THE MOISTURE CONTENT OF FOREST FUELS - I

A REVIEW OF THE BASIC CONCEPTS

By

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INTRODUCTION

It is a well known fact that the presence of moisture exerts considerable influence on the difficulty of ignition and the subsequent rate of combustion of forest fuels. Its effect on ignition is primarily a result of the fact that the water must be heated to the Boiling Point and then vaporized before the fuel will reach its ignition temperature. If enough moisture is present the heat required may be greater than that available in the firebrand, and ignition will not occur. Once ignited, water must be continuously driven from adjacent fuels, if the fire is to spread. This absorbs some of the heat energy which is emitted by the fire and reduces the rate of combustion. It is also possible that the presence of moisture affects combustion due to the water vapor which surrounds the fuel and dilutes the available oxygen.

The correlation between fire behavior and fuel moisture is an important part of every fire danger rating system in use today. In every system, measurements of certain factors which are presumed to influence fuel moisture are made. These are, in turn, correlated with fire behavior to produce the desired index.

There are as many methods for estimating fuel moisture as there are fire danger rating systems. These differences are the result of two problems. The first is the difficulty of obtaining direct measurements of fuel moisture in the field. The second is the fact that the factors which influence fuel moisture, and the methods by which they work, are only partially understood.

In an effort to avoid the second problem, the moisture content of some type of pre-calibrated indicator fuel (sticks, slats, duff baskets, etc.) is often measured. There are several advantages to this approach:

1 - The indicator acts as an integrator of all the environmental factors. It is not necessary to understand the complex mechanism through which fuel moisture is controlled as the fuel sample provides a direct reading.

2 - It is not necessary to have continuous readings - the actual moisture content is always available. Without the indicator, the previous day's moisture content must be known.

3 - Because of their uniform exposure, it is possible to compare fuel moisture at different localities.

On the other hand, several problems present themselves:

1 - A fire danger rating system which depends on measurement of a fuel sample can be used on a current basis only. It is impossible to predict future fire danger because the indicator reading is valid only at the time at which it is measured. This argument also holds when evaporation is measured to give an indication of the drying potential of a day. Although the problems associated with the fuel have been eliminated, the system is still tied to current measurements.

2 - All fuel samples are different. For this reason, every indicator used must be pre-calibrated so that its readings will correspond to those of any other sample.

3 - Pre-calibration involves oven-drying prior to use. Leroy

(1954) discussed problems involved with oven-drying fuel samples with respect to moisture content:

a) Rapidly heating the material to a high temperature may induce changes in some of the component substances. When accompanied by boiling water, this may permanently fix some of that water to the substance.

b) All water cannot be driven from a material regardless of the temperature used. An amount is retained in proportion to the bonding force even at temperatures at which many substances decompose.

c) Volatile substances are driven from wood at high temperatures. Their loss is difficult to separate from the loss of water, and may influence subsequent absorption of water.

In addition to the problems mentioned by Leroy (1954), others present themselves:

d) Rapid heating can cause surface cracks and checks in wood which will allow water to enter much more rapidly than if the surface was continuous.

e) Rapid heating may cause case hardening of the sample, with the resultant uneven distribution of tension and compression stresses. These may in turn affect moisture relationships.

f) High temperatures may affect the outer cuticle wax layer present on coniferous needles and hardwood leaves. Removal or alteration of these layers will subsequently affect moisture gain and loss.

4 - As fuel samples are exposed to the environment, weathering causes deterioration which necessitates frequent changes in order to insure accuracy of measurements.

5 - Lastly, and perhaps most important, is the fact that the sample represents itself only. A sample of wood cannot be expected to behave as a layer of duff. Experimental evidence indicates that bare wood behaves quite differently from wood with a tight bark covering. Therefore, even if it were possible to solve all the previous problems, the possibility that the fuel moisture indicator might not represent the actual forest fuel could not be overlooked.

In the past, and also in many systems today, it has generally been accepted that the ability of the indicator to act as an integrator of environmental conditions was sufficient advantage to warrant its use. Presently, however, it is becoming increasingly desirable to forecast fire danger for some period in the future. Interest is also being expressed in tying fire danger more closely to particular fuel types. It is for these reasons that this study was undertaken.

The purpose of this study is threefold:

1 - To determine which environmental factors influence fuel moisture.

2 - To determine the extent of their influence.

3 - To develop regression equations, where possible, to predict fuel moisture directly from meteorological observations.

The factors which influence fuel moisture content changes vary, depending on whether the source of water is in a liquid or vapor form. For this reason the two types of sources will be discussed separately. To further facilitate the investigation, fuels will be divided into two broad classes: fast reacting and slow reacting. These names are used because they refer to the rate of response to environmental change. The more conventional classes of fine and heavy fuels imply size, although they are usually intended to refer to the rate of response. It is hoped that this inconsistency will be eliminated by the use of the first terms.

The present paper is the first of a series dealing with forest fuel moisture. The ultimate purpose of the study is to provide regression equations for estimating forest fuel moisture directly from antecedent and current weather observations. This paper is concerned only with a discussion of some of the basic concepts involved in moisture content variations. Subsequent papers in the series will deal with actual experimental observations.

BASIC FUEL MOISTURE CONCEPTS

The moisture content of dead plant material is governed by two major groups of factors. The first group consists of those meteorological factors which influence the gain or loss of water from a non-absorbing surface such as water or glass. Atmospheric vapor pressure, temperature, (from which saturation vapor pressure is obtained), water vapor transfer coefficient, precipitation, and the formation of dew are generally agreed to be the most influential of the various meteorological factors.

The second group consists of those factors within the fuel which influences the total amount of moisture which the material can gain or lose, and the rate at which this change can occur under the existing meteorological conditions. Among the more important factors which are known to influence this are: equilibrium moisture content, heat of desorption, specific heat, rate of diffusion, size of the material, and numerous species differences.

The method by which each factor influences the moisture content of dead plant material will now be discussed in detail.

1. METEOROLOGICAL FACTORS

A. Water Loss

Saturation vapor pressure is directly related to temperature. Therefore, as temperature increases, the amount of water vapor that may be present in a particular space also increases. The temperature at the surface of the fuel depends on several variables, among which are: solar radiation reaching the surface, evaporation from the surface, amount of heat radiated from the surface, the last two of which are influenced by wind speed. The difference between the temperature measured in a standard instrument shelter and at the surface of the fuel must not be overlooked.

The difference between the partial vapor pressure of air and the saturation vapor pressure (vapor pressure deficit) is a measure of the evaporation potential between the surface of the material and the atmosphere. As the vapor pressure deficit increases, the amount of water loss per unit length of time will increase proportionally, assuming that the other factors remain unchanged.

The water vapor transfer coefficient is a function of the variables which influence the removal of the evaporated water molecules from the region immediately surrounding the surface of the material and thereby maintain a vapor pressure deficit. The transfer coefficient increases with increasing surface roughness and wind speed.

While the factors which influence the rate of evaporation from a water surface, such as a pan, are fairly well known, the exact method by which they operate is very complex. Several methods of determining the rate of evaporation were compared by Sellers (1965). The results of this comparison can be seen in Table 1.

Table 1 Comparison of several methods for estimating potential evapotranspiration from short grass. From: Sellers (1965)

Month	Evapotranspiration (grass)	Evaporation (pan)	Budyko-Penman	Blaney, Criddle	Thornthwaite	McIlroy	
Jan.	8.68	7.78	6.86	4.91	4.47	7.57	(All units in mm/day)
Feb.	6.82	5.62	5.44	4.34	3.51	6.12	
Mar.	4.81	4.21	3.76	3.82	2.80	4.22	
Apr.	3.35	2.94	2.76	3.24	2.04	3.16	
May	1.83	1.31	1.48	2.52	1.09	1.67	
June	1.35	0.99	1.01	2.26	0.80	1.19	
July	1.39	0.93	1.17	2.25	0.74	1.34	
Aug.	2.00	1.37	1.60	2.55	0.89	1.76	
Sep.	3.02	2.30	2.64	3.02	1.50	2.88	
Oct.	4.61	4.07	4.00	3.64	2.09	4.28	
Nov.	5.53	5.16	4.76	4.17	2.73	5.27	
Dec.	7.41	5.99	5.99	4.62	3.47	6.56	
Annual Totals	1,543	1,296	1,260	1,257	793	1,398	

Of these methods, the equations proposed independently by Penman (1956) and Budyko (1956) appear to give the best results. Budyko uses the energy balance equation for a land surface:

$$R = H + LE + G \qquad (1)$$

where R is the total incoming radiation, which is balanced by H - the sensible heat; the product of L - the latent heat of vaporization, and E - evaporation; and G - the heat absorbed by the surface.

This is combined with the sensible heat equation:

(2) $$H = \rho \cdot Cp \cdot D_h \cdot (T_s - T)$$

and the equation for determining the upward flux of water vapor:

(3) $$LE = 0.622 \cdot \frac{\rho \cdot LD_w}{p} \cdot (e_s - e)$$

Where: ρ = air density

Cp = specific heat of air at constant pressure

D_w, D_h = transfer coefficient of water vapor, and heat, respectively

T_s, T = surface and air temperature, respectively (°C.)

p = total pressure

e_s, e = surface vapor pressure and air vapor pressure, respectively.

The result of this combination is:

(4) $$R_o - G_o = A \cdot (e_s - e) + B \cdot (T_s - T)$$

where:

(5) $$A = 0.622 \cdot \frac{\rho \cdot LD}{p}, \text{ and } \quad B = \rho \cdot Cp \cdot D$$

While this equation gives the most accurate results, (Sellers, 1965) its use is cumbersome, and some of the variables, such as net radiation and transfer coefficient, are difficult to determine in the field. A simpler (but less accurate) method was proposed by Eagleman (1967) using the relationship:

(6) $$E_r = 0.035 \cdot e_{sat} \sqrt{RHD}$$

where: E_r = maximum evapotranspiration in inches per month

e_{sat} = saturation vapor pressure (mb) of the mean monthly temperature

RHD = average monthly relative humidity deficit in per cent

He noted that the amount of water that a plant will transpire is roughly related to its vegetative state from green to cured. He then demonstrated a correlation between vegetative state and the average monthly temperature. He then adjusted equation (2.6) by the vegetative cycle C where:

(7) $C = 0.20 + 0.133 \cdot T_m$ (for values of T between 30° and 70°F.)

where T_m is the mean monthly temperature. This equation appears to give better results than those of Thornthwaite (1948) or Blaney & Criddle (1962) which use the same number of variables.

B. Water Gain

The two basic factors which cause an increase in the amount of water present on a non-absorbing surface are rainfall and the formation of dew. Rainfall is the most important and also the easiest to work with. Its effect is direct - the amount of water added is equal to the amount of rain.

Dew formation, on the other hand, is a complex function involving several variables. In order for dew to form, the leaves of grass (or surface litter) must cool to the dew point of the surrounding air. This will permit the partial condensation of water vapor escaping from the still warmer soil below. This, in turn, reduces the total flux of water vapor from the soil to the atmosphere by the amount of the condensation. Another effect is an increase in the surface temperature, which in turn restricts further condensation. Monteith (1957) indicates that this period, during which there is a delicate heat balance on the surface, is a transition period which never lasted for more than an hour during his investigations.

He then gives evidence to show that the formation of dew is the summation of two separate sources of condensation. He calls them distillation (upward movement from warmer soil) and dewfall (downward movement from the atmosphere). He believes that wind speed is the main factor which determines the relative importance of each of these on a given night. Without wind there will be very little turbulent transfer of water vapor downward from the atmosphere. On the other hand, distillation will continue, essentially independent of wind speed. He proposes two equations by which distillation and dewfall can be estimated. They are:

$$D = K_v \cdot (T_s - T) \cdot \left(\frac{\partial x}{\partial T}\right)_s \tag{8}$$

where: D = rate of distillation in mg/cm²/hr

K_v = diffusion coefficient (assuming $K = 0.24$ cm²/sec)

T_s = mean surface temperature (°C)

T = air temperature at 1 cm. (°C)

$\left(\frac{\partial x}{\partial T}\right)_s$ = rate of change of saturated absolute humidity at $T = \frac{T_s + T}{2}$

and

$$F = \frac{K^2 \cdot z^2 \cdot (\partial u/\partial z) \cdot (\partial x/\partial z)}{1 + \sigma Ri} \tag{9}$$

where: F = eddy flux of water vapor from the atmosphere

K = von Karman's constant = 0.4

z = height of wind measurement

σ = a constant (assumed = 10)

Ri = Richardson's number or stability parameter

$(\partial u/\partial z)$ = wind gradient at height z

$(\partial x/\partial z)$ = absolute humidity gradient at height z

and

$$Ri\ (z) = 3.5 \cdot (\partial T/\partial z)/(\partial u/\partial z)^2 \qquad (10)$$

Using these equations Monteith (1957) calculated what the distillation and dewfall should have been during a number of periods and compared these with actual observations. He found that he was able to predict dewfall to within ±30%, and also the general magnitude of distillation. He attributed the discrepancies to the difficulty of obtaining accurate measurements, and a possible incomplete knowledge of all the factors which influence dew formation. In general, he found that distillation of water vapor from the soil varied from 1 to 2 $mg/cm^2/hr$ on calm nights. Dewfall, on the other hand, was negligible when the wind at 2 m was less than 0.5 m/sec,[1/] but reached 3 - 4 $mg/cm^2/hr$ with stronger winds. Total dewfall for one night was as high as 13 mg/cm^2, which is equivalent to about 0.005 inches of rain. Whether the effect of dew is the same as an equivalent amount of rain or greater, due to its extended duration, remains to be determined from experimental data.

2. INTERNAL FACTORS

If it were possible to accurately determine the effect of the various meteorological factors, we would still not have the fuel moisture content problem solved. There are numerous internal factors within the fuel which govern the amount of moisture which it can absorb or lose under specific meteorological conditions.

A. Heat of Desorption

As the moisture content of a piece of wood is lowered the holding forces become stronger, and the energy necessary to

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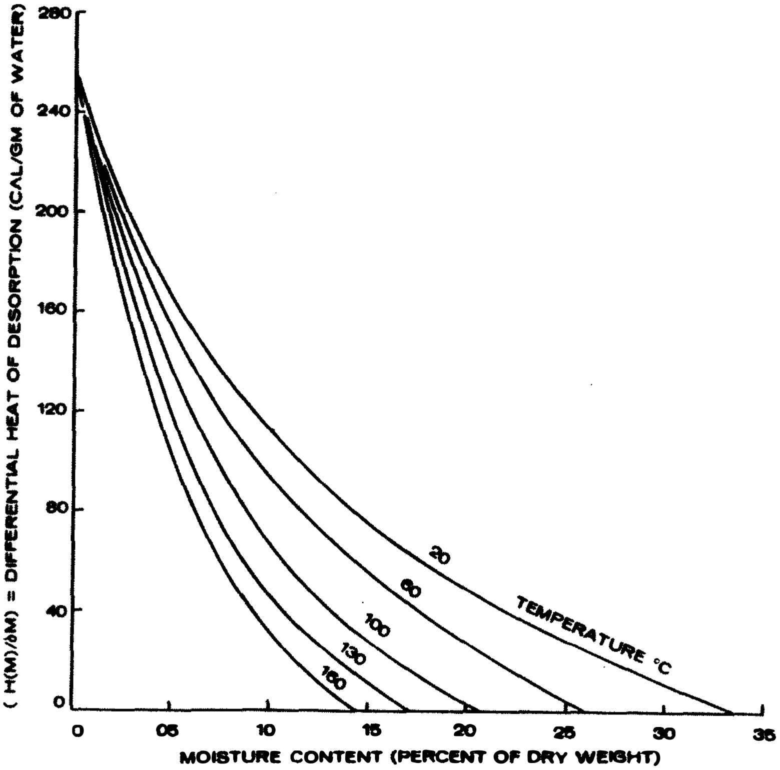
[1/] *About 1 mile/hr.*

decrease the moisture content by a proportional amount becomes correspondingly greater. This increase, called the differential heat of desorption was computed on a theoretical basis by Byram et al. (1952) using the formula:

(11)
$$\left[\frac{\partial H(M)}{\partial M}\right]_T = - \frac{R}{m} \cdot \left[\frac{\partial (ln\ f)}{\partial (1/T)}\right]_T$$

where: $H(M)$ represents the amount of heat necessary to release all water from one gram of wood initially at moisture content M and absolute temperature T. R is the gas constant, m the molecular weight of water, and f the relative humidity. The computed results are plotted for various constant temperatures in Figure 1.

Figure 1 Differential Heat of Desorption



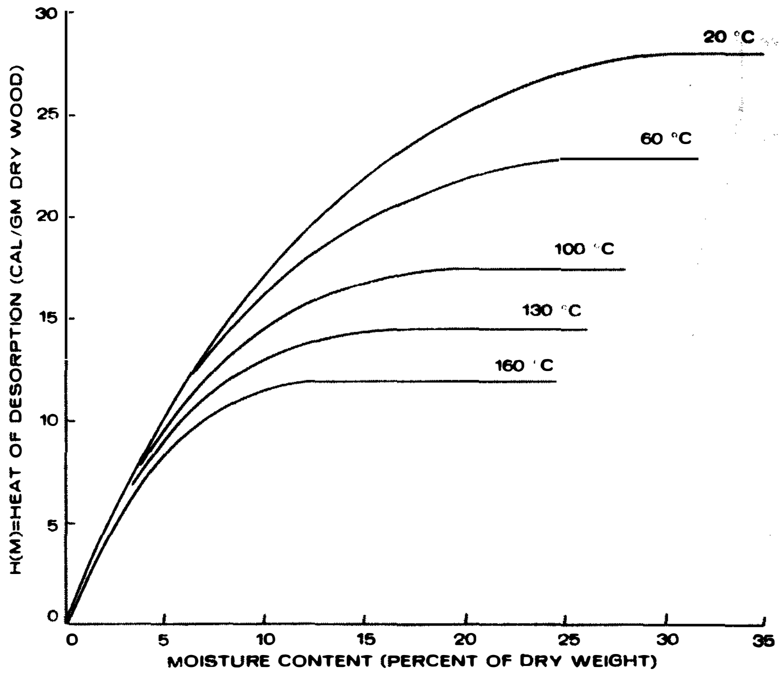
From: Byram, et al (1952), Thermal Properties of Forest Fuels

The heat of desorption, or total amount of heat needed to remove all water from one gram of wood at a constant temperature, was then computed on a theoretical basis using the relationship:

$$H(M)_T = \int_0^M \left[\frac{\partial H(M)}{\partial M}\right]_T \cdot dM \tag{12}$$

The heat of desorption can therefore be determined for a specific temperature T by measuring the area under the corresponding curve in Fig. (1) between M = 0 and M = M. Heats of desorption computed in this manner are shown in Fig. (2).

Figure 2 Heat of Desorption



From: *Byram, <u>et al</u> (1952), Thermal Properties of Forest Fuels*

B. Specific Heat

Specific heat is defined as the amount of heat required to raise the temperature of a unit mass of a substance a unit amount. It is generally expressed either as cal/g . or B.T.U./lb. The specific heat of a wood-water system is considerably greater than the sum of the heat capacities of the individual substances (Byram, 1952). This increase is believed to be related in a complex manner to the forces which bind the water to the wood. The specific heat of the entire system Cp can be determined by the equation:

$$Cp = Cf + MC_w + \Delta Cp \qquad (13)$$

where: Cf = specific heat of wood

MC_w = moisture content of wood

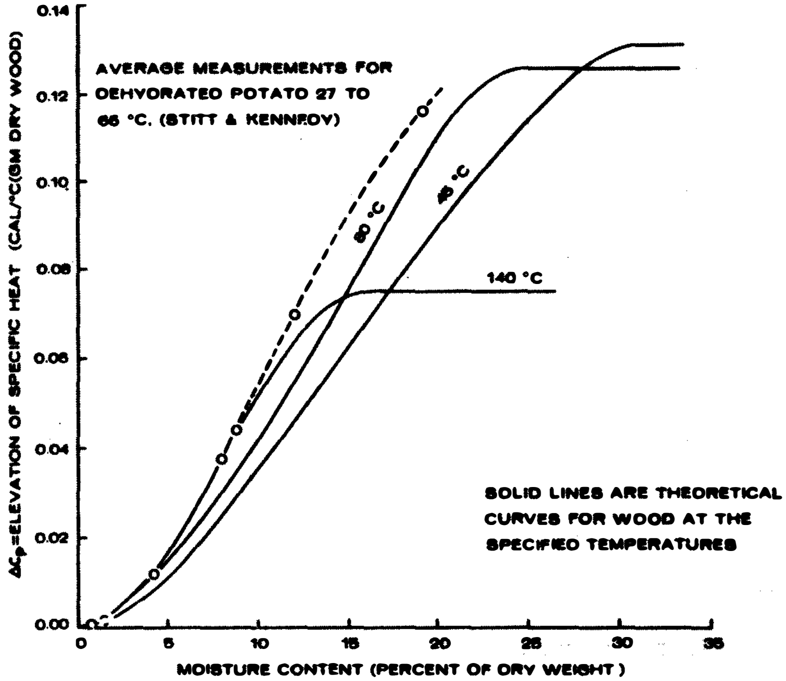
ΔCp = elevation of Cp due to the bonding forces.

Elevation of the specific heat ΔCp due to the bonding forces is given by Byram <u>et al</u>. (2) as:

$$\Delta Cp = -\left[\frac{\partial H(M)}{\partial T}\right]_M \qquad (14)$$

Since H(M) decreases with increasing temperature, $-\left[\frac{\partial H(M)}{\partial T}\right]_M$ is positive and the specific heat of the system will be increased by ΔCp. Values for ΔCp were computed for temperatures of 45°C., 80°C., and 140°C., from equation (14) and are shown as solid lines in Figure (3). Actual measurements of Stitt and Kennedy (1945) for dehydrated potatoes are shown as plotted points for comparison.

Figure 3 Increase in specific heat due to bonding forces.



From: Byram, et al (1952), Thermal Properties of Forest Fuels.

C. Equilibrium Moisture Content

All wood fibers contain moisture. A piece of wood which is initially saturated contains both free water and bound water. As long as the vapor pressure of the water in the wood exceeds that of the atmosphere, water will continue to escape. Eventually, a point will be reached where a certain amount of water is held in by a combination of capillary action in the minute openings of the cell walls and secondary valence bonds which link the water to the cellulose molecules. This is known as bound water. As a result of these bonding forces, the vapor pressure of the water is lowered and the boiling point raised. The net result is that the vaporization of bound water requires more heat than does an equal amount of free water.

When the vapor pressure of the water in the wood equals that of the atmosphere, the system is said to be at the equilibrium moisture content. The amount of water that a specific volume or weight of wood can hold at the E.M.C. is governed primarily by the relative humidity of the surrounding atmosphere. It should be noted that the E.M.C. for absorptive conditions is lower than the E.M.C. for desorptive conditions. This is known as sorption hysteresis. Several theories of the reason for this have been advanced, one of which may be explained with the aid of Fig. 4 which represents an enlarged view of a tiny cavity in a piece of wood. When the wood is adsorbing moisture (A.), a combination of adhesion between the water and the surface of the wood, and surface tension of the water combine to bridge the narrow gap, leaving the larger cavity empty. The forces maintaining this bridge are sufficient to balance the adsorption forces within the wood and maintain a lower moisture content. When desorption is taking place, the same mechanism allows water to be held in the cavity (B.). The forces at the opening offset the evaporative potential and maintain a higher equilibrium moisture content. Stamm (1964) gives a detailed explanation of several other theories which attempt to explain sorption hysteresis.

Figure 5 shows the E.M.C. as a function of relative humidity. Temperature also influences the E.M.C., but to a much lesser degree. The relationship between the three is shown in Figure 6.

Figure 4 Diagram of a theoretical explanation for sorption hysteresis

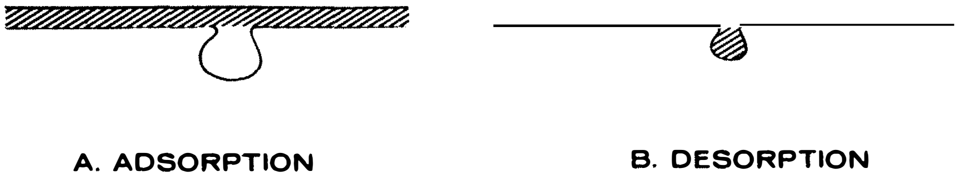
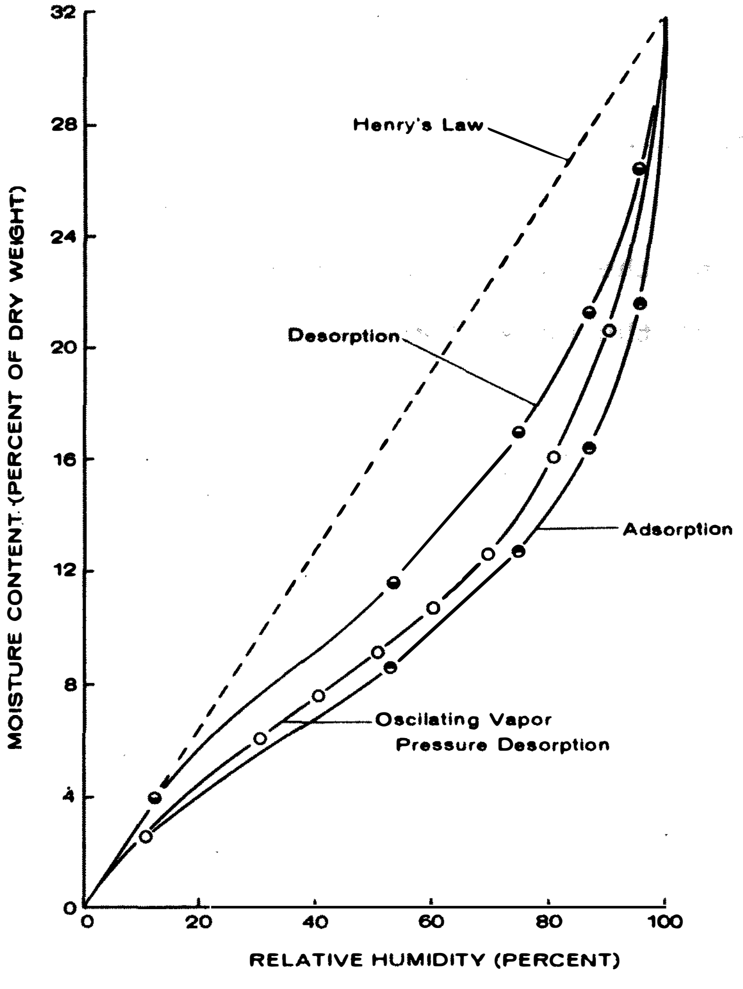
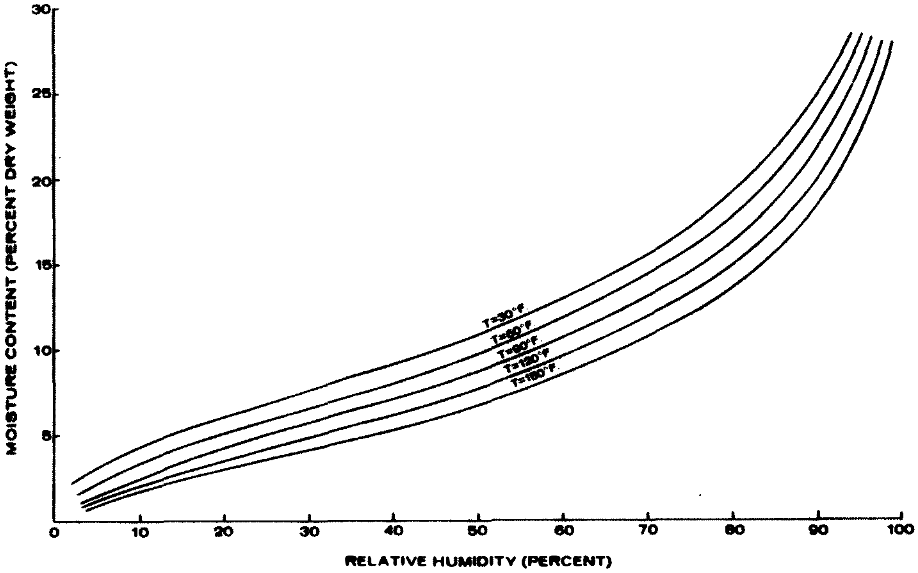


Figure 5 Equilibrium moisture content as a function of relative humidity



From: Stamm and Loughborough (1934), Thermodynamics of the Swelling of Wood.

Figure 6 Equilibruim moisture content as a function of temperature and relative humidity (data taken from Canadian Woods pp 136-137



Using values published by Millett (1951) and the U.S.F.S. Forest Products Laboratory (1955), regression equations (a. and b. respectively) were derived for calculating the average E.M.C. of wood.

For R.H. less than 10%:

(15 a) E.M.C. = .003983 + 4.54531 (LOG RH) - .018958 T

(15 b) E.M.C. = .03229 + .281073 RH - .000578 T

where E.M.C. = equilibrium moisture content (% of dry weight)

R.H. = relative humidity in per cent

T. = temperature (°F.)

For R.H. between 11% and 50%:

(16 a) E.M.C. = 3.63954 + .157206 RH - .029478 T

(16 b) E.M.C. = 2.22749 + .160107 RH - .014784 T

and for R.H. greater than 50%:

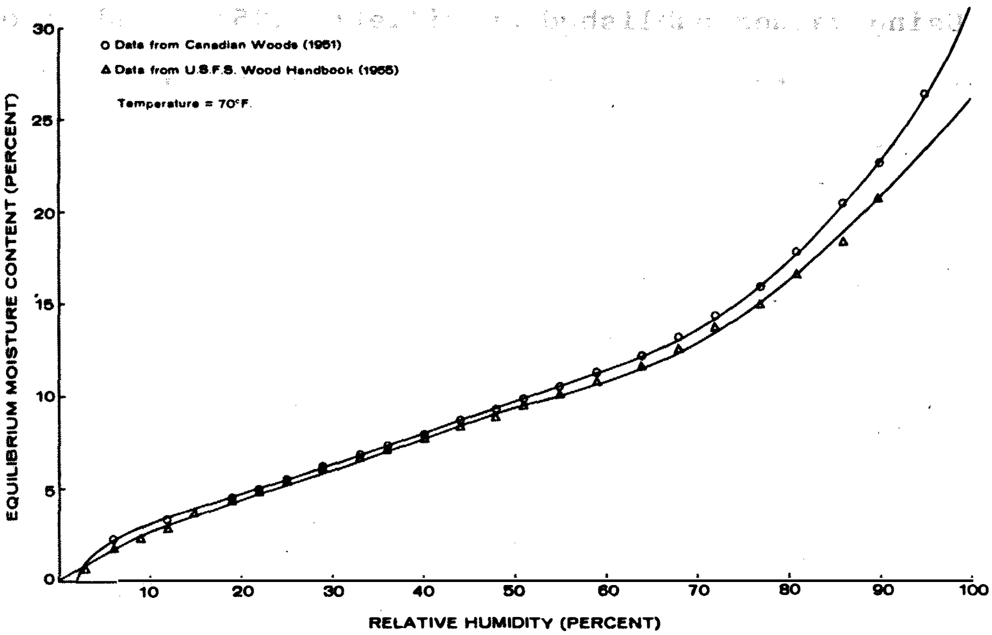
(17 a) $E.M.C. = 28.2771 + .007493\ RH - .000532\ (RH\ T) - .69806\ RH$

(17 b) $E.M.C. = 21.0606 + .005565\ RH - .0003505\ (RH\ T) - .483199\ RH$

Further information concerning these equations and their derivation can be found in Appendix I.

Figure 7 compares calculated values for E.M.C. with those obtained from the tables. Both visual and statistical inspection indicate that these equations can be used with considerable confidence to calculate the average equilibrium moisture content of wood.

Figure 7 Comparison of the use of equations and tables to determine equilibrium moisture content.



D. Moisture Transport Through the Fuel

The factors which govern the transport of moisture through a fuel are, at best, only partially understood. It may be temperature gradient, or vapor pressure gradient, or the actual moisture content gradient. On the other hand it is also influenced by mechanical factors such as capillary flow, gravity, and shrinkage, or swelling which induce mechanical pressure gradients. Most likely, it is a complex function of all of these factors, and possibly more. It is possible that transport of moisture through the fuel may be brought about by both molecular diffusion of water vapor, and viscous flow of water in the liquid state. The process may involve alternate phases of vapor and liquid transfer, with condensation and evaporation phases intervening.1/

Most investigators have used the term diffusion to denote the transport of water through a substance. For simplicity, the same convention is followed throughout this paper. For the purpose of this paper, diffusion is defined as the movement of moisture through a substance. This should not be interpreted as being necessarily limited to molecular diffusion of water vapor.

Linton (1962) found that the variation in diffusion coefficient was as high as 7:1 due to temperature variations, 3:1 with moisture content variations, and 2:1 with oven drying. The total range in the variability between the various samples tested was about 3 orders of magnitude. Further, with complex fuel arrangements, such as litter, the rate of diffusion through the internal air spaces must also be considered.

1/ *P. R. Day, Dept. Soils and Plant Nutrition, University of California, Berkeley, Personal Communication, 1968.*

Another problem is that diffusion for a particular fuel does not seem to be constant even under apparently uniform environmental conditions. Experimental evidence indicates that the rate of drying is greater than that of wetting. In the field this might be explained by reasoning that temperatures tend to be cooler when wetting occurs. The same results appear to hold true, however, when constant conditions are maintained in the laboratory. This leads us to a consideration of the basic factors which drive the diffusion process.

In drying, diffusion is being influenced by the moisture and vapor pressure gradients which evaporation creates at the surface. During the wetting phase, absorptive forces within the fuel would be responsible for the gradients. In the special case of a fuel covered with a surface film of water, moisture content gradient alone may be the driving factor (especially if the fuel is above the fiber saturation point). From this, it would seem logical to assume that since these processes no doubt create different gradients with different potentials, the final result - diffusion - should be different.

The structural characteristics of the fuel affects diffusion in several ways:

(1) Spring and summer wood: Rees (1938) found that the rate of diffusion decreased with increasing density, therefore the diffusion coefficient in spring wood should be greater than denser summer wood.

(2) Bark on twigs: Reifsnyder (1967) found that moisture diffusivity in bark was one-quarter to one-eighth that of wood.

(3) The presence of cuticle wax on the surface of some materials will reduce the rate of diffusion into the fuel.

(4) Structure of wood: the ratio of diffusion parallel to the grain to across the grain has been measured as high as 26:1 and 13:1 for pine at 10 and 20% moisture content respectively (Kuebler (1957)). Pidgeon and Maass (1930) found a ratio of 20:1 for both heartwood and sapwood of white spruce. Yokota and Goto (1963) found ratios as high as 10:1 and 21:1 for diffusion in the longitudinal direction relative to the radial and tangential directions for Nara-wood, when approaching a low E.M.C. value (4.5%) from zero. When approaching a high value (24%), the ratios were reduced to less than 2:1. This indicates that the factors which drive diffusion at higher moisture conditions are not greatly hindered by wood structure and therefore must be different from the factors which play an influential role at low moisture conditions. Lastly, the presence of rays may speed diffusion into wood.

(5) Sapwood and heartwood: Diffusion in sapwood has been reported to be twice that in heartwood (Kuebler (1957)). Pidgeon and Maass (1930) found the same ratio in a radial direction, and for white spruce as much as a 7 magnitude difference in the longitudinal direction.

(6) Swelling of the material: As moisture is absorbed the material swells, causing a corresponding change in dimensions, moisture content, and diffusion coefficients.

(7) Byram (1963) found that moisture diffusion in wood varies inversely with the square of the half thickness of the

material (r), for all but the thinnest samples where it appears to be more closely related to the first power of r.

(8) The effect of bonding forces at low moisture contents on the rate of diffusion should not be overlooked. These forces should tend to speed diffusion when water is being gained, and reduce it when it is being lost.

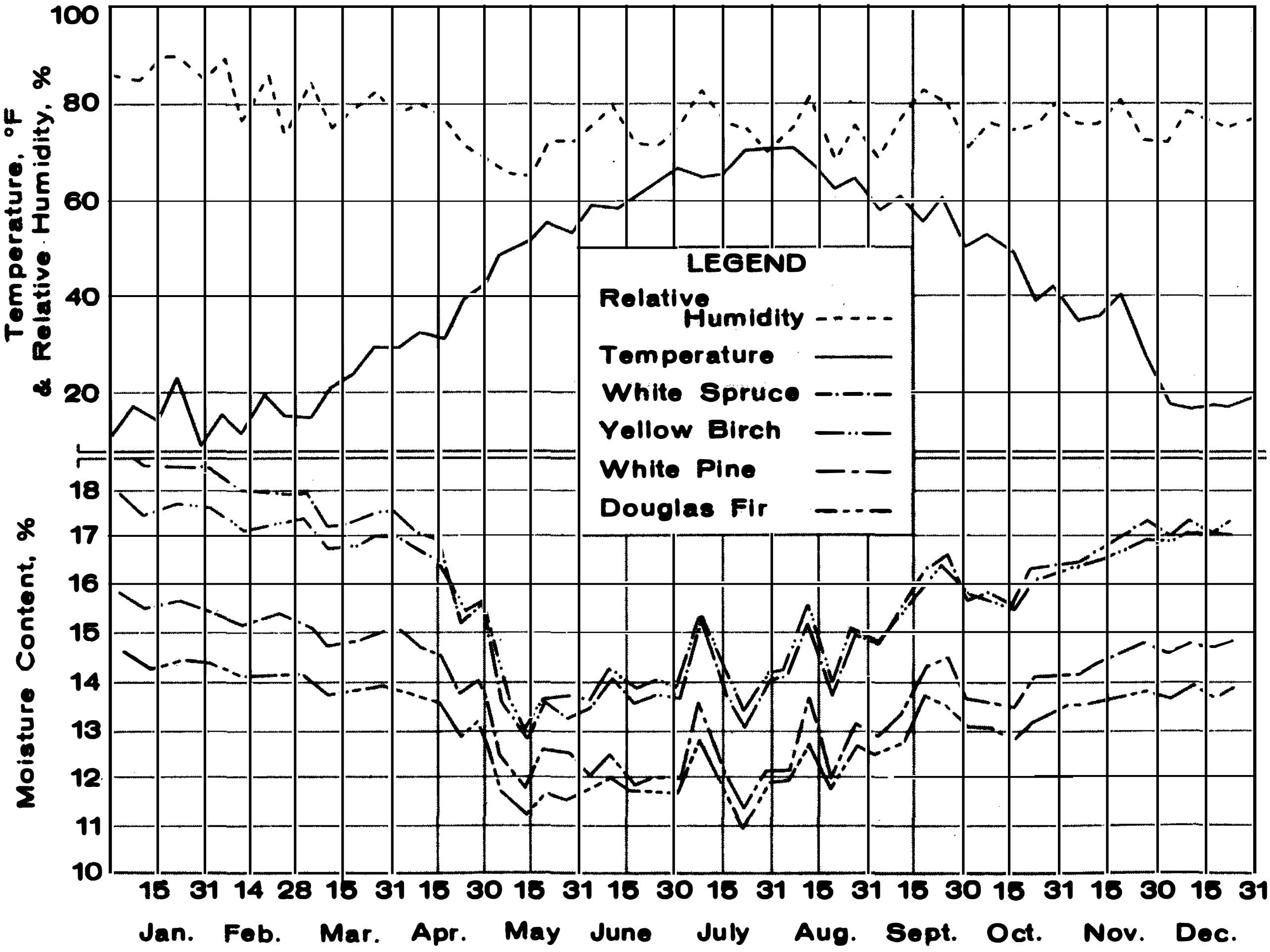
(9) Volume to surface ratio - in fuels with a large volume to surface ratio (logs, heavy duff) the rate of evaporation would serve mainly to affect the temperature and moisture gradients at the surface, and thereby indirectly influence the rate of diffusion. On the other hand, in fuels with a small volume to surface area (twigs, surface litter), diffusion would cease to be limiting owing to the small distances to be travelled by the water and the rate of evaporation would become an increasingly important factor. Therefore, it can be seen that the drying rate of a particular fuel would depend on both diffusion and evaporation. The relative importance of each would in turn be dependent on the volume to surface ratio of that fuel.

It can be seen from the foregoing discussion that diffusion in itself is a complex variable which is not well understood. As was the case with evaporation and dew formation, more work needs to be done before it can be adequately defined.

E. Species Differences

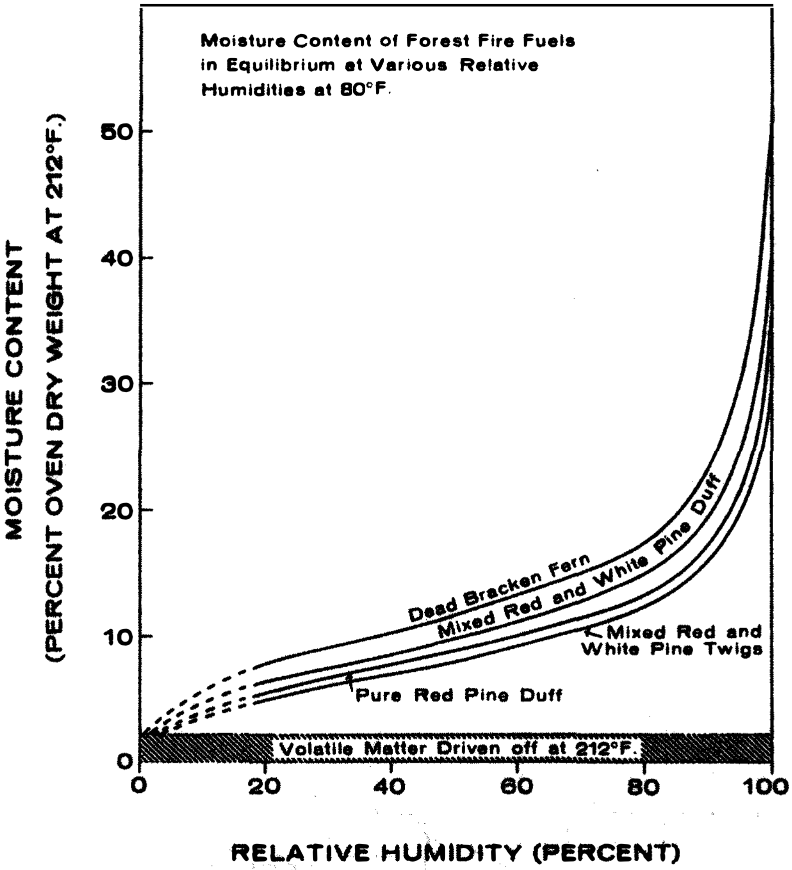
Most published E.M.C. tables list only average values for wood. It has been found by several investigators that different types of fuel, and even different species of the same fuel type, have different E.M.C. curves. This is believed to be caused by the minute differences in chemical composition and anatomical construction of the various species. Figures 2.8 and 2.9 compare some different fuel types and species of wood. It can be seen that in order to accurately predict the moisture content of a specific forest fuel the differences between the various types and species cannot be overlooked. A survey of the literature, however, has failed to disclose data for more than a very limited number of species.

Figure 8 Comparison of the moisture content of selected species of lumber.



From: Millet (1951) The Seasoning of Lumber, Canadian Woods (Chapter 5).

Figure 9 Comparison of the moisture contents of selected fuel types



From: Wright (1932), Forest Fire Hazard Research as Developed and Conducted at the Petawawa Forest Experiment Station.

F. Increase in Moisture Content

When dealing with moisture movements above the fiber saturation point, we are concerned primarily with liquid water. The methods by which liquid water comes in contact with the fuel have already been discussed. Once the fuel is wet, it matters not whether it is through dew formation or rainfall, or snow melt, (except that environmental conditions naturally vary under each of these methods of wetting).

Initially, the forces of adhesion between the water and the individual piece of fuel form a surface film of water. Through diffusion, the water then begins to be absorbed by the fuel. If the water is applied at a rate faster than the fuel can absorb it, the surface film begins to form drops. When surface tension can no longer hold the drop, gravity begins to pull it downward. This is offset in complex litter layers by the proximity of several individual fuel components which in combination can hold a considerable number of large drops. This will be called structural water as opposed to surface water of the individual component. Finally, when the water is in excess of all that which can be held against the pull of gravity, it begins to percolate downward through the litter. If water is applied at a rate greater than that at which it can be absorbed by the entire layer of litter, it will simply reach the ground and not be absorbed at all. From another point of view, as the rate of application decreases, the efficiency of retention increases since less water percolates through the litter to the ground.

The water will be absorbed by the fuel at a particular rate of diffusion (other environmental conditions remaining equal), regardless of the rate of application. This is because only the surface film of water plays an active part in diffusion. Therefore, the total amount of water absorbed by an individual component (needle, twig, etc.) or a layer of litter is governed by the length of time that the surface film is present and the diffusion potential, (assuming that the rate of application exceeds that of diffusion). The surface film lasts as long as water is being applied plus the length of time needed to evaporate it. The difference between an individual component and a complex litter layer is due primarily to the length of time that the surface film of water is retained. Due to the reduced evaporative potential of the atmosphere in the lower layers, the surface film will remain for longer periods of time and water will continue to be absorbed by that layer despite the fact that the upper layer may be losing water.

G. Time-Lag Constant

The concept of time-lag constant is gaining acceptance as an empirical method of measuring the combined effects of fuel size and exposure. This concept defines fuels according to the time required to lose 63% of the moisture which would be lost if left for a sufficient length of time to reach the E.M.C. Therefore, a particular layer of duff would have the same time-lag constant as a certain size branch, if both lost water at the same rate.

While time-lag constant is a useful method for describing the loss of water by various fuels, difficulties arise when using it to describe an increase in moisture content above the E.M.C.[1/] The wetting which takes place during a rain or in the presence of dew is governed by a number of factors in addition to size and exposure. These factors include: initial moisture content, duration of precipitation or dew, and mechanical arrangement of the fuel. A material which is initially dry may have some difficulty in absorbing moisture at first. This effect is governed primarily by the surface tension of the water drop. The presence of moisture in the fuel will speed up the initial adsorption of water. The ability of different types of fuels (logs vs. litter layer) to retain water varies considerably with

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[1/] *C. E. Van Wagner, Petawawa Forest Experiment Station, Personal Communication, 1967.*

their structure. Most of the applied rain will run off a log, except for small pools and the surface film. This is in contrast to a litter layer which can retain much of what is applied if the rate of application is not excessive.

These weaknesses in the time-lag constant concept must be considered prior to its use for defining the behaviour of the moisture content of forest fuels. If only drying is considered important, time-lag constant is probably the best method available today. In the case of wetting, other restrictions such as fuel type and initial moisture content must be added to make the constant representative of a particular fuel.

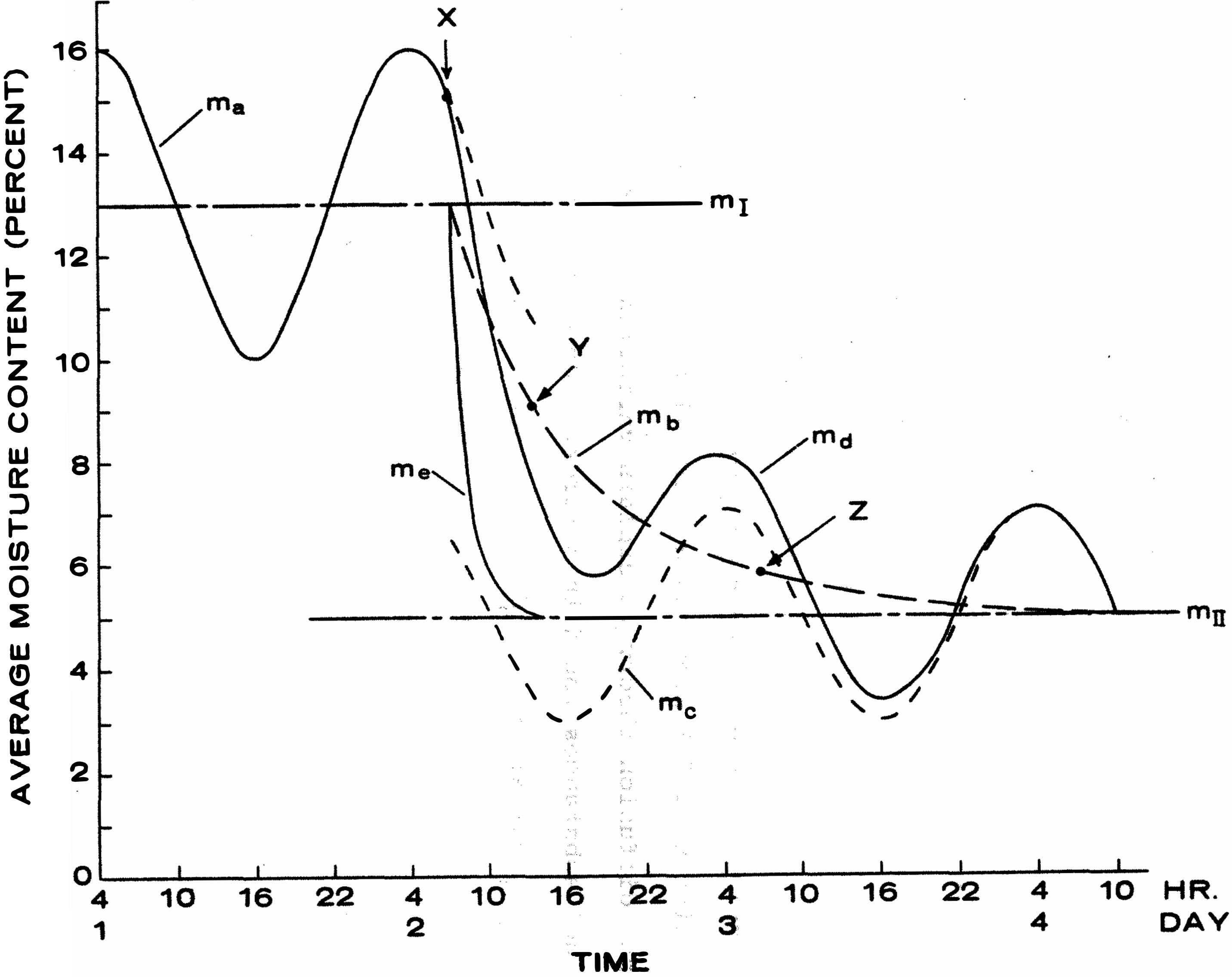
3. THEORETICAL FUEL MOISTURE CONTENT MODEL

A. Below the Fiber Saturation Point

Linton (1962) presents a discussion on the possibility of using diffusion theory to explain variations in moisture content in fuel substances found in a forest. He considers two types of climatic variations, and is concerned with variations below the fiber saturation point (in the absence of rain). The first variation is that which would be associated with the passage of a frontal system. He assumes it to be of an exponential rather than stepwise (instantaneous) nature. In addition to this, he considers diurnal climatic variations, which he approximates with a sinusoidal function. Figure 10 is a graphical representation of these two climatic functions operating together. He gives an equation for the approximate prediction of moisture content from the functions described in Fig. 10:

$$md = mb + \frac{m_I - m_e}{m_I - m_{II}} \cdot (m_a - m_I) + \frac{m_e - m_{II}}{m_I - m_{II}} \cdot (m_c - m_{II}) \tag{18}$$

Figure 10 Diagram of the theoretical effect of an exponential environmental change on fuel moisture



From: Linton (1962) Report on Moisture Variation in Forest Fuels.

Where: md = actual moisture content of fuel sample changes

m_e = exponential climatic change

x = time of initiation of m_e

m_a = diurnal fluctuation of average moisture content about average m_I

m_c = diurnal fluctuation of average moisture content about average m_{II}

mb = response of fuel to change m_e

Basically, this equation calculates the moisture response as the difference between the response to the exponential change and the sinusoidal variation about the initial and final mean values.

The magnitude of the diurnal fluctuations of the average moisture content of a fuel sample are given by:

$$\bar{C} = C_o + C_s \cdot \bar{A} \cdot \sin(\omega t + \psi) \tag{19}$$

Where: $\bar{C}$ = average moisture content (between the maximum fluctuation at the surface and the minimum in the center)

$\bar{A}$ = amplitude of sinusoidal average moisture content fluctuation

C_o, C_s = average moisture content (about which $\bar{C}$ fluctuates), surface moisture content at time t (gm/cm^2)

t = time of measurement (sec.)

ψ = phase lag (in radians)

ω = $2 \cdot \pi \cdot \tau$ = angular velocity (radians/sec.)

τ = period of a sinusoidal cycle (sec.)

He then goes on to define $\bar{A}$ and ψ as complex sinusoidal functions[1/] of a dimensionless diffusion coefficient kl where:

$$Kl = (\omega/2D) \cdot 1/2 \tag{20}$$

[1/] *See Appendix II.*

and: D = diffusion coefficient for the material (cm^2/sec.)

l = half thickness of slab in diffusion direction (cm)

This assumes that the diffusion coefficient varies inversely as the square root of the thickness of the slab. Basically, this series of equations calculates the average moisture content as the mean about which the actual moisture content varies plus a fraction of the surface variation, based on the size and diffusion coefficient of the material.

The "time lag" (difference between the time required for the climatic change (or change in surface moisture content) and time required for the entire fuel to respond by a given fraction of the total change) is given as:

$$\lambda_i = \lambda \cdot (t_{ei} - t_{ci}) \tag{21}$$

Where: λ_i = time lag (sec.)

t_{ei} = time corresponding to a given fraction i of the ultimate change in moisture content of the entire fuel sample in response to an exponential change (sec.)

t_{ci} = time for surface moisture (or climate) to change by a fraction i of the ultimate change in an exponential manner.

Calculation of t_{ei} and t_{ci} is through the use of several complex exponential functions[1] involving among other things the values of D and l.

An attempt to test a modification of this theory with some experimental evidence proved moderately successful. He lists a number of reasons which may account for some of the difficulties.

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[1] *See Appendix II.*

1 - Experimental error

2 - Oven drying the samples

3 - Simplifying assumptions,

    a. simple shape of material (slab or cylinder)

    b. uniform structure and composition

    c. that the transfer is directly proportional to moisture content gradient. This may not be entirely valid because we are dealing with both liquid water and water vapor

    d. a constant diffusion coefficient.

4 - Simultaneous diffusion of heat and moisture must take place.

5 - Sorption hysterisis under equilibrium condition must also be considered.

6 - The approximation of climatic changes with exponential and sinusoidal functions is an ideal solution. In fact, the weather exhibits practically an infinite number of ways of changing.

B. Above the Fiber Saturation Point

With respect to the movement of liquid water some simplifying assumptions must be made. They are:

1 - Loss of water will follow an exponential pattern.

2 - A constant rate of application of water.

3 - There is no delay in the percolation of excess water through the fuel complex. (i.e. when rain stops, only surface and structural water remains).

4 - The rate of diffusion of water through the fuel and evaporative potential of the atmosphere can be quantitatively defined.

5 - Environmental conditions remain unchanged (other than the normal sinusoidal variation) during the application of water and subsequent drying.

6 - Water will continue to be lost regardless of the diurnal cycle until the fiber saturation point is reached, although the rate of loss may vary. In the absence of additional liquid water, the moisture content will not rise until a minimum is reached which will be below the fiber saturation point, (and also below the E.M.C. which would exist under the prevailing environmental condition).

If the rate of rainfall is less than the rate at which water can diffuse into the fuel, the amount absorbed (D_t) can be simply defined as follows:

(22) $$D_t = TW_a$$

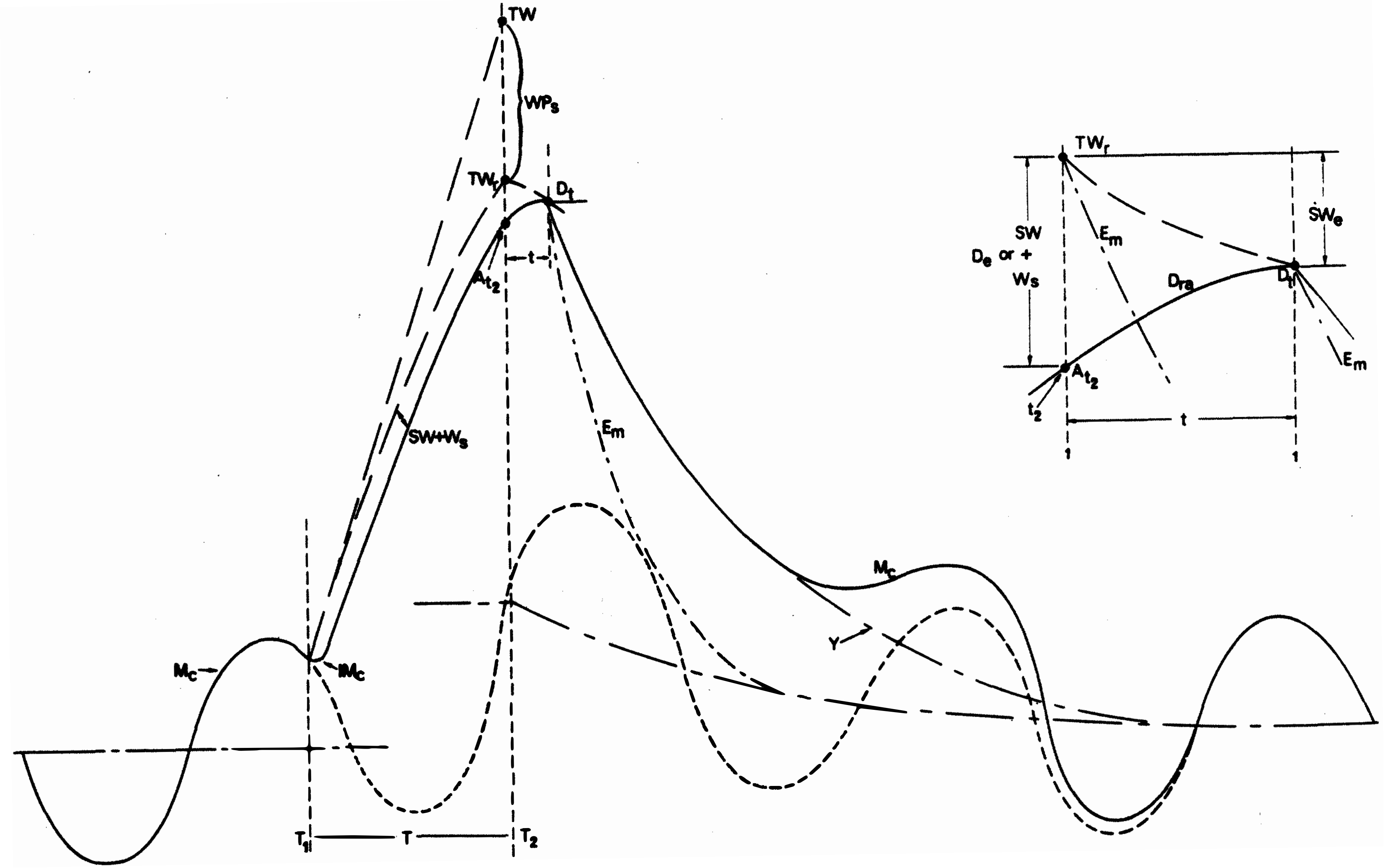
Where: D_t = total diffusion into the fuel complex

TW_a = total water applied to the fuel complex

If, on the other hand, the rate of application exceeds the maximum possible rate of diffusion, a far more complicated picture presents itself. Referring to Fig. 2.11, we see a sinusoidal cycle being interrupted by a rain. The following relationships are apparent: (all units in the following series of equations should be consistent).

(23) $TW_r = TW - WP_s$, and (24) $RW_a = \frac{TW}{T}$

Figure 11 Diagram of the theoretical effect of rainfall on fuel moisture.



where: TW_r = total water retained by the fuel complex

WP_s = amount of water percolated through the fuel complex

RW_a = rate of water application

T = duration of rainfall

The amount retained can be further defined as:

$$TW_r = A_{t2} + SW + W_s \tag{25}$$

where: A_{t2} = amount of water absorbed by the fuel at time t2

SW = surface water

W_s = structural water

(26) $SW = (ad + ST_w) - g$, and (27) $W_s = \Delta ad - g$

where: ad = force of adhesion between the water layer and fuel surface

ST_w = surface tension of water

g = force of gravity

Δad = additional adhesion forces created by the proximity of several individual fuel components

Lastly, the amount of water absorbed when rain stops A_{t2} is given by:

(28) $A_{t2} = D_{ra} T$, where: $D_{ra} = D_{rm} + IMC_r$

where: D_{ra}, D_{rm} = actual and maximum rate of diffusion through the fuel

IMC_r = reduction of rate of diffusion due to low initial moisture content

Of greater importance is the total amount of water absorbed D_t as a result of the rain. Looking at the inset in Fig. 2.11, we can study the behavior of the moisture content immediately after the rain stops. We note that:

(29) $$D_t = D_{ra} \cdot (T - t)$$

where: t = time interval after the rain stops during which surface water remains on the fuel

The interval t can be determined as follows:

(30) $$E_a = E_m - C$$

where: E_a = actual rate of evaporation

E_m = maximum possible rate of evaporation (evaporative potential)

C = time lag constant for the fuel in question

and since:

(31) $$Dl = SW + W_s \text{ , or}$$ (32) $$Dl = (D_{ra} \cdot t) - (E_a \cdot t)$$

where: Dl = diffusion lag between the water available and that absorbed (note that E_a is always negative)

rearranging (2.32) we get:

(33) $$t = \frac{Dl}{D_{ra} - E_a}$$

It should be noted that as long as the rate of rainfall exceeds the rate of diffusion, the amount which falls has no direct bearing on the amount absorbed. Rather it is the duration of surface water which influences the total absorption (D_t). The duration, and to a lesser extent, the amount of rain affect the duration of surface water, and thus indirectly influence total absorption.

When the point D_t is reached, water will be lost in a manner similar to that described by Linton with certain exceptions.

1 - The rate of loss may not be the same as for water vapor.

2 - The mean of the sinusoidal cycle will gradually decrease after water is no longer being applied. This is because humidities normally gradually become lower, as the time since rain stopped increases. This is of minor importance with respect to dew.

This is intended only as a basic summary of a theoretical discussion of moisture content changes above the fiber saturation point. Any further elaboration would require an effort far beyond the scope of this paper.

In reviewing a possible theoretical approach, it can be seen at once that a large portion of the theory consists of assumptions and unknowns. Determination of each of the individual components of the model are major research projects in themselves. The problem is so complex that it is very doubtful that a purely theoretical approach will be able to provide the answer to the fuel moisture content problem without a great deal of additional research. It is for this reason that an empirical approach will be used in this study. Attempts will be made to explain some of the results obtained on the basis of the theory previously discussed. The more complex relationship will simply be stated as observations under given conditions. While this will leave many questions unanswered, it will, it is hoped, provide some relationships which will prove to be useful for the purpose of estimating the moisture content of forest fuels.

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APPENDIX I

REGRESSION EQUATIONS
FOR
EQUILIBRIUM MOISTURE CONTENT

As part of this paper it was intended to test the possibility of using E.M.C. to predict the actual fuel moisture content. Further, since much of the work was to be done on a computer, it was determined that regression equations would be superior to the existing tables for this purpose.

A preliminary examination of two standard tables (Millett 1955, U.S.F.S. 1955) indicated slight differences between them. It was decided to derive equations for both tables so that they could be compared. A standard stepwise regression program developed by the Biometrics Research Branch of the Department of Forestry and Rural Development was used.

Visual examination of the data showed that the function was roughly "S" shaped and could be divided into three sections to facilitate mathematical calculations as follows:

1. Relative humidity less than 10% -- the function is increasing at a decreasing rate.
2. Relative humidity between 10% and 50% -- the function is approximately linear.
3. Relative humidity greater than 50% -- the function is increasing at an exponential rate.

A series of equations were derived for each of these conditions using various combinations of temperature and relative humidity. They are summarized in Table 1.

The only series which require additional comments are those for relative humidity less than 10%. Theoretical considerations suggest that at zero relative humidity the moisture content should also be zero. Experimental proof of this theory is difficult to obtain. While the above equations do not pass through the origin, they represent the best practical mathematical fit to the available data. It can also be noted that a linear equation is being used to estimate a curved function for the F.P.L. data. The use of a log(RH) and RH was also attempted but the resulting equations were only slightly more accurate than the linear ones. Further complications arose with the use of a lograthmic function of a value which approaches zero. Therefore, due to its simplicity, and sufficient accuracy it was decided to use the linear equations.

In the case of the data from "Canadian Woods" the fit with the linear function was considerably poorer than the use of log(RH). Since very little data was available for values of R.H. less than 5%, this equation gives an excellent fit. Due to the characteristics of the log function, it will not give valid results as R.H. approaches zero. Therefore, this equation can not be used below RH = 4%.

Both sets of data give similar but not equal values for E.M.C. The values from the Forest Products Laboratory are slightly lower than those of Millett. This difference is probably caused by the use of different species of wood. Since both tables give the average E.M.C. of wood, the actual value is less important than the shape of the curve. The actual value will have to be adjusted according to the species used.

TABLE 1

DATA SOURCE	VARIABLES	CONSTANT	COEFFICIENTS				R^2 1)	$(\text{RESIDUAL MEAN})^2$
R.H. less than 10%								
F.P.L.	RH	.052660	.230900				.93997	.03980
F.P.L. 2)	$\sqrt{RH}$	-.011662	1.10773				.92080	.04545
C.W.	T	3.08094	-.008643				.24276	.25352
C.W. 3)	log(RH)	.022979	2.82849				.93996	.43666
* F.P.L.	RH, RH x T	.032290	.281073	-.000578			.96459	.02391
C.W.	T,$\sqrt{RH}$	.052510	-.018941	1.41379			.60426	.13691
* C.W.	log(RH), T	.003983	4.54531	-.018957			.98614	.00398
F.P.L.	RH, RH x T, $\sqrt{RH}$	.255226	.230113	-.000574	.251163		.96602	.02338
F.P.L.	RH, log(RH), RH x T	-2.19469	2.52834	-2.85702	-.000654		.96700	.01969
C.W.	T,$\sqrt{RH}$, RH	-8.39678	-.017462	7.75149	-1.18606		.63490	.130664
C.W.	T,$\sqrt{RH}$, RH, RH x T	-13.6475	-.038744	12.8382	-2.32608	-.002614	.66220	.12521
R.H. between 11% and 50%								
C.W.	RH	1.40799	.158665				.80582	.96145
F.P.L.	RH	.982030	.162639				.91947	.32965
* C.W.	RH, T	3.63954	.157206	-.029478			.99329	.03384
* F.P.L.	RH, T	2.22749	.160107	-.014784			.97874	.08853

...2

DATA SOURCE	VARIABLES	CONSTANT	COEFFICIENTS				R^2 1)	$\left(\text{RESIDUAL MEAN}\right)^2$
R.H. greater than 50%								
C.W.	RH^2	1.84150	.002511				.92520	2.3014
F.P.L.	RH^2	3.34725	.001969				.94784	.74925
C.W.	RH^2, RH x T	3.04920	.002809	-.000508			.97232	.86629
F.P.L.	RH^2, RH x T	3.89389	.002208	-.000298			.97556	.35567
* C.W.	RH^2, RH x T, RH	28.2771	.007493	-.000532	-.698061		.99388	.19505
* F.P.L.	RH^2, RH x T, RH	21.0606	.005565	-.000350	-.483199		.99277	.10661
C.W.	RH^2, RH x T, RH, T	29.7908	.007437	-.000232	-.713661	-.022866	.99445	.17990
F.P.L.	RH^2, RH x T, RH, T	21.7450	.005477	-.000179	-.486134	-.012532	.99305	.10386

1) R^2 = Coefficient of Determination

2) Forests Products Lab (U.S.F.S. 1955)

3) Canadian Woods (Millett 1951)

* Considered best in terms of simplicity and accuracy

APPENDIX II

DEFINITION OF FUNCTIONS
IN THE
MOISTURE CONTENT MODEL

Further definition of the functions discussed in Chapter II, Section III, are quite complex. It was decided not to include them in the section itself because it was only intended to present the basic concept - not the method by which it would be carried out. They are presented here for those who wish to carry this concept further.

With respect to sinusoidal variations:

(II.1) $\Lambda = \sqrt{a^2 + b^2}$

(II.2) $\psi = artan \ (a/b)$

(II.3) $a = \dfrac{sinh \ 2kl - sin \ 2kl}{4kl \ (cos^2kl + sin \ h^2kl)}$

(II.4) $b = \dfrac{-(sinh \ 2kl + sin2kl)}{4kl(cos^2kl + sinh^2kl)}$

With respect to exponential changes, a dimensionless time lag L_i is given as:

(II.5) $L_i = (T_{ei} - T_{ci}) = \lambda_i \dfrac{D}{l^2}$ (II.6)

where

(II.7) $T = t \dfrac{D}{l^2}$

A dimensionless time constant B is defined as:

(II.8) $B = \beta \dfrac{l^2}{D}$

where β is a time constant

An exponential climate change (or surface concentration) is defined by:

(II.9) $$\phi = \frac{C_s - C_o}{C_\infty - C_o} = 1 - e^{BT}$$

Where C_s is surface moisture at time t, C_o is the initial and C_∞ the final moisture content. By substituting B, T_{ci}, and i

(where $0 < i < 1$) in eq. (II.9) and rearranging he obtains:

(II.10) $$T_{ci} = \frac{1}{B} \ln\left(\frac{1}{1 - i}\right)$$

For the fractional gain or loss in moisture content of a slab fuel, B and T are substituted in equations given by Crank (1956):

(II.11)
$$\frac{M}{M_\infty} = \frac{m - m_o}{m_\infty - m_o} = \frac{1 - \tan(B^{\frac{1}{2}}) - e^{-BT}}{B^{\frac{1}{2}}} - \frac{8}{\pi^2} \sum_{n=0}^{\infty} \frac{e^{-(2n+1)^2 \cdot \frac{\pi^2}{4} T}}{(2n+1)^2 \left[1 - \frac{(2n+1)^2 \cdot \pi^2}{4B}\right]}$$

where m is the average moisture content at time t and $M/M_\infty = i$

Finally, the fractional change for a cylinder is given as:

(II.12) $$\frac{M}{M_\infty} = 1 - \frac{2J_1 \cdot (B^{\frac{1}{2}}) \cdot e^{-BT}}{B^{\frac{1}{2}} \cdot J_o \cdot (B^{\frac{1}{2}})} + 4 \sum_{n=1}^{\infty} \frac{e^{-(x_n)^2 T}}{(a_n)^2 \cdot \left(\frac{\alpha_n^2}{B} - 1\right)}$$

where α_n are the roots of $J_o(\alpha_n) = 0$ and where J_o, J_1 are Bessel functions of order zero and unity respectively.

With these equations, it is possible to calculate the time required for various fuels with known diffusion coefficients to go through given fractions of the total change which will occur in response to exponential climatic changes.