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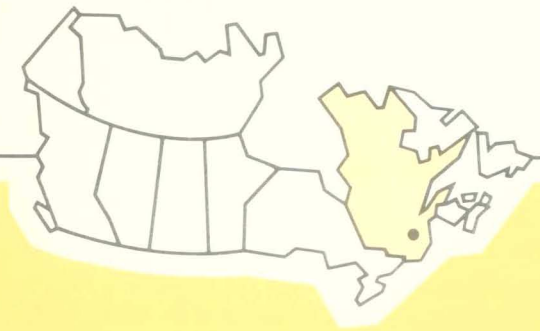
Tree and bole standard volume tables for yellow birch (*Betula alleghaniensis*, Britton) and American beech (*Fagus grandifolia*, Ehrh.) for the Great Lakes - St. Lawrence Forest Region, Quebec

S. Popovich

Information Report LAU-X-66E

Laurentian Forest Research Centre



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ABSTRACT

From 491 yellow birch trees and 548 American beech trees sampled (half felled and half measured by the Barr and Stroud dendrometer) standard bole and tree volume tables were constructed for both species for the Great Lakes - St. Lawrence Forest Region, Quebec.

Four standard volume tables for each species, distinguishing between bole and tree volumes outside and inside bark, are presented. These tables are exceptionally accurate, with an error rate ranging between plus or minus 5.9 and 9.8 percent of the true value 95 percent of the time.

RÉSUMÉ

À partir des données recueillies sur 491 arbres de bouleau jaune et 548 hêtres à grandes feuilles, dont environ 50 % furent mesurés sur des parterres de coupe et le reste à l'aide du dendromètre Barr et Stroud, des tarifs de cubage généraux furent établis pour les arbres et les troncs de ces deux espèces pour la région forestière des Grands Lacs et du Saint-Laurent, au Québec.

L'étude présente quatre tarifs de cubage généraux pour chacune de ces espèces établis grâce aux méthodes qui prennent séparément les volumes des arbres et des troncs avec écorce et sans écorce. La précision de ces tarifs est particulièrement bonne avec un taux d'erreur se situant entre plus ou moins 5,9 et 9,8 % de la réalité pour 95 % du temps.

INTRODUCTION

Yellow birch (*Betula alleghaniensis*, Britton) and American beech (*Fagus grandifolia*, Ehrh.) are not only a characteristic constituent of, but, along with sugar maple (*Acer saccharum*, Marsh.), compose the dominant forest cover of hardwood stands in the Great Lakes - St. Lawrence Forest Region (Rowe 1972). These two species are often associated with other forest species in small private woodlots. Private woodlots supply simultaneously forest industry needs as well as society's energy requirements, implying the need for the complete use of the felled tree.

American beech is used for widely differing purposes, from parquet blocks to railway ties, while yellow birch, considered the most valuable Eastern Canadian hardwood species (Hosie 1969), finds its application in the manufacture of furniture, veneer and plywood, and parquet flooring. Therefore, the Quebec government is very interested in the optimal use of yellow birch, in order to establish a range of products as functions of lumber quality (Larrivée 1981). Of all the merchantable or mature populations of standing material in Quebec, 23 percent is represented by hardwoods; yellow birch and sugar maple form a substantial fraction of this total volume (Lussier 1976).

Accordingly, construction of volume tables for yellow birch and American beech were constructed. As in a previous study (Popovich 1983), these tables are constructed separately giving total volume and bole volume outside and inside bark for that part of the Great Lakes - St. Lawrence Forest Region situated in Quebec.

DATA

The data used in the construction of these tables came from trees of differing ages harvested from various places and from different types of forest cover in the forest sections of Quebec (Figure 1): Algonquin-Pontiac

(L.4.b); (2) Upper St. Lawrence (L.2); (3) Eastern Townships (L.5); and (4) Middle St. Lawrence (L.3) of the Great Lakes - St. Lawrence Forest Region (Rowe 1972).

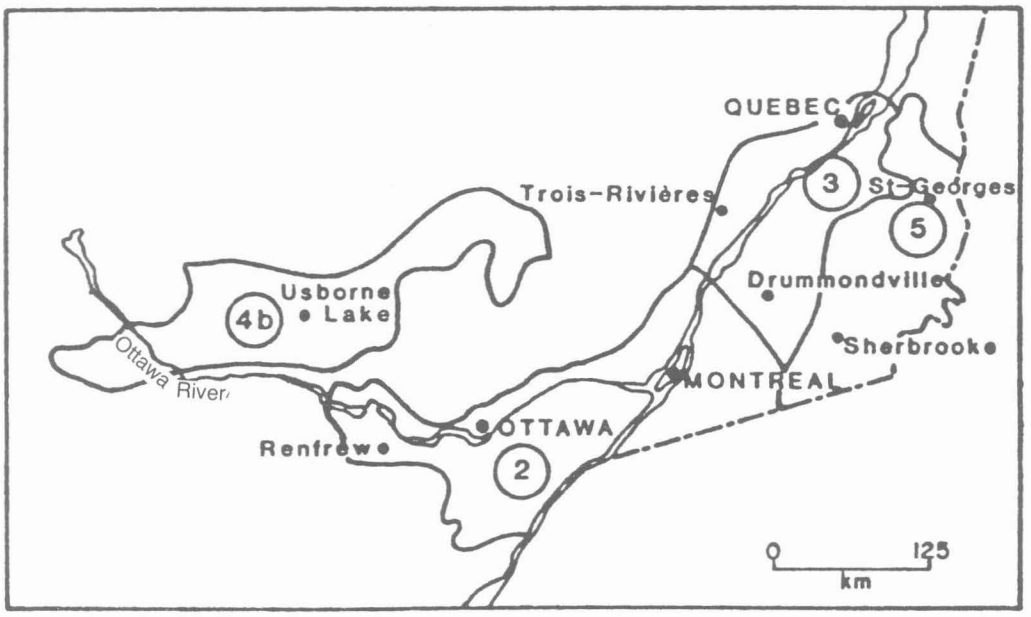

Figure 1. Map of the Great Lakes - Saint Lawrence Forest Region, Quebec, Sections (L.4.b) (L.2) (L.5) (L.3)

Four hundred and ninety-one yellow birch and 548 American beech were available for this analysis (Tables 1 and 2). Of these, 232 yellow birch (47.3%) and 304 American beech (55.5%) were felled for commercial or silvicultural purposes. The Barr and Stroud dendrometer was used to measure 259 (52.7%) yellow birch and 244 (44.5%) American beech. For each tree the following measurements were recorded: total height and height of bole to the closest decimetre; and diameters, outside and inside bark, at breast height (dbh), at the stump (d_0), at 5 percent ($d_{0.05}$), then at every tenth of the bole height to the nearest centimetre. For the 259 and 244 standing yellow

birch and American beech, diameter inside bark was measured at breast height only with a Pressler's borer.

The branches of the crown were measured on 105 and 226 felled yellow birch and American beech. Each primary and secondary branch with a minimum diameter of 2 cm at the small end was measured outside and inside bark at the mid-point of each one metre length.

METHODS

Bole volume

The first step to construct yellow birch and American beech bole standard volume tables was to express, according to Honer and Alemdag (1972): (a) the diameter inside bark as a function of the diameter outside bark for all trees (standing trees included); and (b) the height of the bole[1] as a function of the total height of each tree.

These functional relationships were solved by linear regressions and their coefficients were determined by the least-squares method.

Thus for (a)

Yellow birch

$$d.i.b. = -0.07689 + 0.94884\ d \qquad (1)$$

$N = 687$ $\qquad Sy.x = 1.38$ cm

American beech

$$d.i.b. = -0.09747 + 0.97124\ d \qquad (2)$$

$N = 188$ $\qquad Sy.x = 0.32$ cm

---

1 Bole height is the distance between soil level and the base of the crown, that is, the height of the stem of the tree without branches. The base of the crown is the position at which the first branches, dead or alive, are found.

where

d.i.b. = diameter inside bark
d = diameter outside bark
N = number of sample trees
Sy.x = standard error of the estimate

and for (b)

Yellow birch

$$h_{(b)} = 0.80701 + 0.49639\ h \qquad (3)$$

N = 491 Sy.x = 1.20 m

American beech

$$h_{(b)} = 1.92030 + 0.42359\ h \qquad (4)$$

N = 548 Sy.x = 1.29 m

where

$h_{(b)}$ = bole height
h = total height of tree
N = number of sample trees
Sy.x = standard error of the estimate

Using the linear regressions (1) and (2) through which the thickness of the bark for sampled standing trees was computed, the true volume of the bole of each individual tree, yellow birch or American beech, outside and inside bark, was calculated by the Golovatchev (1967) formula, as used by Popovich (1972 and 1983).

$$V = \left(\frac{\pi d^2_{0.05}}{4} + \frac{\pi d^2_{0.1}}{8} + \frac{\pi d^2_{0.2}}{4} + \ldots \frac{\pi d^2_{0.9}}{4} \right) \cdot \frac{h_{(b)}}{10} \qquad (5)$$

V = actual volume of bole

$d_{0.05}$; $d_{0.1}$.... $d_{0.9}$ = diameter at 5, 10, ... 90% of the bole height $h_{(b)}$

$h_{(b)}$ = bole height.

The normal bole form quotients $(q_{0\cdot5})^2$ and normal bole form factors $(f_{0\cdot1})^3$ were obtained for each sample tree outside and inside bark, from the following formulas:

a) $$\frac{d^2_{0\cdot5}}{d^2_{0\cdot1}} = q^2_{0\cdot5/0\cdot1} \qquad (6)$$

b) $$f_{0\cdot1} = \frac{q^2_{0\cdot05} + \frac{q^2_{0\cdot1}}{2} + q^2_{0\cdot2} + \ldots q^2_{0\cdot9}}{10} \qquad (7)$$

$d_{0\cdot1}$; $d_{0\cdot5}$; = diameter at 10%; 50% of bole height

$q_{0\cdot05}$; $q_{0\cdot1}$; ..... $q_{0\cdot9}$ = normal bole form quotients of each of the sections at 5, 10, ... 90% of bole height.

Bole volume table construction

Only the necessary elements for constructing volume tables based on the normal form factor are presented. The construction method of bole form tables based on the normal bole form factor was described by Popovich (1972) and Popovich and Forgues (1973).

The three linear regressions needed for the construction of volume tables by this method were calculated for the sample trees of each species, in order to establish the relations existing between dbh and $d_{0\cdot1}$; dbh and $d_{0\cdot5}$; and $q_{0\cdot5}$ $(d_{0\cdot5}/d_{0\cdot1})$ and $f_{0\cdot1}$ (the normal form factor).

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2 The normal bole form quotient is the ratio between diameter at one-half bole or stem height and diameter at one-tenth height of the same bole.

3 The normal bole form factor is defined as the ratio between the volume of a bole or stems and the volume of a cylinder of equal height, with reference diameter taken at one-tenth bole height (Popovich 1972).

These regressions are:

a) Yellow birch inside bark

$$\overline{d_{0 \cdot 1}}\ (\text{i.b.}) = 1.08530 + 0.94238 \cdot \overline{dbh} \qquad (8)$$
N = 491 Sy.x = 1.06 cm

$$\overline{d_{0 \cdot 5}}\ (\text{i.b.}) = 1.37321 + 0.77155 \cdot \overline{dbh} \qquad (9)$$
N = 491 Sy.x = 2.02 cm

$$\overline{f_{0 \cdot 1}}\ (\text{i.b.}) = -0.18616 + 1.07725 \cdot \overline{q_{0 \cdot 5}}\ (\text{i.b.}) \qquad (10)$$
N = 491 Sy.x = 0.0256

b) Yellow birch outside bark

$$\overline{d_{0 \cdot 1}} = 1.02605 + 1.00133 \cdot \overline{dbh} \qquad (11)$$
N = 491 Sy.x = 1.07 cm

$$\overline{d_{0 \cdot 5}} = 1.30088 + 0.82299 \cdot \overline{dbh} \qquad (12)$$
N = 491 Sy.x = 2.11 cm

$$\overline{f_{0 \cdot 1}} = -0.18274 + 1.07392 \cdot \overline{q_{0 \cdot 5}} \qquad (13)$$
N = 491 Sy.x = 0.0253

c) American beech inside bark

$$\overline{d_{0 \cdot 1}}\ (\text{i.b.}) = 1.10860 + 0.95601 \cdot \overline{dbh} \qquad (14)$$
N = 548 Sy.x = 1.67 cm

$$\overline{d_{0 \cdot 5}}\ (\text{i.b.}) = 0.86900 + 0.82766 \cdot \overline{dbh} \qquad (15)$$
N = 548 Sy.x = 1.46 cm

$$\overline{f_{0 \cdot 1}}\ (\text{i.b.}) = -0.20333 + 1.10185 \cdot \overline{q_{0 \cdot 5}}\ (\text{i.b.}) \qquad (16)$$
N = 548 Sy.x = 0.025 cm

d) American beech outside bark

$$\overline{d_{0 \cdot 1}} = 0.64943 + 1.00424 \cdot \overline{dbh} \qquad (17)$$
$$N = 548 \qquad Sy.x = 0.93 \text{ cm}$$

$$\overline{d_{0 \cdot 5}} = 0.97016 + 0.85303 \cdot \overline{dbh} \qquad (18)$$
$$N = 548 \qquad Sy.x = 1.75 \text{ cm}$$

$$\overline{f_{0 \cdot 1}} = -0.20059 + 1.09920 \cdot \overline{q_{0 \cdot 5}} \qquad (19)$$
$$N = 548 \qquad Sy.x = 0.025$$

$\overline{d_{0 \cdot 1}}$ (i.b.) and $\overline{d_{0 \cdot 5}}$ (i.b.) = mean diameter at 10% and 50% of the height of boles inside bark

$\overline{d_{0 \cdot 1}}$ and $\overline{d_{0 \cdot 5}}$ = mean diameter at 10% and 50% of the height of boles outside bark

$\overline{dbh}$ = mean diameter, breast height, outside bark

$\overline{f_{0 \cdot 1}}$ (i.b.) = mean normal bole form factor of boles inside bark

$\overline{f_{0 \cdot 1}}$ = mean normal bole form factor of boles outside bark

$\overline{q_{0 \cdot 5}}$ (i.b.) = mean normal bole form quotient of boles inside bark

$\overline{q_{0 \cdot 5}}$ = mean normal bole form quotient of boles outside bark

N = number of sample trees
Sy.x = standard error of estimate

To obtain the mathematical expressions that give bole volume in cubic metres, the normal bole form quotient ($\overline{q_{0 \cdot 5}}$), the normal bole form

factor ($\overline{f_{0 \cdot 1}}$), the parameter F (the ratio between $\overline{q_{0 \cdot 5}}$ and $\overline{f_{0 \cdot 1}}$), and the parameter A (A = π: 4F) were calculated for each species outside and inside bark.

These mean coefficients and parameters are:

a) Yellow birch

	inside bark	outside bark
$\overline{q_{0 \cdot 5}}$	0.833	0.836
$\overline{f_{0 \cdot 1}}$	0.711	0.715
F	1.172	1.169
A	0.670	0.672

b) American beech

	inside bark	outside bark
$\overline{q_{0 \cdot 5}}$	0.863	0.861
$\overline{f_{0 \cdot 1}}$	0.748	0.746
F	1.154	1.154
A	0.681	0.681

These elements being available, the formulas used for the construction of the bole volume tables (Tables 3, 5, 7, and 9) can be introduced.

Thus, for example, the formula for the yellow birch may be calculated as follows:

$$V = A.\overline{q_{0 \cdot 5}} \cdot (\overline{d_{0 \cdot 1}})^2 \cdot h_{(b)} \qquad (20)$$

$$V = 0.670 \cdot 0.833 \cdot (1.08530 + 0.94238\ dbh)^2 \cdot h_{(b)} \qquad (21)$$

$$V = 0.558\ (1.08530 + 0.94238\ dbh)^2 \cdot h_{(b)} \qquad (22)$$

On solving this last formula (22) the final formula, shown in Table 3, bole volume (m^3) inside bark, of yellow birch, is obtained.

Tree volume

The volume of an entire tree in this study is the sum of the volumes of the bole and the branches of the crown.

$$V = V_{(b)} + V_{(Br.)} \qquad (23)$$

V = volume of total tree
$V_{(b)}$ = bole volume
$V_{(Br.)}$ = volume of the crown branches

The volume of all primary and secondary branches measurable in section lengths of 1 m was calculated by Huber's formula (Husch 1963). The apparent volume of irregular branches was measured. This volume (including bark) was converted into cubic metres taking into account that one stere equals 0.20 m^3 (Baur in Emrovic 1949).

The constant form factor formula (Spurr 1952) was used to estimate the volume of branches for each sample tree.

$$V_{(Br.)} = aD^2H \qquad (24)$$

$V_{(Br.)}$ = volume of branches

D = diameter breast height
H = total height of the tree, and
a = constant

Thus the formulas for estimating the volumes of the branches of yellow birch and of American beech, outside and inside bark, become:

a) Yellow birch

$$V_{(Br.)} = 0.086398 \cdot D^2H \qquad (25)$$

$$V_{(Br.\ outside\ bark)} = 0.104662 \cdot D^2H \qquad (26)$$

b) American beech

$$V_{(Br.)} = 0.117077 \cdot D^2H \qquad (27)$$

$$V_{(Br.\ outside\ bark)} = 0.126773 \cdot D^2H \qquad (28)$$

$V_{(Br.)}$ = volume of branches inside bark

$V_{(Br.\ outside\ bark)}$ = volume of branches outside bark

On adding the bole volume to that of the branches, outside and inside bark, calculated from formulas (25), (26), (27), and (28), the total volumes for 491 yellow birch and 548 American beech were obtained. The accuracy of formulas (25) and (27) were estimated according to Freeze's test (1960). Thus, the chi-squared (χ^2) values expressed as a percentage at the 0.05 probability level are respectively 6.6 percent for yellow birch and 9.8 percent for beech. The estimated values are within plus or minus 6.6 percent and 9.8 percent of the true values in 95 percent of the cases.

Tree volume table construction

Since tree height and diameter (dbh) are the two independent variables used in practically all volume tables (Spurr 1952, Husch 1963), volume, as a dependent variable, was expressed as a function of these two variables. Hence, the same type of equation as formula (24) was used to construct the total tree volume tables, stump and branches included.

ACCURACY

The limits of accuracy for each volume table are given, calculated by Freeze's method (1960). The limits are expressed as a percentage at the 5 percent probability level.

For tables 3, 4, 5, 6, 7, 8, 9, and 10, the blocks indicate the extent of the basic data on which the chi-square tests were performed. For the data used in constructing the tables, the estimates of the volumes in the tables are within plus or minus P percent of the true volume e.g., the estimates of bole cubic volume (m^3) inside bark for yellow birch in the Great Lakes - St. Lawrence Forest Region are within plus or minus 7.7 percent of actual volume 95 percent of the time.

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Appendix 1

Table 1. Distribution of sample yellow birch trees by diameter and height classes

dbh (cm)	Total height (m)																	Total dbh
	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	
15	1		4	15	10	10	3	3	2	2								50
20		2	5	9	9	12	6	5	10	3		1						62
25			2	6	8	16	12	14	7	7	6	3		1				82
30				3	3	11	18	13	13	3	4	4	2	1				75
35				2	4	1	12	15	16	11	7	2	5	2		1		78
40						8	9	11	12	12	8	5	6	3	1			75
45						2	1	4	5	6	4	8	6	7	1	1		45
50							1	2	3	1	3	2			3		2	17
55											1		1	1		1		4
60													1			1		2
65											1							1
Total (height)	1	2	11	35	34	60	62	67	68	45	34	25	21	15	5	4	2	491

Table 2. Distribution of sample American beech trees by diameter and height classes

dbh (cm)	Total height (m)																	Total dbh
	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	
15	2	3	6	9	5	5	2		1	1								34
20			2	7	8	6	9	4	3	7	2	1	1					50
25			1	4	12	15	19	12	10	12	10	6	6	3	2			112
30				1	3	14	13	21	12	14	26	11	9	6	8	2		140
35					1	10	10	16	20	7	13	16	13	12	5	5	1	129
40						1	4	7	10	5	6	10	3	5	4	2		57
45						1	3	1	1	3	3	2	4	2	2	2		24
50											2							2
55																		-
60																		-
65																		-
Total (height)	2	3	9	21	29	52	60	61	57	49	62	46	36	28	21	11	1	548

Table 3. Bole volume (m^3) inside bark for yellow birch

	Total height (m)												
	9	10	11	12	13	14	15	16	17	18	19	20	21
dbh	Height of bole (m)												
(cm)	5.3	5.8	6.3	6.8	7.3	7.8	8.3	8.7	9.2	9.7	10.2	10.7	11.2
15	0.069	0.075	0.081	0.088	0.094	0.101	0.107	0.112	0.119	0.125			
20		0.129	0.140	0.151	0.162	0.173	0.184	0.192	0.204	0.215			
25			0.214	0.230	0.247	0.264	0.281	0.295	0.312	0.329	0.346	0.363	
30				0.327	0.351	0.375	0.399	0.418	0.442	0.466	0.491	0.515	0.539
35				0.440	0.473	0.505	0.538	0.563	0.596	0.628	0.661	0.693	0.725
40						0.655	0.697	0.730	0.772	0.814	0.856	0.898	0.940
45						0.823	0.876	0.918	1.056	0.942	1.077	1.129	1.182
50							1.076	1.128	1.193	1.258	1.323	1.387	1.452
55								1.359	1.437	1.516	1.594	1.672	1.750
60									1.705	1.798	1.890	1.983	2.075
65										2.103	2.212	2.320	2.429

N = 491 Accuracy at the 5% level, + or - 7.7%

$$V_{(b)} = \left(\frac{0.65725 + 1.14141\ (dbh) + 0.49555\ (dbh)^2}{10\ 000} \right) \cdot h_{(b)}$$

Table 4. Total volume (m^3) inside bark for yellow birch
(stumps and crowns included)

dbh (cm)	Total height (m)												
	9	10	11	12	13	14	15	16	17	18	19	20	21
15	0.076	0.084	0.093	0.101	0.109	0.118	0.126	0.135	0.143	0.151			
20		0.150	0.165	0.180	0.195	0.209	0.224	0.239	0.254	0.269			
25			0.257	0.281	0.304	0.327	0.351	0.374	0.397	0.421	0.444	0.468	
30				0.404	0.438	0.471	0.505	0.539	0.572	0.606	0.640	0.673	0.707
35				0.550	0.596	0.642	0.687	0.733	0.779	0.825	0.871	0.916	0.962
40						0.839	0.898	0.958	1.017	1.077	1.137	1.197	1.257
45						1.060	1.136	1.212	1.288	1.363	1.439	1.515	1.591
50							1.403	1.496	1.590	1.683	1.777	1.870	1.964
55								1.810	1.924	2.037	2.150	2.263	2.376
60									2.289	2.424	2.559	2.693	2.828
65										2.845	3.003	3.161	3.319

N = 491 Accuracy at the 5% level, + or - 8.8%

$$V = 0.37407 \cdot \left(\frac{dbh}{100}\right)^2 \cdot h$$

Table 5. Bole volume (m^3) outside bark for yellow birch

	Total height (m)												
	9	10	11	12	13	14	15	16	17	18	19	20	21
dbh	Height of bole (m)												
(cm)	5.3	5.8	6.3	6.8	7.3	7.8	8.3	8.7	9.2	9.7	10.2	10.7	11.2
15	0.077	0.084	0.091	0.098	0.106	0.113	0.120	0.126	0.133	0.140			
20		0.144	0.157	0.169	0.182	0.194	0.207	0.217	0.229	0.242			
25			0.240	0.260	0.279	0.298	0.317	0.332	0.351	0.370	0.389	0.408	
30				0.369	0.396	0.423	0.450	0.472	0.499	0.526	0.553	0.580	0.607
35				0.497	0.534	0.570	0.607	0.636	0.673	0.709	0.746	0.782	0.819
40						0.740	0.787	0.825	0.873	0.920	0.967	1.015	1.062
45						0.931	0.991	1.038	1.098	1.158	1.218	1.277	1.337
50							1.218	1.276	1.350	1.423	1.496	1.570	1.643
55								1.539	1.627	1.716	1.804	1.893	1.981
60									1.931	2.036	2.140	2.245	2.350
65										2.383	2.506	2.628	2.751

N = 491 Accuracy at the 5% level, + or - 7.4%

$$V_{(b)} = \left(\frac{0.59166 + 1.15481\ (dbh) + 0.56350\ (dbh)^2}{10\ 000} \right) \cdot h_{(b)}$$

Table 6. Total volume (m^3) outside bark for yellow birch
(stumps and crowns included)

dbh	Total height (m)												
(cm)	9	10	11	12	13	14	15	16	17	18	19	20	21
15	0.086	0.095	0.105	0.114	0.124	0.133	0.143	0.152	0.162	0.171			
20		0.169	0.186	0.203	0.220	0.237	0.254	0.271	0.288	0.305			
25			0.291	0.317	0.344	0.370	0.396	0.423	0.449	0.476	0.502	0.529	0.555
30				0.457	0.495	0.533	0.571	0.609	0.647	0.685	0.723	0.761	0.799
35				0.622	0.674	0.725	0.777	0.829	0.881	0.933	0.984	1.036	1.088
40					0.880	0.947	1.015	1.083	1.150	1.218	1.286	1.353	1.421
45						1.199	1.285	1.370	1.456	1.542	1.627	1.713	1.799
50							1.586	1.692	1.797	1.903	2.009	2.115	2.220
55								2.047	2.175	2.303	2.431	2.559	2.687
60								2.436	2.588	2.741	2.893	3.045	3.197
65								2.859	3.038	3.216	3.395	3.574	3.752

N = 491 Accuracy at 5% level, + or - 8.7%

$$V = 0.42293 \cdot \left(\frac{dbh}{100}\right)^2 \cdot h$$

Table 7. Bole volume (m^3) inside bark for American beech

	Total height (m)												
	12	13	14	15	16	17	18	19	20	21	22	23	24
dbh	Height of bole (m)												
(cm)	7.0	7.4	7.9	8.3	8.7	9.1	9.5	10.0	10.4	10.8	11.2	11.7	12.1
15	0.098	0.104	0.111	0.116	0.122	0.128	0.133						
20	0.168	0.178	0.190	0.200	0.209	0.219	0,229	0.241	0.250	0.260			
25	0.257	0.272	0.291	0.305	0.320	0.335	0.349	0.368	0.382	0.397	0.412	0.430	0.445
30		0.386	0.412	0.433	0.454	0.475	0.496	0.522	0.543	0.564	0.584	0.610	0.631
35			0.548	0.583	0.611	0.639	0.668	0.703	0.731	0.759	0.787	0.822	0.850
40				0.756	0.792	0.828	0.865	0.910	0.947	0.983	1.020	1.065	1.102
45				0.950	0.996	1.042	1.088	1.145	1.191	1.237	1.282	1.340	1.386
50					1.224	1.280	1.336	1.407	1.463	1.519	1.575	1.646	1.702
55						1.542	1.610	1.695	1.763	1.831	1.898	1.983	2.051
60							1.910	2.010	2.091	2.171	2.251	2.352	2.432
65							2.235	2.352	2.446	2.540	2.635	2.752	2.846

N = 548 Accuracy at 5% level, + or - 5.9%

$$V_{(b)} = \left(\frac{0.72265 + 1.24636\ (dbh) + 0.53741\ (dbh)^2}{10\ 000} \right) \cdot h_{(b)}$$

Table 8. Total volume (m^3) inside bark for American beech
(stumps and crowns included)

dbh (cm)	Total height (m)												
	12	13	14	15	16	17	18	19	20	21	22	23	24
15	0.114	0.124	0.133	0.143	0.152	0.162	0.171						
20	0.203	0.220	0.236	0.253	0.270	0.287	0.304	0.321	0.338	0.355			
25	0.317	0.343	0.370	0.396	0.422	0.449	0.475	0.501	0.528	0.554	0.581	0.607	0.633
30	0.456	0.494	0.532	0.570	0.608	0.646	0.684	0.722	0.760	0.798	0.836	0.874	0.912
35			0.724	0.776	0.828	0.879	0.931	0.983	1.035	1.086	1.138	1.190	1.242
40				1.014	1.081	1.149	1.216	1.284	1.351	1.419	1.487	1.554	1.622
45				1.283	1.368	1.454	1.539	1.625	1.710	1.796	1.881	1.967	2.052
50					1.689	1.795	1.900	2.006	2.112	2.217	2.323	2.428	2.534
55						2.172	2.299	2.427	2.555	2.683	2.810	2.938	3.066
60							2.737	2.889	3.041	3.193	3.345	3.497	3.649
65							3.212	3.390	3.569	3.747	3.925	4.104	4.282

N = 548 Accuracy at 5% level, + or - 9.8%

$$V = 0.42231 \cdot \left(\frac{dbh}{100}\right)^2 \cdot h$$

Table 9. Bole volume (m^3) outside bark for American beech

	Total height (m)												
	12	13	14	15	16	17	18	19	20	21	22	23	24
dbh	Height of bole (m)												
(cm)	7.0	7.4	7.9	8.3	8.7	9.1	9.5	10.0	10.4	10.8	11.2	11.7	12.1
15	0.101	0.107	0.114	0.120	0.126	0.132	0.137	0.145	0.150	0.156	0.162		
20	0.176	0.186	0.199	0.209	0.219	0.229	0.239	0.252	0.262	0.272	0.282	0.295	
25	0.272	0.288	0.307	0.323	0.338	0.354	0.369	0.389	0.404	0.420	0.435	0.455	0.470
30	0.389	0.411	0.438	0.461	0.483	0.505	0.527	0.555	0.577	0.599	0.622	0.649	0.672
35		0.556	0.593	0.623	0.653	0.683	0.713	0.751	0.781	0.811	0.841	0.879	0.909
40		0.723	0.771	0.810	0.849	0.889	0.928	0.976	1.015	1.055	1.094	1.142	1.181
45			0.973	1.022	1.071	1.121	1.170	1.231	1.281	1.330	1.379	1.441	1.490
50			1.198	1.258	1.319	1.379	1.440	1.516	1.577	1.637	1.698	1.774	1.834
55						1.665	1.739	1.830	1.903	1.976	2.050	2.141	2.214
60							2.065	2.174	2.261	2.348	2.434	2.543	2.630
65							2.419	2.547	2.649	2.751	2.852	2.980	3.082

N = 548 Accuracy at the 5% level, + or - 7.2%

$$V_{(b)} = \left(\frac{0.24716 + 0.76436\ (dbh) + 0.59098\ (dbh)^2}{10\ 000} \right) \cdot h_{(b)}$$

Table 10. Total volume (m^3) outside bark for American beech (stumps and crowns included)

dbh (cm)	Total height (m)												
	12	13	14	15	16	17	18	19	20	21	22	23	24
15	0.122	0.132	0.142	0.152	0.162	0.172	0.182	0.192	0.203				
20	0.216	0.234	0.252	0.270	0.288	0.306	0.324	0.342	0.360	0.378			
25	0.338	0.366	0.394	0.422	0.450	0.479	0.507	0.535	0.563	0.591	0.619	0.647	0.676
30	0.486	0.527	0.567	0.608	0.649	0.689	0.730	0.770	0.811	0.851	0.892	0.932	0.973
35		0.717	0.772	0.828	0.883	0.938	0.993	1.048	1.103	1.159	1.214	1.269	1.324
40		0.937	1.009	1.081	1.153	1.225	1.297	1.369	1.441	1.513	1.585	1.657	1.729
45			1.277	1.368	1.459	1.550	1.642	1.733	1.824	1.915	2.006	2.098	2.189
50				1.689	1.802	1.914	2.027	2.139	2.252	2.365	2.477	2.590	2.702
55						2.316	2.452	2.589	2.725	2.861	2.997	3.134	3.270
60							2.919	3.081	3.243	3.405	3.567	3.729	3.891
65							3.425	3.616	3.806	3.996	4.186	4.377	4.567

N = 548 Accuracy at the 5% level, + or - 8.8%

$$V = 0.45039 \cdot \left(\frac{dbh}{100}\right)^2 \cdot h$$

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