1987 SIN87-A

PREDICTION ERROR IN TREE BIOMASS REGRESSION FUNCTIONS FOR WESTERN CANADA

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T6H 3S5

Abstract

Biomass equations were derived for eight commercial tree species in the Boreal Forest Region of western Canada, using two regression functions based on diameter, and a combination of diameter and height. The model based on diameter and height regression function gave excellent predictions when validated using independent data; R^2 obtained for the predictions based on this regression function for regional and Alberta applications were 0.99 and 0.97 for trembling aspen, and balsam fir had R^2 values of 0.96 for both the regional and Alberta applications.

Introduction

Biomass equations are increasingly being used for deriving forest biomass tables and inventories. Individual species equations are available also to provide biomass information pertaining to all or part of a tree as a renewable energy source. Extensive biomass data have been accumulated in North America over recent years for most of the forest tree species, however, little attention has been given to assessment of estimation errors and bias inherent in the use of biomass equations for such predictions.

Estimation errors can occur due to many sources. Sampling methods used for collection of data, choice of a regression function on which the prediction equation is to be based, and the lack of fit contribute to most of these errors. The precision and accuracy of prediction equations need to be known to avoid misleading interpretation.

A study was undertaken in the Boreal Forest Region of western Canada to determine and compare errors of estimation when prediction equations derived from two different regression functions are used to obtain biomass information. The objectives of the study were as follows:

1. Determine the accuracy of prediction equations as derived from individual prairie provinces (Alberta, Saskatchewan, and Manitoba) and the combined provinces (regional) data sets.

2. Compare the errors of prediction when equations from an individual province are used in other prairie provinces.

3. Assess the accuracy and bias inherent in the use of prediction equations based on two different regression functions when validated using an independent data set in Alberta.

Methods

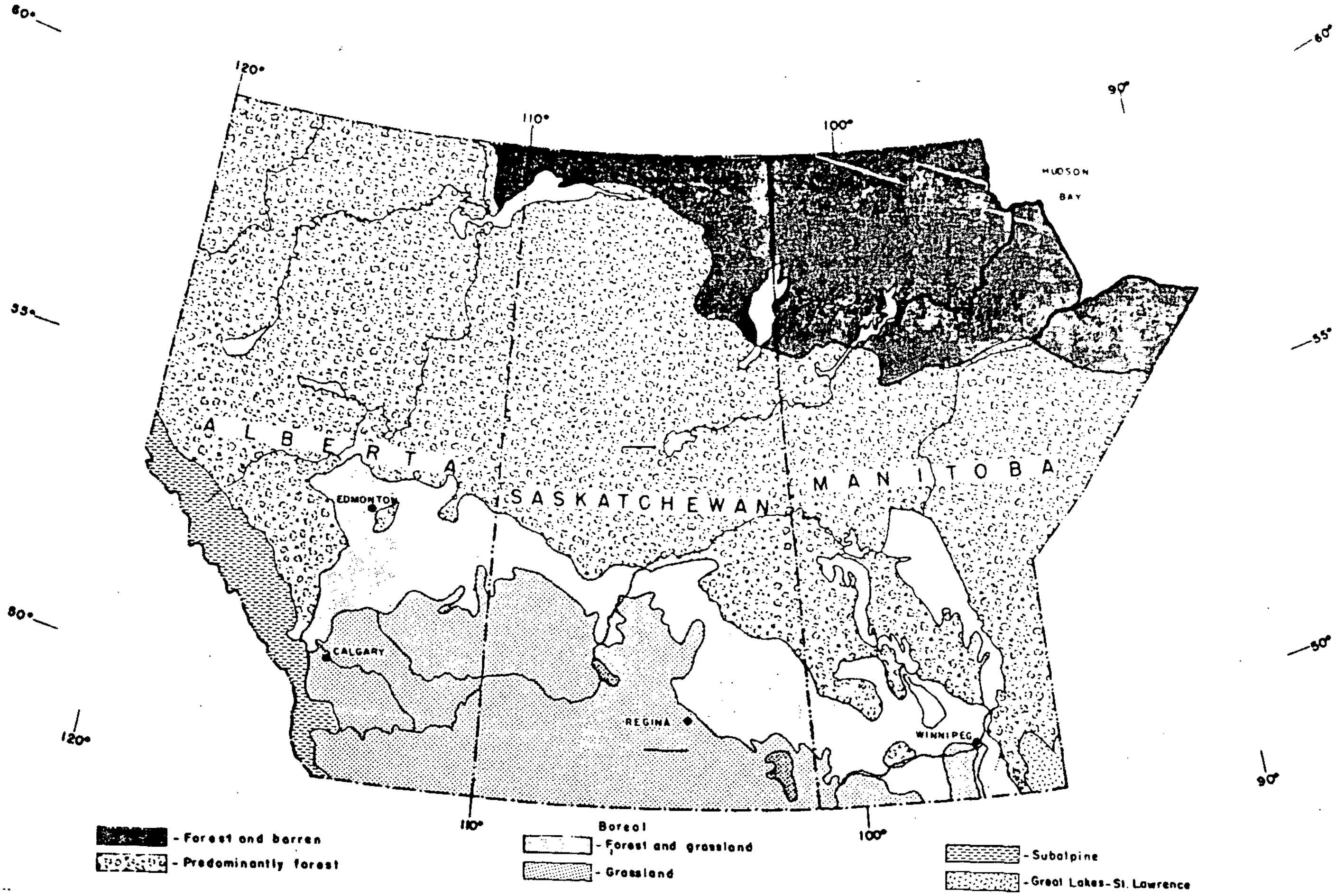
Field and Laboratory Procedures

The tree species tested in the study were five softwoods and three hardwoods. The softwoods were jack pine (*Pinus banksiana* Lamb.), black spruce (*Picea mariana* (Mill.) BSP), white spruce (*Picea glauca* (Moench) Voss), balsam fir (*Abies balsamea* (L.) Mill.), and tamarack (*Larix laricina* (du Roi) K. Koch). The three hardwoods were trembling aspen (*Populus tremuloides* Michx.), balsam poplar (*P. balsamifera* Marsh.), and white birch (*Betula papyrifera* Marsh.).

Twenty trees of each species were sampled in each of the three prairie provinces (Alberta, Saskatchewan, and Manitoba), for a total of 60 trees per species on a regional basis for the predominantly Boreal Forest Region in western Canada (Fig. 1). The field information recorded included diameter at breast height outside bark (dbhob) at 1.3 m (D), and total height of the tree (H). Sampling was done according to diameter size, by felling five trees per species per province for each of the four diameter classes (0-11, 11-20, 21-30, and 31^+ cm).

After felling the trees, merchantable (dbhob 10 cm or greater) and nonmerchantable (dbhob less than 10 cm) sections of the tree stem were marked. The merchantable stem was cut into four and the nonmerchantable stem into three equal subsections. Length and dbhob measurements were made for each of the subsections, and the tree stem was cut and weighed for fresh weights of individual components. These weights were also determined for the remaining aboveground parts of the felled tree. Subsamples consisting of 1-cm thick disks were obtained for the tree stem sections, and representative subsamples were similarly taken for foliage and live branches. Debarking to separate wood and bark was done for all subsamples, except for live small branches. Ovendry weights were determined after all subsamples were dried at 103°C for 24 hours or until a constant weight was attained. Details on field and laboratory procedures are provided by Singh (1982).

Figure 1. Boreal Forest Region in the prairie provinces of Alberta, Saskatchewan, and Manitoba.



Computer Procedures

Dry/fresh weight and bark/wood dry weight ratios were computed from the volume-weighted data. The

the Smalion formula:

$$V = L(A_t + A_b)/2$$

where L = length of section,

A_t = cross-sectional area at top, and

A_b = cross-sectional area at bottom.

The cross-sectional areas were computed from dbhob measurements. Total volume of the tree stem was obtained by summing the volumes of all component stem sections. Computer subroutines were written in FORTRAN for biomass computations based on field and laboratory data collected in the study (Singh and Campbell 1983).

Regression Functions and Analysis

Two regression functions were used, one based on a combination of D and H, and the other based on D only:

Model I: $W = a_0 + a_1 D^2 H$

Model II: $W = a_0 + a_1 D + a_2 D^2 + a_3 D^3$

where W = ovendry biomass of live tree aboveground, and a_0, a_1, ... , a_3 are regression coefficients.

Mean, standard deviation, R^2, SEE (standard error of estimate), % SEE (standard error expressed as percentage of the mean), and residuals (difference between the actual and the fitted value) were calculated for the derived versus the applied regression equations.

Comparisons were made for testing the accuracy of the individual province and regional prediction equations using the data collected for the study and an independent data set available for two hardwood species in Alberta.

Results and Discussions

The R^2 values for the individual province equations for the eight tree species ranged from 0.883 (white birch) to 0.997 (trembling aspen) for Model I, and from 0.934 to 0.997 for Model II (Appendix I). Both species for the highest and the lowest R^2 values were sampled in Saskatchewan. The corresponding range of % SEE values was 35.2 to 5.3, and 28.0 to 6.4, respectively. The regional equations had a R^2 range of 0.943 to 0.988, with the same species in Saskatchewan making up the highest and the lowest ranking. Black spruce (Model I) also showed the highest fit in Alberta (R^2 = 0.997, % SEE = 5.1).

The regional equations based on the combined sample of 60 trees per species showed that the R^2 ranged from 0.943 to 0.988 for Model I, and from 0.956 to 0.977 for Model II (Appendix II). The corresponding % SEE values ranged from 24.2 to 11.2, and from 21.8 to 15.8, respectively. White birch had the lowest and trembling aspen the highest fit in this case as well. The goodness of fit and the associated errors seemed to have averaged out when the three data subsets from the prairie provinces were combined into a single regional set for deriving biomass prediction equations for each species.

The prediction and errors involved in the individual province equations when used in each province are shown in Table 1. A test of the prairies regional equations on the individual province data for each species showed that the provincial equations were slightly better than the regional equations in predicting the actual biomass values (Table 2).

Table 1. Regional equations applied to estimation of ovendry biomass of eight tree species in three individual provinces[a]

Species	Province	Actual Mean	Predicted Mean Model I	Predicted Mean Model II	Predicted R^2 Model I	Predicted R^2 Model II	% SEE Model I	% SEE Model II	Mean Residuals Model I	Mean Residuals Model I
Softwoods:										
Jack pine	Alberta	157.5	162.9	174.7	0.97	0.95	17.8	23.0	-5.4	-17.1
	Saskatchewan	189.1	192.5	178.8	0.98	0.97	12.8	18.0	-3.4	10.3
	Manitoba	197.9	189.0	190.8	0.97	0.97	17.0	17.6	8.9	7.0
Black spruce	Alberta	177.5	182.8	180.5	0.99	0.99	7.5	9.2	-5.4	-3.0
	Saskatchewan	181.6	178.5	180.0	0.997	0.996	5.6	6.7	3.1	1.6
	Manitoba	198.0	195.8	196.6	0.96	0.97	20.0	19.2	2.2	1.4
White spruce	Alberta	220.3	215.5	215.0	0.99	0.99	13.6	13.5	4.7	5.3
	Saskatchewan	183.1	190.1	188.4	0.98	0.97	13.8	18.6	-7.0	-5.3
	Manitoba	212.1	209.7	211.9	0.99	0.99	12.2	12.1	2.3	0.2
Balsam fir	Alberta	191.3	205.8	187.7	0.99	0.99	11.0	12.0	-14.4	3.6
	Saskatchewan	181.5	190.8	180.9	0.97	0.96	18.6	20.5	-9.3	0.6
	Manitoba	161.4	137.6	165.5	0.88	0.98	34.8	14.7	23.8	-4.1
Tamarack	Alberta	157.7	158.8	155.3	0.99	0.99	11.2	10.5	-1.1	2.4
	Saskatchewan	156.5	155.0	159.7	0.97	0.96	15.2	19.8	1.6	-3.2
	Manitoba	158.5	159.0	157.9	0.99	0.99	11.4	11.1	-0.6	0.6
Hardwoods:										
Trembling aspen	Alberta	220.2	233.6	208.2	0.99	0.97	12.6	20.5	-13.4	12.0
	Saskatchewan	195.9	189.2	203.8	0.99	0.99	7.6	9.0	6.7	-7.9
	Manitoba	212.7	206.1	216.9	0.98	0.97	13.1	18.2	6.6	-4.3
Balsam poplar	Alberta	165.5	173.8	161.8	0.97	0.97	17.7	17.8	-8.4	3.7
	Saskatchewan	141.8	142.0	146.2	0.97	0.97	16.7	18.2	-0.2	-4.4
	Manitoba	145.7	137.2	145.1	0.95	0.97	21.4	16.6	8.5	0.6
White birch	Alberta	294.9	288.0	281.6	0.98	0.99	16.1	14.0	6.9	13.4
	Saskatchewan	226.9	235.8	229.6	0.87	0.90	37.3	35.1	-8.9	-2.8
	Manitoba	234.0	231.9	244.9	0.95	0.96	22.6	21.4	2.1	-10.9

a/ Model I: $W = a_0 + a_1 D^2H$

Model II: $W = a_0 + a_1 D + a_2 D^2 + a_3 D^3$

where W is the ovendry weight (kg) of living tree above ground without dead branches, D is diameter (cm outside bark at breast height, H is total tree height (m), $a_1 \ldots a_3$ are regression coefficients, Residual is actual minus predicted, and % SEE is standard error of estimate expressed as percentage of the mean.

b/ Calculated from actual measurements from species sampled in Alberta, Saskatchewan, and Manitoba.

Table 2. Comparative estimates and errors in prediction when prairies general equations and individual province equations based on two regression functions are used to predict ovendry biomass of eight species in each province[a]

Species	Equation used for prediction	Alberta Predicted Mean	Alberta R^2	Alberta % SEE	Saskatchewan Predicted Mean	Saskatchewan R^2	Saskatchewan % SEE	Manitoba Predicted Mean	Manitoba R^2	Manitoba % SEE
Model I:										
Softwoods										
Jack pine	Regional	162.94	0.97	17.2	192.48	0.98	12.5	188.95	0.97	17.8
	Provincial	157.52	0.97	16.8	189.08	0.98	12.3	197.89	0.98	15.3
Black spruce	Regional	182.83	0.99	7.2	178.51	0.997	5.7	195.81	0.96	20.2
	Provincial	177.48	0.997	5.8	181.61	0.997	5.1	197.97	0.96	19.9
White spruce	Regional	215.53	0.99	14.0	190.06	0.98	13.3	209.74	0.98	12.4
	Provincial	220.29	0.99	13.1	183.00	0.98	12.1	212.12	0.99	12.2
Balsam fir	Regional	205.77	0.99	10.2	190.83	0.97	17.7	137.55	0.88	40.9
	Provincial	191.31	0.99	7.4	181.50	0.97	16.9	161.39	0.95	23.3
Tamarack	Regional	158.76	0.99	11.1	154.96	0.97	15.3	159.03	0.99	11.3
	Provincial	157.68	0.99	10.8	156.52	0.98	14.3	158.50	0.99	11.2
Hardwoods										
Trembling aspen	Regional	233.60	0.99	11.9	189.18	0.99	7.8	206.08	0.98	13.5
	Provincial	220.20	0.99	10.7	195.90	0.997	5.3	212.63	0.98	12.7
Balsam poplar	Regional	173.84	0.97	16.9	141.97	0.97	16.7	137.16	0.95	22.7
	Provincial	165.43	0.98	16.3	141.78	0.97	16.7	145.68	0.97	17.8
White birch	Regional	288.01	0.98	16.5	235.78	0.87	35.9	231.92	0.95	22.8
	Provincial	294.93	0.98	15.6	226.90	0.88	35.2	233.99	0.95	21.8
Model II:										
Softwoods										
Jack pine	Regional	174.66	0.95	20.7	178.75	0.97	19.1	190.84	0.97	18.3
	Provincial	157.53	0.98	14.5	189.08	0.98	15.5	197.92	0.97	16.8
Black spruce	Regional	180.52	0.99	9.0	180.02	0.996	6.7	196.62	0.97	19.4
	Provincial	177.50	0.995	7.7	181.67	0.99	6.5	197.98	0.97	18.3
White spruce	Regional	215.02	0.99	13.8	188.39	0.97	18.1	211.89	0.99	12.1
	Provincial	220.27	0.99	10.5	183.14	0.97	17.7	212.10	0.99	10.6
Balsam fir	Regional	187.73	0.99	12.2	180.90	0.96	20.5	165.47	0.98	14.3
	Provincial	191.33	0.99	8.0	181.44	0.97	18.7	161.34	0.98	14.2
Tamarack	Regional	155.30	0.99	10.7	159.72	0.96	19.4	157.91	0.99	11.1
	Provincial	157.76	0.99	10.1	156.54	0.96	19.6	158.46	0.99	10.3
Hardwoods										
Trembling aspen	Regional	208.16	0.97	21.7	203.75	0.99	8.7	216.95	0.97	17.9
	Provincial	220.18	0.98	16.2	195.87	0.997	6.4	212.75	0.98	15.4
Balsam poplar	Regional	161.78	0.97	18.2	146.20	0.97	17.6	145.08	0.97	16.6
	Provincial	165.55	0.97	17.5	141.86	0.98	15.4	145.67	0.98	15.2
White birch	Regional	281.55	0.99	14.7	229.65	0.90	34.7	244.90	0.96	20.5
	Provincial	294.87	0.99	11.7	226.86	0.93	28.1	234.00	0.96	20.4

a/ Model I: $W = a_0 + a_1 D^2H$

Model II: $W = a_0 + a_1 D + a_2 D^2 + a_3 D^3$

where W is ovendry weight (kg) of living tree above ground (excluding dead branches), D is diameter outside bark at breast height (cm), and H is total tree height (m). Mean is predicted mean, and % SEE is standard error of estimate expressed as a percentage of predicted mean.

It is natural to expect that the individual province equations should fit the input data of the same province better than those derived elsewhere. A test was therefore made to see how good the fit for each species was when the equations derived from one province were applied to the independent data sets from the other two provinces Table 3 shows the results of Alberta equations applied to Saskatchewan and Manitoba. In Saskatchewan, the prediction equations based on Model I showed that all species had R^2 for the estimated values of 0.97 or greater, with the exception of white birch (0.86). For Model II, all species had prediction R^2 values of 0.92 or greater, with the exception of white birch (0.88). In Manitoba, the corresponding Model I values were 0.94 or greater, with the exception of balsam fir (0.84); and the Model II prediction R^2 values were 0.92 or higher for all species. The highest fit (R^2 = 0.99 and % SEE = 8.3, as averaged for both models) for the use of Alberta equations in Saskatchewan and Manitoba was for black spruce in Saskatchewan.

The results of Saskatchewan equations applied to the independent data sets for Alberta and Manitoba are shown in Table 4. In Alberta, the R^2 for the estimated values ranged from 0.96 to 0.99 for all species for Model I, and from 0.93 to 0.99 with the exception of white spruce (0.87) for Model II. The corresponding values in Manitoba were as follows: Model I: 0.93 to 0.98, with the exception of balsam fir (0.85); and Model II: 0.93 to 0.99 for all species. Black spruce in Alberta showed the best predictions averaged for the two models (R^2 = 0.99, % SEE = 9.1).

Manitoba equations applied to prediction in other provinces showed results as listed in Table 5. In Alberta, Model I prediction R^2 values ranged from 0.93 to 0.99 for all species, with the exception of balsam fir (0.82). The Model II values ranged from 0.93 to 0.98 for all species. For Saskatchewan, Model I had R^2 values of 0.95 to 0.997 for all species, with the exception of balsam fir (0.79) and white birch (0.85). The corresponding values for Model II ranged from 0.94 to 0.99 for all species, with the exception of white birch (0.88). On the basis of the average taken for both models, trembling aspen in Saskatchewan gave the best predictions (R^2 = 0.99, % SEE = 9.7).

There are two obvious inferences from the error analysis presented in Tables 3 to 5: (1) with the exception of balsam fir and white birch in most cases, the equations from one province applied to another province yielded nearly as good results as the equations for the specific province; and (2) the species with the lowest fit in all such applications over the three individual provinces was balsam fir, based on regression function of Model I. A test on the identity of individual province equations using regression functions of Models I and II showed that the Model I predictions equations were highly significantly different ($P < 0.01$) for both slope and intercept (Singh 1986). There was no such combined difference, at this probability level, for any of the remaining seven species for either model. It was therefore concluded that: (1) selection of a suitable regression function was an important consideration in deriving prediction equations, and (2) most of the separate equations could be combined into single biomass prediction equations for regional application over the Boreal Forest in western Canada.

Fresh data collection for validation purposes is a costly undertaking. A search for biomass data showed that information required for independent testing was available for two hardwoods (trembling aspen and balsam poplar), as reported by Johnstone and Peterson (1980) for six different locations in Alberta. Four of these locations were in the Boreal Forest Region and one each in the Montane Forest Region and the Forest-grassland transition (Rowe 1972). The errors involved in predicting known population parameters for the 254 trees of trembling aspen and 60 trees of balsam poplar are shown in Table 6. It is evident that the regional equations tested in the study showed better or as good a prediction for trembling aspen as the predictions obtained from the individual province (Alberta) equations. The accuracy and the errors for regional vs. Alberta equations in the case of trembling aspen were: R^2 = 0.99 vs. 0.97 (Model I), and 0.95 (Model I and Model II); % SEE = 15.4 vs. 22.2 (Model I), and 30.2 vs. 29.1 (Model II). For balsam poplar, the predictions for regional vs. Alberta equations were almost identical: R^2 = 0.96 vs. 0.96 (Model I), and 0.95 vs. 0.96 (Model II); % SEE = 23.3 vs. 24.8 (Model I), and 28.5 vs. 26.0 (Model II).

The Model I regression function gave better biomass estimates compared to the Model II regression function. For trembling aspen, Model I provided R^2 values of 0.99 vs. 0.95 and % SEE as 15.4 vs. 30.2 for regional equation, and R^2 values of 0.97 vs. 0.95 and % SEE as 22.2 vs. 29.1 for the Alberta equation. The Model I regression function also gave slightly better results for the validation data predictions of balsam poplar, mainly because the regression function of Model I is based on two measurements (D and H), rather than on a single measurement (D), as is the Model II regression function.

Table 3. Alberta equations applied to estimation of ovendry biomass of tree species in other prairie provinces[a]

Species	Province	Actual Mean[b]	Predicted Mean Model I	Predicted Mean Model II	Predicted R^2 Model I	Predicted R^2 Model II	% SEE Model I	% SEE Model II	Mean Residuals Model I	Mean Residuals Model II
Softwoods:										
Jack pine	Saskatchewan	189.1	185.8	163.1	0.98	0.92	12.5	29.4	3.3	26.0
	Manitoba	197.9	182.4	174.0	0.96	0.97	19.9	25.5	5.5	23.9
Black spruce	Saskatchewan	181.6	173.3	175.6	0.99	0.99	8.4	8.1	8.3	6.0
	Manitoba	198.0	190.0	194.3	0.95	0.96	20.9	20.8	8.0	3.7
White spruce	Saskatchewan	183.1	194.1	202.2	0.97	0.94	15.9	24.9	-11.0	-19.2
	Manitoba	212.1	214.3	223.8	0.99	0.97	12.7	18.1	-2.3	-11.8
Balsam fir	Saskatchewan	181.5	176.6	183.4	0.97	0.94	17.6	26.0	4.9	-1.9
	Manitoba	161.4	124.1	166.4	0.84	0.98	40.2	16.6	37.3	-5.0
Tamarack	Saskatchewan	156.5	154.0	162.5	0.97	0.96	16.2	20.3	2.6	-6.0
	Manitoba	158.5	157.9	160.7	0.99	0.99	11.2	11.8	0.5	-2.2
Hardwoods:										
Trembling aspen	Saskatchewan	195.9	176.3	214.3	0.98	0.97	12.9	18.0	19.6	-18.4
	Manitoba	212.7	193.0	231.4	0.98	0.92	16.0	30.1	19.7	-18.7
Balsam poplar	Saskatchewan	141.8	134.9	149.7	0.97	0.97	17.7	19.3	7.0	-7.9
	Manitoba	145.7	130.2	148.4	0.94	0.97	25.0	16.8	15.5	-2.7
White birch	Saskatchewan	226.9	241.0	237.0	0.86	0.88	38.8	37.9	-14.1	-10.1
	Manitoba	234.0	237.0	252.4	0.95	0.95	22.0	22.8	-3.0	-18.4

a/ Model I: $W = a_0 + a_1 D^2H$
Model II: $W = a_0 + a_1 D + a_2 D^2 + a_3 D^3$
where W is the ovendry weight (kg) of living tree above ground without dead branches, D is diameter (cm) outside bark at breast height, H is total tree height (m), and $a_1 \ldots a_3$ are regression coefficients; % SEE is standard error of estimate expressed as percentage of the actual mean.
b/ Calculated from actual measurements from species sampled in Alberta, Saskatchewan, and Manitoba.

Table 4. Saskatchewan equations applied to estimation of ovendry biomass of tree species in other prairie provinces[a]

Species	Province	Actual Mean[b]	Predicted Mean Model I	Predicted Mean Model II	Predicted R^2 Model I	Predicted R^2 Model II	% SEE Model I	% SEE Model II	Mean Residuals Model I	Mean Residuals Model II
Softwoods:										
Jack pine	Alberta	157.5	160.4	186.6	0.97	0.91	17.0	31.7	-2.9	-29.0
	Manitoba	197.9	185.7	202.4	0.97	0.97	18.6	17.3	12.2	-4.6
Black spruce	Alberta	177.5	186.0	182.8	0.99	0.99	9.2	9.0	-8.5	-5.3
	Manitoba	198.0	199.2	199.1	0.96	0.96	19.9	19.6	-1.1	-1.1
White spruce	Alberta	220.3	207.1	210.2	0.98	0.98	17.1	15.3	13.1	10.0
	Manitoba	212.1	201.6	206.1	0.98	0.98	14.2	14.0	10.4	6.0
Balsam fir	Alberta	191.3	195.7	187.8	0.99	0.97	8.7	18.4	-4.3	3.5
	Manitoba	161.4	131.0	167.7	0.85	0.98	39.4	15.6	30.4	-6.3
Tamarack	Alberta	157.7	160.5	152.1	0.98	0.99	13.7	11.2	-2.8	5.6
	Manitoba	158.5	160.8	154.7	0.98	0.99	13.4	11.7	-2.4	3.8
Hardwoods:										
Trembling aspen	Alberta	220.2	242.1	201.3	0.98	0.96	16.1	22.4	-21.9	18.9
	Manitoba	212.7	213.5	210.9	0.98	0.97	13.6	19.3	-0.8	1.8
Balsam poplar	Alberta	165.5	173.2	153.5	0.97	0.93	17.3	28.9	-7.8	11.9
	Manitoba	145.7	137.0	143.0	0.95	0.97	21.9	19.2	8.7	2.7
White birch	Alberta	294.9	273.6	243.4	0.96	0.87	22.4	41.4	21.3	51.5
	Manitoba	234.0	223.4	243.2	0.93	0.93	27.2	28.4	10.5	-9.2

a/ Model I: $W = a_0 + a_1 D^2H$
Model II: $W = a_0 + a_1 D + a_2 D^2 + a_3 D^3$
where W is the ovendry weight (kg) of living tree above ground without dead branches, D is diameter (cm) outside bark at breast height, H is total tree height (m), and $a_1 \ldots a_3$ are regression coefficients; % SEE is standard error of estimate expressed as percentage of the actual mean.
b/ Calculated from actual measurements from species sampled in Alberta, Saskatchewan, and Manitoba.

Table 5. Manitoba equations applied to estimation of ovendry biomass of tree species in other prairie provinces[a]

Species	Province	Actual Mean[b]	Predicted Mean Model I	Predicted Mean Model II	Predicted R^2 Model I	Predicted R^2 Model II	% SEE Model I	% SEE Model II	Mean Residuals Model I	Mean Residuals Model II
Softwoods:										
Jack pine	Alberta	157.5	170.3	181.7	0.95	0.93	21.7	28.4	-12.8	-24.2
	Saskatchewan	189.1	201.6	185.8	0.97	0.98	16.8	16.0	-12.5	3.3
Black spruce	Alberta	177.5	184.7	182.6	0.99	0.98	9.2	14.7	-7.3	-5.2
	Saskatchewan	181.6	180.3	183.8	0.997	0.99	5.2	10.9	1.3	-2.2
White spruce	Alberta	220.3	217.9	218.2	0.99	0.98	13.6	17.1	2.4	2.1
	Saskatchewan	183.1	192.5	186.7	0.98	0.96	14.2	19.0	-9.4	-3.6
Balsam fir	Alberta	191.3	247.8	182.5	0.82	0.98	43.3	15.4	-56.4	8.8
	Saskatchewan	181.5	228.8	176.1	0.79	0.97	46.5	19.8	-47.3	5.5
Tamarack	Alberta	157.7	158.2	156.4	0.99	0.98	10.8	13.6	-0.5	1.3
	Saskatchewan	156.5	154.5	160.2	0.97	0.96	15.8	21.3	2.0	3.7
Hardwoods:										
Trembling aspen	Alberta	220.2	239.9	205.7	0.98	0.95	14.3	25.6	-19.8	14.4
	Saskatchewan	195.9	195.9	202.0	0.995	0.99	7.1	12.3	0.03	-6.2
Balsam poplar	Alberta	165.5	186.6	169.4	0.93	0.93	27.0	28.6	-21.1	-3.9
	Saskatchewan	141.8	151.0	149.2	0.95	0.94	22.2	25.8	-9.2	-7.4
White birch	Alberta	294.9	293.6	276.7	0.98	0.98	15.9	14.2	1.4	18.2
	Saskatchewan	226.9	238.1	219.4	0.85	0.88	40.0	37.4	-11.2	7.5

a/ Model I: $W = a_0 + a_1 D^2H$

Model II: $W = a_0 + a_1 D + a_2 D^2 + a_3 D^3$

where W is the ovendry weight (kg) of living tree above ground without dead branches, D is diameter (cm) outside bark at breast height, H is total tree height (m), and $a^1 \ldots a^3$ are regression coefficients; % SEE is standard error of estimate expressed as percentage of the actual mean.

b/ Calculated from actual measurements from species sampled in Alberta, Saskatchewan, and Manitoba.

Table 6. Comparative estimation errors for regional and individual province (Alberta) equations when validated on an independent data set to estimate ovendry biomass of two poplar species in Alberta[a]

Species	Prediction Equation	N	Actual Mean[b]	Predicted Mean Model I	Predicted Mean Model II	Predicted R^2 Model I	Predicted R^2 Model II	% SEE Model I	% SEE Model II	Mean Residuals Model I	Mean Residuals Model II
Trembling aspen	Regional	254	68.0	68.6	69.8	0.99	0.95	15.4	30.2	-0.6	-1.8
	Alberta	254	68.0	57.2	69.1	0.97	0.95	22.2	29.1	10.8	-1.2
Balsam poplar	Regional	60	49.7	54.3	44.3	0.96	0.95	23.3	28.5	-4.6	5.4
	Alberta	60	49.7	50.7	47.1	0.96	0.96	24.8	26.0	-1.0	2.6

a/ Model I: $W = a_0 + a_1 D^2H$

Model II: $W = a_0 + a_1 D + a_2 D^2 + a_3 D^3$

where W is the ovendry weight (kg) of living tree above ground without dead branches, D is diameter (cm) outside bark at breast height, H is total tree height (m), and $a_1 \ldots a_3$ are regression coefficients; % SEE is standard error of estimate expressed as percentage of actual mean.

b/ Calculated from actual measurements from species sampled in Alberta, Saskatchewan, and Manitoba.

Conclusions

The conclusions drawn from the study are:

1. The biomass regression function based on diameter provided prediction equations with low estimation errors; the regression function based on a combination of diameter and height gave slightly better estimates.

2. The range of application of prediction equations derived from a large sample taken from widely distributed sampling locations within an ecoregion is likely to be as good or better than the sample taken from limited locations.

3. Independent data are essential for validation of a model. Validation results showed very high fit, in addition to providing more reliable estimates on the performance of regression functions tested in the study.

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APPENDIX I

Biomass prediction equations for tree species in individual provinces of west-central Canada, based on 20 samples for each species per province

Species	Province	Biomass prediction equation	R^2	% SEE
Softwoods:				
Jack pine	Alberta	$W_1 = -25.75 + 6.498D - 0.1989D^2 + 0.01194D^3$	0.981	14.5
		$W_2 = 5.62 + 0.01749D^2H$	0.971	16.8
	Saskatchewan	$W_1 = 61.22 - 15.310D + 1.2275D^2 - 0.01198D^3$	0.978	15.5
		$W_2 = 6.25 + 0.01775D^2H$	0.985	12.3
	Manitoba	$W_1 = 14.24 - 5.986D + 0.7082D^2 - 0.00371D^3$	0.975	16.8
		$W_2 = 1.71 + 0.01941D^2H$	0.977	15.3
Black spruce	Alberta	$W_1 = 30.18 - 10.322D + 0.9862D^2 - 0.00888D^3$	0.995	7.7
		$W_2 = 4.54 + 0.01810D^2H$	0.997	5.8
	Saskatchewan	$W_1 = 20.84 - 5.755D + 0.6744D^2 - 0.00322D^3$	0.996	6.5
		$W_2 = 4.74 + 0.01897D^2H$	0.997	5.1
	Manitoba	$W_1 = -23.51 + 6.655D - 0.1590D^2 + 0.01201D^3$	0.969	18.3
		$W_2 = 2.33 + 0.01909D^2H$	0.958	19.9
White spruce	Alberta	$W_1 = 47.21 - 13.851D + 1.1396D^2 - 0.01035D^3$	0.992	10.5
		$W_2 = 7.61 + 0.01656D^2H$	0.987	13.1
	Saskatchewan	$W_1 = -1.07 - 0.633D + 0.3165D^2 + 0.00245D^3$	0.969	17.7
		$W_2 = 11.39 + 0.01524D^2H$	0.984	12.1
	Manitoba	$W_1 = 5.56 - 1.944D + 0.3273D^2 + 0.00366D^3$	0.991	10.6
		$W_2 = 11.76 + 0.01605D^2H$	0.986	12.2
Balsam fir	Alberta	$W_1 = -15.64 + 3.657D - 0.0632D^2 + 0.01011D^3$	0.995	8.0
		$W_2 = 2.07 + 0.01532D^2H$	0.995	7.4
	Saskatchewan	$W_1 = 18.23 - 7.804D + 0.8077D^2 - 0.00689D^3$	0.970	18.7
		$W_2 = 13.59 + 0.01474D^2H$	0.972	16.9
	Manitoba	$W_1 = 11.76 - 4.565D + 0.5177D^2 - 0.00118D^3$	0.983	14.1
		$W_2 = 4.42 + 0.01970D^2H$	0.947	23.3
Tamarack	Alberta	$W_1 = 20.22 - 6.754D + 0.7244D^2 - 0.00601D^3$	0.991	10.1
		$W_2 = 8.89 + 0.01784D^2H$	0.988	10.8
	Saskatchewan	$W_1 = 23.93 - 7.219D + 0.7110D^2 - 0.00556D^3$	0.962	19.6
		$W_2 = -0.93 + 0.01936D^2H$	0.977	14.3
	Manitoba	$W_1 = -18.18 + 3.723D + 0.0287D^2 + 0.00649D^3$	0.989	10.3
		$W_2 = 8.28 + 0.01798D^2H$	0.985	11.2
Hardwoods:				
Trembling aspen	Alberta	$W_1 = 24.26 - 6.433D + 0.5800D^2 + 0.00247D^3$	0.981	16.2
		$W_2 = -9.11 + 0.01909D^2H$	0.990	10.7
	Saskatchewan	$W_1 = 5.02 - 2.278D + 0.3691D^2 + 0.00411D^3$	0.997	6.4
		$W_2 = 0.55 + 0.02011D^2H$	0.997	5.3
	Manitoba	$W_1 = 11.32 - 5.253D + 0.7389D^2 - 0.00479D^3$	0.980	15.4
		$W_2 = 9.44 + 0.01919D^2H$	0.985	12.7
Balsam poplar	Alberta	$W_1 = 0.54 - 1.921D + 0.4346D^2 - 0.00200D^3$	0.975	17.5
		$W_2 = 10.38 + 0.01324D^2H$	0.975	16.3
	Saskatchewan	$W_1 = 31.85 - 10.205D + 0.9447D^2 - 0.01114D^3$	0.979	15.4
		$W_2 = 13.74 + 0.01362D^2H$	0.972	16.7
	Manitoba	$W_1 = -5.16 + 1.235D + 0.1149D^2 + 0.00513D^3$	0.979	15.2
		$W_2 = 6.27 + 0.01540D^2H$	0.968	17.8
White birch	Alberta	$W_1 = 24.48 - 7.202D + 0.7906D^2 - 0.00160D^3$	0.990	11.7
		$W_2 = 0.53 + 0.02602D^2H$	0.979	15.6
	Saskatchewan	$W_1 = 110.55 - 39.002D + 3.1177D^2 - 0.04645D^3$	0.934	28.0
		$W_2 = 18.50 + 0.02255D^2H$	0.883	35.2
	Manitoba	$W_1 = 13.04 - 5.423D + 0.6400D^2 + 0.00075D^3$	0.964	20.4
		$W_2 = -9.31 + 0.02677D^2H$	0.954	21.8

a/ Model I: $W_1 = a_0 + a_1 D^2H$
Model II: $a_0 + a_1 D + a_2 D^2 + a_3 D^3$
where W is the ovendry weight (kg) of living tree above ground (excluding dead branches), D is the diameter outside bark at breast height (cm), and H is total tree height (m).

APPENDIX II

Prairies regional equations for biomass prediction derived from 60 trees per species in the three provinces

Species	Biomass Prediction Equation[a]	R^2	% SEE
Softwoods:			
Jack pine	$W_1 = 4.09 + 0.01829D^2H$	0.975	15.3
	$W_2 = 13.87 - 4.998D + 0.5954D^2 - 0.00159D^3$	0.967	17.9
Black spruce	$W_1 = 4.06 + 0.01871D^2H$	0.982	12.9
	$W_2 = 6.70 - 3.040D + 0.5131D^2 - 0.00047D^3$	0.984	12.4
White spruce	$W_1 = 8.88 + 0.01609D^2H$	0.984	12.8
	$W_2 = 10.39 - 3.749D + 0.4906D^2 + 0.00018D^3$	0.983	13.5
Balsam fir	$W_1 = 13.57 + 0.01556D^2H$	0.952	21.5
	$W_2 = 8.15 - 3.034D + 0.4074D^2 + 0.00112D^3$	0.978	14.9
Tamarack	$W_1 = 5.89 + 0.01833D^2H$	0.982	12.2
	$W_2 = 16.08 - 5.135D + 0.5945D^2 - 0.00358D^3$	0.980	13.3
Hardwoods:			
Trembling aspen	$W_1 = 1.41 + 0.01933D^2H$	0.988	11.2
	$W_2 = 21.73 - 7.304D + 0.7545D^2 - 0.00307D^3$	0.977	15.8
Balsam poplar	$W_1 = 12.23 + 0.01380D^2H$	0.966	18.0
	$W_2 = 6.54 - 3.432D + 0.5021D^2 - 0.00295D^3$	0.974	16.3
White birch	$W_1 = 2.89 + 0.02520D^2H$	0.943	24.4
	$W_2 = 30.26 - 10.591D + 1.0458D^2 - 0.00686D^3$	0.956	21.8

a/ Model I: $W_1 = a_0 + a_1 D^2H$
Model II: $W_2 = a_0 + a_1 D + a_2 D^2 + a_3 D^3$
where W is ovendry weight (kg) of living tree above ground (excluding dead branches), D is the diamete outside bark at breast height (cm), and H is total tree height (m).

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May 26-30, 1986
Syracuse, New York

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