LINEAR PROGRAMMING, RESOURCE ALLOCATION AND NON-MARKET BENEFITS

Forest Economics Research Institute

Résumé en français

DEPARTMENT OF THE ENVIRONMENT
CANADIAN FORESTRY SERVICE
PUBLICATION NO. 1298
1971

ABSTRACT

A basic problem of resource management is the allocation of land and other scarce resources among competing uses. The problem is compounded in the case where some uses provide no market-valued outputs, or in which some of the outputs are non-market valued. The opportunity cost of providing these uses is one method of providing a surrogate value. Linear programming, or any standard allocation procedure, is then appropriate for the actual allocation of resources.

RÉSUMÉ

Un problème fondamental dans l'aménagement des ressources est la répartition des terres et d'autres ressources rares parmi des usages compétitifs. Le problème est de plus amplifié dans le cas où certains usages des terres n'ont pas de valeur marchande ou même quand certains des produits n'ont pas de valeur. Le coût alternatif de pourvoir à ces usages est une méthode de pourvoir une valeur succédanée. Une programmation linéaire, ou n'importe quel autre moyen normal de répartition, est donc appropriée à la répartition actuelle des ressources.

Published with the authority of the
Minister of the Environment
Ottawa, 1971

INFORMATION CANADA
OTTAWA, 1971

Catalogue No. FO. 47-1298

PRINTED IN CANADA

LINEAR PROGRAMMING, RESOURCE ALLOCATION AND NON-MARKET BENEFITS

by

G.H. Manning[1]

INTRODUCTION

A basic resource problem is allocation of land and other resources among competing uses. The jargon for such a problem is "multiple-use resource allocation". Multiple-use is defined variously. One concept implies optimal single-product use for each unit area of land in an ownership. The competing view is that every unit area of land should be managed for multiple products. Every forester learns (and usually blindly accepts) one or the other of these definitions in his first forestry course, if not before. There is also disagreement as to whether multiple-use is fact or philosophy. Duerr (1964) and Zivnuska (1961) claim the latter.

One major problem in allocating resources among multiple uses is that many outputs of forest management do not have market value, yet provide important social and economic benefits. This paper proposes a system for the allocation of resources among multiple uses that takes into account the presently unmeasurable benefits produced by some uses. A portion of the system consists of a well known technique, linear programming; the other introduces into the system the opportunity cost of providing non-market benefits.

The resource analyst has historically approached the problem of evaluating the benefits of non-market outputs in a number of ways. These fall into two general classes: (1) derivation of a demand curve for the product, and hence of value, and (2) direct estimation of value.

Clawson (1959), Clawson and Knetsch (1966), Trice and Wood (1958), Wennergren (1964) and Dainte (1966) have proposed solutions of the first type. There have been dissenters from this approach, an example being Seckler (1966). A more direct approach was tried by Davis (1963), who used a simulated bidding game to establish demand curves for recreation. Lerner (1962) provides a list and explanation of several attempts at direct estimation of recreational benefits. These range from consumer expenditure studies to the *a priori* value judgement that no matter what the cost of providing recreation, it is worth this amount. Once evaluation has been made, conventional resource allocation tools such as capital budgeting, and benefit-cost analysis have been used.

[1]Economist, Department of the Environment, Canadian Forestry Service, Forest Economics Research Institute, Ottawa, Ontario.

One very desirable type of allocation tool is mathematical optimization. Linear programming, dynamic programming, and other similar methods may be applicable. Use of mathematical programming in forestry problems is described by Curtis (1962), Thompson and Richards (1969), and Teeguarden and Von Sperber (1968). This tool is a portion of the allocation system described herein.

NON-MARKET BENEFITS

The problem remains of providing an adequate pricing mechanism for non-market outputs. One possible approach is to determine the opportunity cost of withholding land from timber production and diverting it to other uses, or reducing timber output of a given unit of land so that it can provide other services.[2] This approach was suggested by Duerr and Vaux (1953). A major advantage of such a method would be that the analyst would be working with timber values, which are generally determined by an established market.

The concept of opportunity cost is valid only under two conditions. These are: (1) when the resource is scarce, and (2) when the resource has alternative revenue-generating uses. In the case of allocation of land resources among various forest uses, the concept is valid. Forest land which is suitable for producing both timber and quality forest recreation is a scarce commodity. The second requirement is met by the observation that a continuum of output combinations is possible from every land unit.

A method similar to opportunity cost was developed in 1956 by Atkinson (1956), who compared timber and recreational benefits for several areas in California. In brief, his method was to compute a timber value/man-day use ratio for a number of recreational areas and timber production areas. Ranking these from lowest to highest, he noted a distinct separation between timber and recreational properties. The marginal ratio, properly discounted, determined a man-day use value for California recreation.

Allocation via opportunity cost is relatively simple if only one scarce resource is under consideration and only two outputs are possible. The result of the analysis for example, of Devine (1966), and Brandl (1968) is a typical transformation curve between recreation (in physical output units) and market outputs (in dollar units). The decision as to proper allocation is then left to the decision maker, who defines his optimal point on the transformation surface.

---

[2]Quite often, agriculture will provide the highest returns, not timber management. However, for this example timber is assumed to be highest value. In fact only those alternative uses which the owner is willing to accept will generally be considered.

Theoretically, opportunity cost has strong appeal. Brandl (1968) satisfactorily demonstrates that up to the point at which output is chosen, non-market and market products may be treated alike. Definition of indifference curves to choose optimal product mix, though, cannot be theoretically demonstrated for this case due to incommensurable outputs.

Opportunity cost is a useful approach to providing the value of non-market forest output in the case of resource allocation conflicts among timber production, forest recreation and multiple uses. For a given unit area of land, the analyst should know:

(1) The resource requirements for the outputs of interest (i.e. physical productivity of land, capital and labor requirements, possible physical output combinations) from a given land unit.

(2) Prices for the market-valued outputs.

It is possible that subjective value judgements will enter the establishment of the resource requirements. However, this is unlikely. Many studies of the silvicultural and economic response to forest practices have been made. Combined with professional judgement, resource requirements for timber, applicable to the specific area under consideration may be derived. With regard to recreation and combinations of recreation and timber, production functions may possibly be derived by leaning heavily on professional judgement and extrapolation from past experience.

Opportunity cost as the value of the non-market benefits from recreation and multiple-use forestry is not the perfect solution. In the case where the efficiency of allocation of resources to non-timber and multiple uses is in doubt, however, it is one step in determining whether such allocation is justified. Given these opportunity costs, the *decision maker* can then, based on his constraints and objectives in each case, decide whether the cost of providing non-timber or multiple use would be excessive.

LINEAR PROGRAMMING

Given the values of the timber activities, and the opportunity costs of the permitted recreation and multiple-use activities, the allocation system may be implemented. A linear programming technique is indicated as an appropriate first approach.

Linear programming[3] consists of three basic components: (1) the objective, (2) a number of alternative products, and (3) constraints, either resource or managerial. Linear programming is a valid allocative technique

[3]A more detailed explanation of linear programming may be found in E.O. Heady and W. Candler (1958).

only when there are a number of resource constraints and when there are numerous alternative methods of attaining a specific objective, such as benefit maximization or minimizing the costs of production. In addition, the caution must be given that linear programming gives *normative* answers. The application of the answers of linear programming may not give the results forecast. This may be due to factors not considered in the allocation program or to invalid assumptions. Linear programming does, however, provide a *guide* to practical management.

The decision making system for multiple use resource allocation (using linear programming) works as follows:

(1) Resource requirements for the basic outputs are determined, as well as those for the combined outputs (e.g. 50% timber - 50% recreation).

(2) Price for timber is determined.

(3) The opportunity cost of providing either the non-market priced output or the derived combination of this with timber is determined, *for a given unit of land area.*[4] This necessitates knowing something of land productivity.

(4) The net value of the non-market and multiple output products are determined.

(5) The decision maker specifies his objective function.

(6) The linear programming matrix is established, with information previously defined and with the appropriate constraints.

An Example

This system should be applicable to a wide range of resource allocation problems where a number of the alternative outputs are not wholly market priced. One such instance follows. In this case two important bits of information are generated: (1) the opportunity cost of providing recreation to the public on specific areas; (2) the optimum resource allocation and output mix to maximize benefits.

We have hypothesized a given land resource of fixed maximum timber and recreation capabilities. This was done in terms of the terminology of the Canada Land Inventory. Resource requirements assumed are as realistic as the experience of the analyst permits. In utilizing the system, experts related to each type of output would be consulted in deriving the resource requirements and transformation curves.

Assumptions concerning availability of land classes and productivity are outlined in Table 1. In addition to these assumptions, a yearly input budget for the example of $2,000 and 1,600 man days of labor was assumed.

---

[4]The decision maker may eliminate any non-market-priced output or multiple use if he deems the opportunity cost to be excessive.

Table 1. Land Availability and Productivity: A Hypothetical Example

Land Area	Yearly Output per Acre		
	Timber	Recreation	Multiple-use
465 Acres	WP[a] 0.9 MBF X_1	H[b] 7.5 days X_2	WP 0.6 MBF - H 4.0 days X_3
320 Acres	WS 0.56 Cords X_4	C[c] 50.0 days X_5	WS 0.2 Cords - C 20.0 days X_6
27 Acres	WP 1.2 MBF X_7	F[d] 135.0 days X_8	WP 0.4 MBF - F 45.0 days X_9
1,275 Acres	Pop 1.00 Cords X_{10}	H 1.0 days X_{11}	Pop 0.4 Cords - H 1.5 days X_{12}

[a] WP - White pine, WS - White spruce, Pop - Poplar
WP is sawtimber, WF and Pop are pulpwood.

[b] Hunting was calculated on the basis of 25 days/month possible, 3 months per year (H).

[c] Camping was calculated on the basis of a maximum possible of 100 days/year (C).

[d] Fishing was calculated on the basis of a maximum of 150 days/year possible (F).

Table 2. Initial Linear Programming Matrix for the Multiple Use Resource Allocation Problem

		X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	X_{11}	X_{12}
Resource Land	1 465A	1	1	1									
	2 320A				1	1	1						
	3 27A							1	1	1			
	4 1275A										1	1	1
Capital	$2000	1.49	0.25	1.20	1.69	23.45	11.50	1.44	2.17	2.00	0.25	0.25	0.25
Labour	1600 days	1.00	1.00	1.25	0.10	13.00	8.50	1.20	2.00	3.00	0.05	0.10	0.10
Net Value of Output/year		3.24	3.24	3.24	0.04	0.04	-9999.99	17.36	17.36	17.36	0.40	0.40	0.40
(Gross Value of[a] Output/year)		(18.73)	(17.48)	(21.95)	(3.13)	(205.50)	(130.52)	(35.60)	(47.25)	(61.40)	(1.34)	(1.84)	(2.64)

[a] Used for demonstration purposes. Not included in actual programming matrix.

The following timber values were used:

Species	Value
White Pine[5]	\$29.70/MBF
White Spruce[6]	\$ 5.60/Cord
Poplar[6]	\$ 1.34/Cord

The initial programming matrix is shown in Table 2. The opportunity costs for recreation for each land class (single and multiple use) are shown in Table 3. For programming purposes these were converted to *net* values (see Table 2). It is, however, the opportunity costs which are initially of interest to the land manager. The gross output values for recreation and multiple-use outputs were derived using the per day opportunity costs. These values were derived via the following formula:[7, 8]

$$\text{Value/day} = \frac{(\text{output}_1)\ \text{price}_1 - \text{cost}_1 + \text{cost}_2}{\text{output}_2}$$

In the case of multiple uses, the market-priced output is subtracted from the first term on the right hand side. In deriving these values, a price of \$14.00 per man-day of labor was assumed (\$1.75/hour).

Table 3. Gross Cost for Recreation

Variable	Activity	Per day gross cost
X_2	Hunting	\$ 2.33
X_3	Hunting	1.53
X_5	Camping	4.11
X_6	Camping	6.47
X_8	Fishing	0.35
X_9	Fishing	1.10
X_{11}	Hunting	1.84
X_{12}	Hunting	1.40

[5]Stumpage for White Pine was derived from Hair, D. and A.H. Ulrich (1967).

[6]Stumpage for White Spruce and Poplar was taken from information in: Manthy, R.S. and L.M. James (1964).

[7]Where: sub 1 indicates the market-priced outputs and sub 2 indicates non-market priced outputs.

[8] $\frac{(\text{output}_1)\ \text{price}_1}{\text{output}_2}$ = opportunity cost/day.

It was assumed in this example that the decision maker rejected the cost of camping in X_6 as too high. Hence, this activity was deleted from the solution matrix by coding the value of output as $-999999.99.

Given these assumptions and values, the problem was solved by the simplex method; the solution provided an implicit net income[9] to the decision maker of $2,498.12. The implicit annual *net* return to acres managed is $1.21 per acre. Capital is completely exhausted, and only 465 units of labor remains.

Land classes 1, 2, 3 and 4 are completely exhausted.

Table 4. Activities of a Hypothetical Forest Enterprise

Activity	Units (Acres)
X_3 - White Pine and Hunting	465
X_4 - White Spruce	296
X_5 - Camping	24
X_9 - White Pine and Fishing	27

SUMMARY

The major problem in resource allocation, of no formal market or of distorted market values for some forest outputs has a number of solutions. The appropriateness of the various solutions depends in large part on the objective of the resource owner.

A method has been proposed in this report which would be most appropriate to the owner who has made the *a priori* decision to provide non-market forest products and wishes to allocate resources in an optimal fashion, within defined cost maxima.

The method provides, within the limits of the assumptions and value judgements used, an optimal economic allocation of scarce land and multiple uses. Some care is needed in making the necessary value judgements due to the long term nature of the allocations. The method is limited mainly by

[9]Due to the use of opportunity cost to value recreation benefits, these are not actual monetary returns. Hence, the results of the program are not valid as an indication of possible changes in input proportions. The program once run gives only an indication of resource allocation under *existing* production functions, no more.

the size of the computer used to solve the simplex matrix. In many cases, the number of alternative activities will probably exceed the capacity of the computer being used to solve the problem. Decomposition algorithms are available to cope with such problems. While not as efficient as the simplex method, the answers given will be mathematically correct.

LITERATURE CITED

Atkinson, W.A. 1956. A method for the recreational evaluation of forest land. Unpublished M.S. Thesis. Department of Forestry, University of California. Berkeley, California.

Brandl, J.E. 1968. On the treatment of incommensurables in cost-benefit analysis. Land Economics 44:523-528.

Clawson, M. 1959. Methods of measuring the demand for and the value of outdoor recreation. Resources for the Future, Inc., Washington, D.C., Reprint 10.

Clawson, M. and J.L. Knetsch. 1966. Economics of outdoor recreation. The Johns Hopkins Press. Baltimore, Md.

Curtis, F.H. 1962. Linear programming the management of a forest property. Journal of Forestry, 60:611-616.

Dainte, R.J. 1966. Methods for determination of demand for outdoor recreation. Land Economics 42:327-338.

Davis, R.K. 1963. Value of outdoor recreation: an economic study of the Maine woods. Unpublished Doctoral Dissertation. Department of Economics, Harvard University. Cambridge, Mass.

Devine, E.J. 1966. The treatment of incommensurables in cost-benefit analysis. Land Economics 42:383-387.

Duerr, W.A. and H.J. Vaux. 1953. Research in the economics of forestry. Charles Lathrop Pack Forestry Foundation. Washington, D.C.

Duerr, W.A. 1964. Economic guides to multiple-use policy. Proceedings of the Society of American Foresters, 1963:163-166.

Hair, D. and A.H. Ulrich. 1967. The demand and price situations for forest products - 1967. U.S.D.A. Forest Service. Miscellaneous Publication 1066.

Heady, E.O. and W. Candler. 1958. Linear programming methods. Iowa State University Press. Ames, Iowa.

Lerner, L. 1962. Quantitative indices of recreational value. Committee on Water Resources and Economic Development of the West, Report 11: 55-80.

Manthy, R.S. and L.M. James. 1964. Marketing pulpwood in selected areas of the North Central Region. Michigan State University. North Central Region Research Publication 156.

Seckler, D.W. 1966. On the uses and abuses of economic science in evaluating public outdoor recreation. Land Economics 42:494.

Teeguarden, D.E. and H.L. Von Sperber. 1968. Scheduling Douglas-fir reforestation investments: a comparison of methods. Forest Science 14:354-368.

Thompson, E.F. and D.P. Richards. 1969. Using linear programming to develop long-term, least-cost wood procurement schedules. Pulp and Paper Magazine of Canada, March, 1969 pp. 172-175.

Trice, A.H. and S.E. Wood. 1958. Measurement of recreation benefits. Land Economics 34:195-207.

Wennergren, E.B. 1964. Valuing non-market price recreational resources. Land Economics 40:303-314.

Zivnuska, J.A. 1961. The multiple problems of multiple use. Journal of Forestry 59:555-560.

APPENDIX

This appendix is included for the benefit of those who wish to use the proposed resource allocation system. Anyone attempting to use the program should have at least token familiarity with computer programming techniques and linear programming.

The program is written in FORTRAN IV, for the IBM 360. Many other machines with at least 100K core storage will also handle the problem. At present the program will handle problems of up to 25 equations in up to 40 variables. This may be increased by changing the appropriate dimension statements in the program. All data is punched right justified.

The input data cards for a simple problem are shown below. They are punched as follows:

Card 1. This indicates the number of problem sets in the computer run and is punched in I10 format. This card appears only once for each run, even if multiple problems are solved.

Card 2. This is the first actual data card for each problem. It contains four data fields and a heading for the problem. The format is (4I10, 10A4). The first field is a problem identification number. The second field is the number of equations (restrictions). The third is the number of variables (activities). If the fourth field is left blank, only the first and last tableaux are printed, if it is "1", all tableaux are printed.

Card 3. As many cards as are necessary should be used. They contain the column indices of the variables in the matrix. These numbers start with indices for the slack variables[10] (note in Figure 1 that slack variables are 5-8). Format is 8I10.

Card 4. As many cards as are necessary should be used. They contain the cost coefficients of the variables in the objective function, listed in the same order as in Card 3 (including slack variables. The format is (8F10.4). In the case of variables to be eliminated, the cost coefficient should be punched as -9999999.99. This will drive the variable from the calculation.

Card 5. Again, as many cards as necessary are punched. A separate set is prepared for each line of the program matrix. Slack variables are not included but are generated by the program. The format is (8F10.4). The first field of the first card of each set contains the restriction (i.e., 800 acres, etc.). A format field may be left blank if the data element equals zero. Remember, each row of the tableau (programming matrix) must have its own set of Card 5 cards.

---

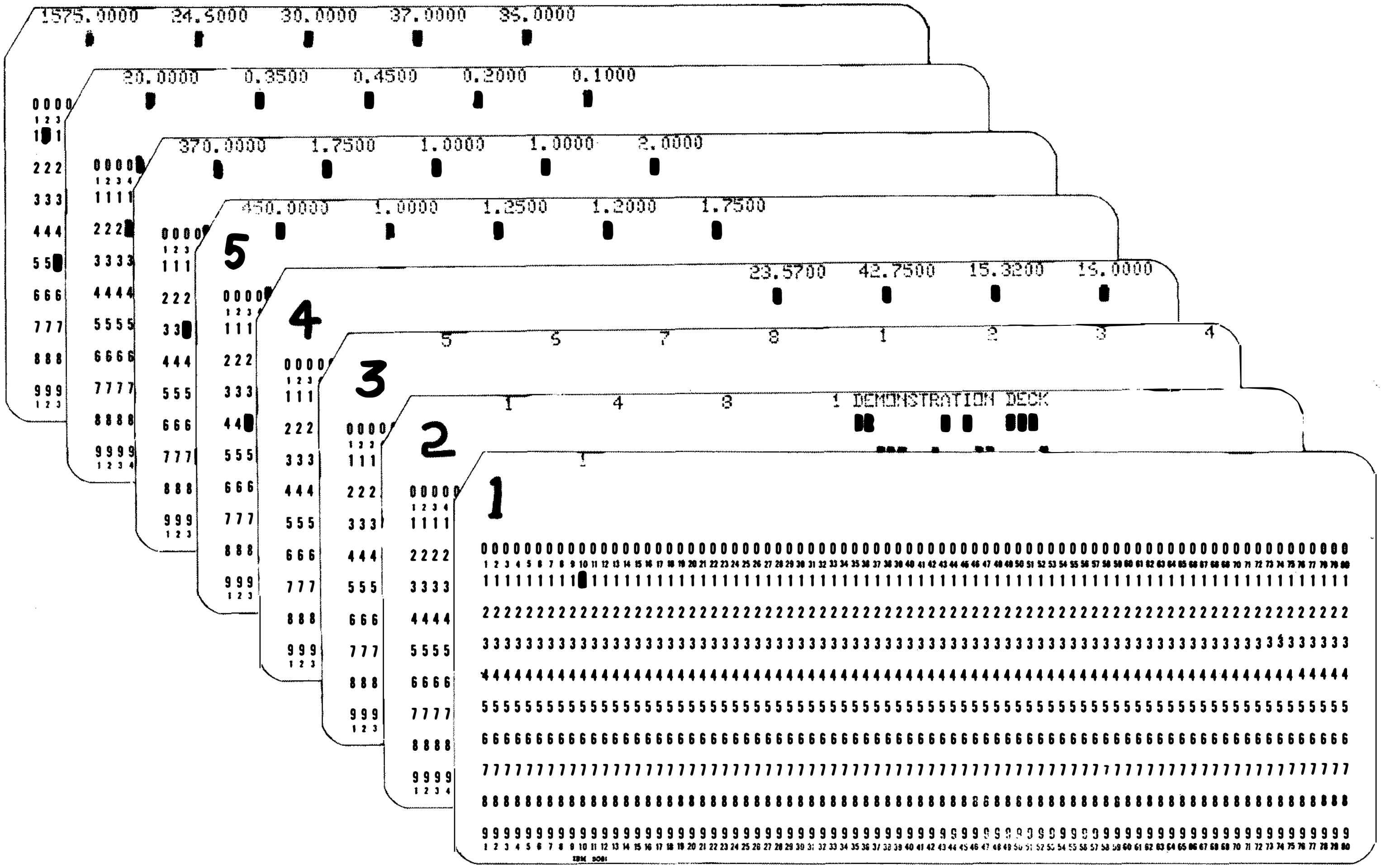
[10]Slack (or disposal) variables are activities which utilize resources not used in real activities. These variables have a cost of zero (Note for card 4). For more complete discussion see E.O. Heady and W. Candler (1958).

The output is as follows:

1. The output of each problem starts on a new page.
2. The first item printed is the problem number and heading information.
3. The cost vector is printed out next. These are listed above the variable indices of output 5.
4. The next output is the iteration number.
5. The next output is the original tableau. This is listed row by row as follows:
   (a) The variable indices are listed first, seven to a row.
   (b) The rows of the tableau proper listed next with the index of the solution variable, the value of that solution variable and the elements of A being listed in the usual order.
   (c) The last two rows are the Z_j and the C_j-Z_j (shadow prices) rows.
   (d) The identity matrix will be the first m columns of A.
6. If the first problem card has called for a tableau at each iteration, outputs 4 and 5 are repeated until the final solution has been printed. If the first problem card did not call for intermediate outputs, output 4 is given for each iteration and outputs 4 and 5 for the final iteration.
7. If tableau is reached in which the objective function proves to be unbounded, the note "THE OBJECTIVE FUNCTION IS NOT BOUNDED" takes the place of output 5 and the next problem is read in or the run ended, depending on whether additional problems exist.

It must be emphasized again that those using this technique should have some knowledge of computer programming and the techniques of linear programming. For those with an adequate background, the previously mentioned volume by Heady and Candler (1958) will be adequate introduction to linear programming. For those with little economics background, a pamphlet by Bennet Foster and Richard Weyrick, entitled *A Modified General Simplex Method for Solving Linear Programming Problems* may be obtained from the New Hampshire Agricultural Experiment Station, University of New Hampshire, Durham, New Hampshire.

Illustrations on following pages


1575.0000 24.5000 30.0000 37.0000 35.0000
20.0000 0.3500 0.4500 0.2000 0.1000
370.0000 1.7500 1.0000 1.0000 2.0000
450.0000 1.0000 1.2500 1.2000 1.7500
23.5700 42.7500 15.3200 15.0000
5 6 7 8 1 2 3 4
1 4 8 1 DEMONSTRATION DECK
5
4
3
2
1
IBM 5081

PROBLEM NUMBER 1 RESOURCE ALLOCATION - N.M. BENEFITS

ITERATION 1

VARIABLE COSTS		0.0	0.0	0.0	0.0	0.0	0.0	3.2400
		3.2400	3.2400	0.0400	0.0400	-999999.9900	17.3600	17.3600
		17.3600	0.4000	0.4000	0.4000			
SOLUTION					TABLEAU			
		13	14	15	16	17	18	1
		2	3	4	5	6	7	8
		9	10	11	12			
13	465.0000	1.0000	0.0	0.0	0.0	0.0	0.0	1.0000
		1.0000	1.0000	0.0	0.0	0.0	0.0	0.0
		0.0	0.0	0.0	0.0			
14	320.0000	0.0	1.0000	0.0	0.0	0.0	0.0	0.0
		0.0	0.0	1.0000	1.0000	1.0000	0.0	0.0
		0.0	0.0	0.0	0.0			
15	27.0000	0.0	0.0	1.0000	0.0	0.0	0.0	0.0
		0.0	0.0	0.0	0.0	0.0	1.0000	1.0000
		1.0000	0.0	0.0	0.0			
16	1275.0000	0.0	0.0	0.0	1.0000	0.0	0.0	0.0
		0.0	0.0	0.0	0.0	0.0	0.0	0.0
		0.0	1.0000	1.0000	1.0000			
17	2000.0000	0.0	0.0	0.0	0.0	1.0000	0.0	1.4900
		0.2500	1.2000	1.6900	23.4500	11.5000	1.4400	2.1700
		2.0000	0.2500	0.2500	0.2500			
18	1600.0000	0.0	0.0	0.0	0.0	0.0	1.0000	1.0000
		1.0000	1.2500	0.1000	13.0000	8.5000	1.2000	2.0000
		3.0000	0.0500	0.1000	0.1000			
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
		0.0	0.0	0.0	0.0	0.0	0.0	0.0
		0.0	0.0	0.0	0.0			
		0.0	0.0	0.0	0.0	0.0	0.0	3.2400
		3.2400	3.2400	0.0400	0.0400	-999999.9900	17.3600	17.3600
		17.3600	0.4000	0.4000	0.4000			

ITERATION 6

SOLUTION		TABLEAU						
		13	14	15	16	17	18	1
		2	3	4	5	6	7	8
		9	10	11	12			
3	465.0000	1.0000	0.0	0.0	0.0	0.0	0.0	1.0000
		1.0000	1.0000	0.0	0.0	0.0	0.0	0.0
		0.0	0.0	0.0	0.0			
4	295.7146	0.0551	1.0777	0.0919	0.0115	-0.0460	0.0	-0.0133
		0.0437	0.0	1.0000	0.0	0.5492	0.0257	-0.0078
		0.0	0.0	0.0	0.0			
9	27.0000	0.0	0.0	1.0000	0.0	0.0	0.0	0.0
		0.0	0.0	0.0	0.0	0.0	1.0000	1.0000
		1.0000	0.0	0.0	0.0			
12	1275.0000	0.0	0.0	0.0	1.0000	0.0	0.0	0.0
		0.0	0.0	0.0	0.0	0.0	0.0	0.0
		0.0	1.0000	1.0000	1.0000			
5	24.2854	-0.0551	-0.0777	-0.0919	-0.0115	0.0460	0.0	0.0133
		-0.0437	0.0	0.0	1.0000	0.4508	-0.0257	0.0078
		0.0	0.0	0.0	0.0			
18	464.9685	-0.5386	0.9019	-1.8143	0.0482	-0.5928	1.0000	-0.4219
		0.3132	0.0	0.0	0.0	2.5843	-1.4680	-1.1008
		0.0	-0.0500	0.0	0.0			
	2498.1200	3.2400	0.0400	17.3600	0.4000	0.0	0.0	3.2400
		3.2400	3.2400	0.0400	0.0400	0.0400	17.3600	17.3600
		17.3600	0.4000	0.4000	0.4000			
		-3.2400	-0.0400	-17.3600	-0.4000	0.0	0.0	0.0000
		0.0000	0.0	0.0	0.0	-1000000.0300	0.0000	0.0000
		0.0	0.0	0.0	0.0			

```
              C      PROGRAM SIMPLEX
              C      BASIC SIMPLEX ALGORITHM                                         70010
              C      THE ALGORITHM IS DESIGNED TO SOLVE THE MAXIMIZING PROBLEM,
              C      THE MINIMIZING PROBLEM MAY BE SOLVED WITH A SIGN INVERSION OF
              C      THE COST EQUATION
              C      MAX. SIZE PROBLEM IS 25 EQUATIONS,  40 VARIABLES
 0001                DIMENSION JVAR(40),IVAR(25) ,TITLE(10)
 0002                DOUBLE PRECISION EP,CMZM,Z,ENT,VALUE,THETA,TH,PIVOT   ,
                    1          CI(25),C(40)        ,
                    2          CMZ(40),A(25,40)       ,
                    3          P(25),ZJ(40)
 0003              1 FORMAT(1H /59X,9HITERATION,I5)
 0004              2 FORMAT (8F10.4)
 0005              3 FORMAT (8I10)
 0006              4 FORMAT(4I10,10A4)
 0007             10 FORMAT(1H )
 0008            102 FORMAT(1H ,20X,37HTHE OBJECTIVE FUNCTION IS NOT BOUNDED)
 0009            112 FORMAT(1H ,21X,7I15/(22X,7I15))
 0010            121 FORMAT(1H ,I5,F14.4,6X,7F15.4/(26X,7F15.4))
 0011            122 FORMAT (1H ,25X,7F15.4)
 0012            123 FORMAT ( 1H ,4X,F15.4,6X,7F15.4/(26X,7F15.4))
 0013            211 FORMAT(1H ,10X,14HPROBLEM NUMBER,I7,10X,10A4///)
 0014            325 FORMAT(1H /7X,8HSOLUTION,67X,7HTABLEAU)
 0015            326 FORMAT(1H ,14HVARIABLE COSTS,11X,7F15.4/(26X,7F15.4))
 0016            335 FORMAT(1H1)
              COMMENT 1      READ NUMBER OF PROBLEMS
 0017                READ (1,3)NPRO
              COMMENT 2      TO STATEMENT 192, ONE PROBLEM IS SOLVED
 0018                DO 192 NP=1,NPRO
 0019                IT=0
 0020                WRITE (3,335)
 0021                EP=.5D-6
              COMMENT 3      READ PROBLEM DATA
              C                   NPROB = PROBLEM NUMBER
              C                   M = NUMBER OF ROWS OF MATRIX A
              C                   N = NUMBER OF COLUMNS INCLUDING IDENTITY MATRIX
              C                   IF IREP = 0, ONLY FIRST AND LAST SOLUTIONS ARE
              C                        PRINTED
              C                   JVAR IS AN ARRAY OF THE NUMBERS OF THE VARIABLES.
              C                        THE NUMBERS OF THE VARIABLES FORMING THE
              C                        THE IDENTITY MATRIX MUST BE INCLUDED. THE JVAR
              C                        VECTOR APPEARS 8 ELEMENTS TO A CARD
              C                   C IS THE COST VECTOR. LISTED 8 ON A CARD, THE COST
              C                        ELEMENTS MUST APPEAR IN THE SAME SEQUENCE AS
              C                        THE ELEMENTS OF JVAR
              C                   A IS THE COEFFICIENT MATRIX. THE IDENTITY MATRIX IS
              C                        ASSUMED TO BE AT THE EXTREME LEFT OF A BUT ITS
              C                        ELEMENTS ARE NOT - REPEAT NOT - READ IN AS PART
```

```
             C                       OF A
             C                  P IS THE VECTOR OF RIGHT HAND CONSTANTS, ALL
             C                       MUST BE NONNEGATIVE
 0022              READ (1,4)NPROB,M,N,IREP,TITLE
 0023              READ (1,3)(JVAR(J),J=1,N)
 0024              READ (1,2)(C(J),J=1,N)
 0025              L=M+1
 0026              DO 300 I=1,M
 0027          300 READ (1,2)P(I),(A(I,J),J=L,N)
             COMMENT 4      THE INITIAL IDENTITY MATRIX IS CREATED ALONG WITH THE
             C                  CI AND IVAR COLUMNS
 0028              DO 301 I=1,M
 0029              DO 302 J=1,M
 0030          302 A(I,J)=0.0
 0031              CI(I)=C(I)
 0032              IVAR(I)=JVAR(I)
 0033          301 A(I,I)=1.0
 0034              WRITE (3,211)NPROB,TITLE
             COMMENT 5      DETERMINE ENTRY COSTS AND SELECT ENTERING VARIABLE
 0035          194 CMZM=-.9D70
 0036              DO 74 J=1,N
 0037              Z=0.0
 0038              DO 75 I=1,M
 0039           75 Z=Z+A(I,J)*CI(I)
 0040              ENT=C(J)-Z
 0041              ZJ(J)=Z
 0042              CMZ(J)=ENT
 0043              IF(CMZM-ENT)201,201,74
 0044          201 CMZM=ENT
 0045              JIN=J
 0046           74 CONTINUE
 0047              IT=IT+1
             COMMENT 6      GO TO PRINTOUT OF PRESENT TABLEAU IF IREP IS NOT ZERO OR
             C                  IF THIS IS EITHER THE FIRST OR THE LAST TABLEAU
 0048              IF(IREP)601,202,601
 0049          202 IF(IT-1)203,601,203
 0050          203 IF(CMZM-EP)601,801,801
             COMMENT 7      COMPUTE PROGRAM VALUE
 0051          601 VALUE=0.0
 0052              DO 110 I=1,M
 0053          110 VALUE=VALUE+P(I)*CI(I)
             COMMENT 8      PRINT ORIGINAL COST VECTOR. THIS IS DONE ONLY ONCE
             C                  PER PROBLEM
 0054              WRITE (3,1)IT
 0055              WRITE (3,10)
 0056              IF(IT-1)204,205,204
 0057          205 WRITE(3,326)(C(J),J=1,N)
 0058          204 WRITE(3,325)
```

1298-5-71-3.8M

```
            WRITE (3,10)
      COMMENT 9      PRINT COMPLETE TABLEAU
            WRITE (3,112)(JVAR(J),J=1,N)
            WRITE (3,10)
            DO 320 I=1,M
            WRITE (3,121)IVAR(I),P(I),(A(I,J),J=1,N)
        320 WRITE (3,10)
            WRITE (3,123)VALUE,(ZJ(J),J=1,N)
            WRITE (3,10)
            WRITE (3,122)(CMZ(J),J=1,N)
      COMMENT 10     IF PRESENT SOLUTION IS OPTIMAL, GO TO END OF PROBLEM LOOP
            IF(CMZM-EP)192,801,801
      COMMENT 11     COMPUTE THETA AND IDENTIFY LEAVING VARIABLE
        801 THETA=.9D70
            DO 812 I=1,M
            IF(A(I,JIN)-EP)812,812,206
        206 TH=P(I)/A(I,JIN)
            IF(THETA-TH)812,207,207
        207 THETA=TH
            IOUT=I
        812 CONTINUE
      COMMENT 12     PRINT ERROR SIGNAL IF OBJECTIVE FUNCTION IS NOT BOUNDED
      C                       THEN ABORT THE PROBLEM
            IF(THETA-.9D50)501,208,208
        208 WRITE(3,102)
            GO TO 192
      COMMENT 13     CREATE THE NEXT TABLEAU
        501 PIVOT=A(IOUT,JIN)
            P(IOUT)=P(IOUT)/PIVOT
            DO 521 J=1,N
        521 A(IOUT,J)=A(IOUT,J)/PIVOT
            DO 522 I = 1,M
            IF(I-IOUT)209,522,209
        209 P(I)=P(I)-P(IOUT)*A(I,JIN)
            DO 523 J=1,N
            IF(J-JIN)210,523,210
        210 A(I,J)=A(I,J)-A(I,JIN)*A(IOUT,J)
        523 CONTINUE
            A(I,JIN)=0.0
        522 CONTINUE
            EP=EP+.50-6
            CI(IOUT)=C(JIN)
            IVAR(IOUT)=JVAR(JIN)
            A(IOUT,JIN)=1.0
      COMMENT 14     GO TO THE START OF THE ITERATION LOOP
            GO TO 194
      COMMENT 15     ONE PROBLEM HAS BEEN SOLVED
        192 CONTINUE
            STOP
            END
```