.2	4031	16.9	0.0	0.0	0.0	127.1	144.1
40	8.2	5.3	7.0	5.3	5.8	11.8	4441	44.7	13.8	0.0	0.0	140.5	185.3
50	10.5	7.4	9.2	6.7	7.4	16.1	3750	74.9	37.1	17.6	0.0	149.6	224.4
60	12.8	9.4	11.4	8.3	9.2	19.4	2956	105.3	64.5	42.3	0.0	154.5	259.8
70	15.0	11.3	13.5	10.0	11.1	22.1	2305	135.3	93.9	70.7	13.1	156.3	291.6
80	17.2	12.8	15.4	11.9	13.0	24.3	1818	164.4	123.8	100.7	40.0	155.7	320.1
90	19.2	14.1	17.2	13.7	15.1	26.2	1465	192.4	153.5	131.3	69.0	153.4	345.8
100	21.0	15.1	18.8	15.6	17.1	27.8	1209	219.3	182.6	161.5	98.8	149.7	369.0
110	22.6	15.8	20.3	17.4	19.1	29.3	1021	245.0	210.7	191.1	128.6	144.9	389.8
120	24.0	16.3	21.6	19.2	21.0	30.6	883	269.5	237.7	219.7	157.8	139.2	408.7
130	25.1	16.5	22.8	20.9	22.8	31.9	779	292.8	263.6	247.2	186.2	132.9	425.1
140	26.0	16.6	23.9	22.4	24.5	33.0	700	315.0	288.2	273.4	213.5	126.0	441.1

APPENDIX 5. YIELD PER HECTARE OF FULLY STOCKED SPRUCE-ASPEN STANDS

Site index 17.5 (@ stump age 70 years)

	SOFTWOOD COMPONENT, MAINLY SPRUCE											HDWD. COMP., MAINLY ASPEN	ENTIRE STAND
Stump age (yr)	Dom. height (m)	Mean height (m)	Lorey's height (m)	A. mean diam. (cm)	Q. mean diam. (cm)	Basal area (m^2)	No. of stems ≥1.5 cm	Cubic-metre volume (ib) of trees with: dbhob ≥1.5 cm	dbhob ≥11.7 cm	dbhob ≥14.2 cm	dbhob ≥21.8 cm	Volume (ib) ≥1.5 cm dbhob (m^3)	Volume (ib) ≥1.5 cm dbhob (m^3)
20	4.4	3.2	3.9	3.5	3.7	1.9	1791	1.0	0.0	0.0	0.0	117.3	118.3
30	7.0	3.8	5.8	4.5	4.9	8.3	4362	26.5	3.0	0.0	0.0	133.3	159.8
40	9.7	5.7	8.2	5.9	6.5	13.8	4089	58.8	24.7	8.1	0.0	146.7	205.5
50	12.5	7.9	10.7	7.6	8.4	17.8	3200	92.2	53.5	32.8	0.0	154.8	247.0
60	15.1	10.0	13.1	9.5	10.5	20.9	2413	125.1	85.2	63.1	9.0	158.6	283.7
70	17.5	12.0	15.4	11.5	12.7	23.3	1836	156.9	117.9	95.8	37.9	159.3	316.3
80	19.7	13.6	17.4	13.6	15.0	25.3	1433	187.4	150.3	129.1	69.6	157.8	345.2
90	21.6	15.0	19.2	15.7	17.3	27.0	1153	216.3	181.8	162.1	102.2	154.6	370.9
100	23.3	16.1	20.8	17.7	19.5	28.6	956	243.9	212.1	194.0	134.6	149.9	393.8
110	24.8	16.9	22.3	19.7	21.6	30.0	815	270.0	241.1	224.8	166.2	144.2	414.2
120	26.0	17.5	23.6	21.5	23.6	31.3	714	294.8	268.6	254.1	196.6	137.6	432.4
130	27.0	17.8	24.7	23.2	25.4	32.5	640	318.3	294.7	281.9	225.5	130.3	448.6
140	27.8	18.0	25.7	24.7	27.0	33.7	586	340.6	319.4	308.2	252.9	122.4	462.9

APPENDIX 6. YIELD PER HECTARE OF FULLY STOCKED SPRUCE-ASPEN STANDS

Site index 20.0 (@ stump age 70 years)

	SOFTWOOD COMPONENT, MAINLY SPRUCE											HDWD. COMP., MAINLY ASPEN	ENTIRE STAND
Stump age (yr)	Dom. height (m)	Mean height (m)	Lorey's height (m)	A. mean diam. (cm)	Q. mean diam. (cm)	Basal area (m^2)	No. of stems ≥1.5 cm	Cubic-metre volume (ib) of trees with: dbhob ≥1.5 cm	dbhob ≥11.7 cm	dbhob ≥14.2 cm	dbhob ≥21.8 cm	Volume (ib) ≥1.5 cm dbhob (m^3)	Volume (ib) ≥1.5 cm dbhob (m^3)
20	4.9	3.4	4.2	3.5	3.8	2.8	2485	4.1	0.0	0.0	0.0	120.0	124.1
30	7.9	4.1	6.5	4.8	5.3	9.9	4462	35.3	8.4	0.0	0.0	138.6	173.9
40	11.2	6.1	9.4	6.5	7.2	15.2	3754	71.0	35.5	17.5	0.0	151.9	222.9
50	14.4	8.4	12.1	8.4	9.3	18.9	2772	106.7	68.5	47.7	0.0	159.4	266.1
60	17.4	10.6	14.7	10.5	11.7	21.7	2022	141.2	103.3	82.2	27.9	162.6	303.8
70	20.0	12.7	17.0	12.8	14.2	23.9	1512	174.1	138.2	117.9	61.4	162.8	336.9
80	22.2	14.4	19.1	15.1	16.7	25.8	1173	205.1	172.0	153.2	96.2	160.8	365.9
90	24.1	15.9	20.9	17.4	19.2	27.4	945	234.5	204.5	187.5	131.0	157.0	391.5
100	25.6	17.1	22.5	19.5	21.6	28.8	790	262.3	235.5	220.4	164.8	151.7	414.1
110	26.9	18.0	24.0	21.6	23.7	30.2	681	288.6	264.8	251.6	197.2	145.3	434.0
120	28.0	18.7	25.2	23.4	25.7	31.5	605	313.5	292.5	281.1	228.0	137.9	451.4
130	29.0	19.1	26.2	25.0	27.5	32.7	551	337.1	318.6	308.9	257.0	129.6	466.7
140	29.7	19.4	27.1	26.4	29.0	33.9	514	359.4	343.1	335.0	284.0	120.7	480.1

APPENDIX 7. YIELD PER HECTARE OF FULLY STOCKED SPRUCE-ASPEN STANDS

Site index 22.5 (@ stump age 70 years)

	SOFTWOOD COMPONENT, MAINLY SPRUCE											HDWD. COMP., MAINLY ASPEN	ENTIRE STAND
								Cubic-metre volume (ib) of trees with:					
Stump age (yr)	Dom. height (m)	Mean height (m)	Lorey's height (m)	A. mean diam. (cm)	Q. mean diam. (cm)	Basal area (m^2)	No. of stems ≥1.5 cm	dbhob ≥1.5 cm	dbhob ≥11.7 cm	dbhob ≥14.2 cm	dbhob ≥21.8 cm	Volume (ib) ≥1.5 cm dbhob (m^3)	Volume (ib) ≥1.5 cm dbhob (m^3)
20	5.4	3.6	4.4	3.6	3.9	3.7	3105	7.4	0.0	0.0	0.0	122.8	130.1
30	8.9	4.4	7.3	5.1	5.6	11.1	4477	42.9	14.0	1.2	0.0	143.3	186.2
40	12.7	6.5	10.4	6.9	7.7	16.1	3468	81.1	45.4	26.9	0.0	156.6	237.7
50	16.4	8.9	13.4	9.0	10.1	19.5	2443	118.4	81.5	61.4	11.9	163.8	282.2
60	19.7	11.2	16.0	11.4	12.7	22.1	1736	153.7	118.4	98.6	45.6	166.9	320.6
70	22.5	13.4	18.4	13.9	15.5	24.1	1283	186.9	154.5	136.1	82.0	167.1	353.9
80	24.7	15.2	20.6	16.4	18.2	25.8	993	218.1	189.0	172.6	118.8	164.9	383.0
90	26.5	16.8	22.4	18.8	20.8	27.3	804	247.6	221.9	207.4	154.6	160.8	408.3
100	27.9	18.1	24.0	21.0	23.2	28.7	677	275.4	253.0	240.6	189.0	155.1	430.4
110	29.1	19.1	25.4	23.0	25.4	30.1	592	301.7	282.4	271.9	221.7	148.0	449.7
120	30.1	19.9	26.5	24.8	27.4	31.4	533	326.6	310.0	301.4	252.3	139.8	466.4
130	30.9	20.4	27.5	26.3	29.0	32.7	494	350.2	336.0	329.0	281.0	130.6	480.8
140	31.6	20.8	28.3	27.5	30.3	34.0	470	372.6	360.3	354.7	307.4	120.5	493.1

APPENDIX 8. YIELD PER HECTARE OF FULLY STOCKED SPRUCE-ASPEN STANDS

Site index 25.0 (@ stump age 70 years)

	SOFTWOOD COMPONENT, MAINLY SPRUCE											HDWD. COMP., MAINLY ASPEN	ENTIRE STAND
Stump age (yr)	Dom. height (m)	Mean height (m)	Lorey's height (m)	A. mean diam. (cm)	Q. mean diam. (cm)	Basal area (m^2)	No. of stems ≥1.5 cm	Cubic-metre volume (ib) of trees with: dbhob ≥1.5 cm	dbhob ≥11.7 cm	dbhob ≥14.2 cm	dbhob ≥21.8 cm	Volume (ib) ≥1.5 cm dbhob (m^3)	Volume (ib) ≥1.5 cm dbhob (m^3)
20	5.8	3.8	4.7	3.6	3.9	4.4	3641	10.4	0.0	0.0	0.0	125.4	135.8
30	9.8	4.7	7.9	5.2	5.8	11.9	4472	49.3	19.3	5.4	0.0	147.4	196.7
40	14.2	6.9	11.3	7.2	8.1	16.7	3238	89.2	54.1	35.7	0.0	161.0	250.2
50	18.4	9.4	14.5	9.5	10.7	19.8	2192	127.2	92.3	73.1	24.4	168.5	295.7
60	22.0	11.8	17.3	12.1	13.6	22.1	1525	162.7	130.3	112.1	61.1	171.9	334.5
70	25.0	14.1	19.7	14.7	16.5	24.0	1118	195.8	166.8	150.4	99.2	172.2	368.0
80	27.3	16.0	21.8	17.3	19.4	25.6	866	226.8	201.5	187.2	136.8	170.1	396.9
90	28.9	17.7	23.7	19.8	22.1	27.0	705	256.0	234.3	222.2	173.0	165.9	421.9
100	30.2	19.1	25.2	22.0	24.5	28.4	600	283.6	265.2	255.2	207.4	159.8	443.4
110	31.2	20.2	26.5	24.0	26.7	29.8	532	309.9	294.5	286.3	239.8	152.2	462.0
120	32.1	21.1	27.6	25.7	28.5	31.1	487	334.8	322.0	315.5	270.1	143.1	477.9
130	32.9	21.7	28.5	27.0	30.0	32.5	460	358.6	347.8	342.9	298.2	132.8	491.4
140	33.4	22.2	29.2	28.0	31.1	33.9	445	381.2	372.0	368.3	324.0	121.5	502.6