A stand density management diagram for balsam fir in New Brunswick

by M. Penner[1], D.E. Swift[2], R. Gagnon[3] and J. Brissette[4]

ABSTRACT

A stand management density diagram (SDMD) is presented for balsam fir (*Abies balsamea* (L.) Mill.) forests in New Brunswick. The SDMD incorporates a maximum size density line, as well as quadratic mean diameter and top height isolines. Several mortality functions are evaluated. The resultant SDMD should be a useful tool for projecting early stand development and determining the timing and intensity of thinnings.

Key words: Acadian Forest Region, mortality curves

RÉSUMÉ

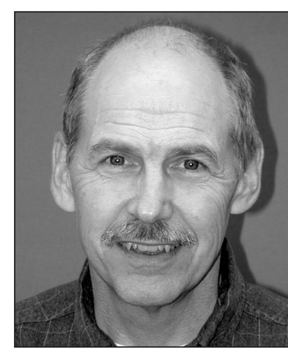
Un diagramme de gestion de densité de peuplement est présenté pour les forêts de sapin baumier (*Abies balsamea* (L.) Mill.) du Nouveau-Brunswick. Le diagramme de gestion de densité de peuplement incorpore une ligne maximum de densité aussi bien que la moyenne quadratique du diamètre et les isolignes de la hauteur maximale. Plusieurs fonctions de mortalité sont évaluées. Le diagramme de gestion de densité de peuplement résultant sera un outil utile pour projeter le développement des peuplements juvéniles et déterminer le moment et l'intensité des éclaircies.

Mots-clés : Région forestière acadienne, courbes de mortalité

M. Penner

D.E. Swift

R. Gagnon



J. Brissette

Introduction

Balsam fir (*Abies balsamea* (L.) Mill.) is one of the most common tree species in New Brunswick, occurring across a variety of site types (Ritchie 1996, Ecological Classification Working Group 2003). The species ranges across New Brunswick, preferring moist, cool sites at high elevations and lowlands along waterways, and forms 19.7% of the forest in New Brunswick, either as pure or mixed species stands. Young balsam fir stands often have a very high number of stems/ha, which results in great intraspecific competition (Ker 1987, Frank 1990, Seymour 1992). Precommercial thinning operations are often conducted in eastern North America to reduce intraspecific competition and increase the merchantable growth rates by accelerating diameter increment on individual tree stems, and to decrease harvest rotation ages (Ker 1981, 1987; Piene 1992; Seymour 1992; Pothier 2002). Often, total yield of the stand is reduced, but the merchantable volume may increase (Assman 1970, Mitchell and Goudie 1997, Varmola and Salminen 2004). Increasingly, commercial thinning is also undertaken in these precommercially thinned stands in eastern North America (Canadian Woodlands Forum 1998, Wagner *et al.* 1999). Until recently, forest management in New Brunswick focused on maximizing merchantable fiber production. Market and economic changes in the forest industry in northeastern North America are changing the emphasis toward management for optimal tree stem size for timber products (Zhang and Liu 2006).

[1]Forest Analysis Ltd., 1188 Walker Lake Dr., R.R. 4, Huntsville, ON P1H 2J6. E-mail:mpenner@vianet.on.ca
[2]Natural Resources Canada, Canadian Forest Service – Atlantic Forestry Centre, P.O. Box 4000 Fredericton, N.B. E3B 5P7.
[3]J.D. Irving, Ltd, Woodlands Division, 181 Aiton Road, Sussex East, N.B. E4G 2V5.
[4]USDA Forest Service, PO Box 640 Durham, NH 03824.

A variety of management tools exists to aid the resource manager in density management decisions, such as the timing and extent of thinning operations. Stand density management diagrams (SDMDs) are a management tool that has been developed for a number of tree species across North America (Drew and Flewelling 1979, Newton and Weetman 1994, Smith and Woods 1997, Farnden 2002). They are based on the theory of self-thinning and the relationships between average diameter, top height, density, and volume for pure species, even-aged stands (Westoby 1984, Jack and Long 1996). Given any two of the following attributes — average diameter, top height, density, and volume — the remaining attributes can be inferred from the diagram (Archibald and Bowling 1995) and used for making stand-level silvicultural decisions (Woods 1998, Farnden 2002). Newton *et al.* (2004, 2005) and Newton and Amponsah (2005) have incorporated diameter distribution models into SDMDs.

Stand density management diagrams are available for a number of commercially important tree species (Farnden 2002). Sturtevant *et al.* (1998) developed an SDMD for balsam fir–black spruce (*Picea mariana* (Mill.) BSP) mixtures and Wilson *et al.* (1999) developed an SDMD for balsam fir–red spruce (*Picea rubens* Sarg.) mixtures. A cooperative research project through the Fundy Model Forest produced three SDMDs for the eastern spruce–balsam fir forest associations of the Acadian Forest Region: 1) balsam fir (Penner *et al.* 2004a), 2) spruce–balsam fir (Penner *et al.*, unpublished report on file at the Atlantic Forestry Centre), and 3) spruce (Penner *et al.* 2004b). This paper builds on the balsam fir SDMD of Penner *et al.* (2004a) by incorporating mortality functions.

The objectives of this paper are to document the construction of the balsam fir SDMD for New Brunswick, illustrate its use, and make it available for testing and application by resource managers throughout the northeastern region of North America.

Data

A total of 127 plots with 584 measurements were used to develop the balsam fir SDMD; these are summarized in Tables 1 and 2. All plots were dominated by balsam fir (>80% balsam fir by basal area).

The Green River spacing trial control plots were established at the time of treatment, but did not have complete tallies until the 5- or 10-year remeasurement because of the very high number of initial stems/ha. The breast height ages at the time of plot establishment were taken from Ker (1987). An average age to breast height of 5 years was applied to the Green River spacing trials data.

The New Brunswick PSP database (Porter *et al.* 2001) did not include heights and volumes for trees <9.1 cm in diameter at breast height (dbh). In order to include the smaller trees in this study, heights and volumes for the smaller trees were estimated using the equations in Appendix II of Porter *et al.* (2001). The height was set to a minimum of 1.3 m.

For the remaining plots, missing heights were estimated using Height (*Ht*)–dbh curves. Height vs. dbh curves were fit separately by plot, measurement, and species, using a Weibull function.

$$\hat{Ht} = 1.3 + \beta_0 (1 - e^{-\beta_1 dbh^{\beta_2}})$$

where

[1] $\hat{Ht}$ = predicted tree height (m)

dbh - diameter at breast height (cm)

β_0, β_1, and β_2 are parameters to be estimated

Basal area (m^2/ha), density (stems/ha), quadratic mean diameter (dbh_q in cm), and mean tree volume (m^3/tree) were summarized by stand, treatment and measurement year. Top height was defined as the average height of the equivalent of the 100 trees/ha with the largest dbh (e.g., for a plot area of 0.04 ha, it is the average height of the four thickest trees). As indicated earlier, for the NB PSPs, the Porter *et al.* (2001) volume equations were used. For the remaining PSP, individual tree gross total volumes were estimated using Honer *et al.* (1983) equation 14. Site index was computed using Ker and Bowling (1991).

Analysis

Maximum size density line

For the maximum size density line, only unthinned plots were used (a total of 203 measurements). All plots showed evidence of mortality between measurements, and thus all measurements were used. A principal component line was fit

Table 1. The data are summarized by source

Source	Number of plots	Number of measurements	Description
Green River			
Lower Belone	8	52	Balsam fir stands spaced at <20 years old.
Lower Chisolm	8	54	The spacing treatments were control, 1.2, 1.8, and 2.4 m^2 spacing, except for Side Hill where the spacing ranged from 0.6 to 4.9 m^2 (Baskerville 1965; Ker 1981, 1987)
Side Hill	16	74	
Summit Road	8	52	
Upper Belone	15	97	
Upper Chisolm	8	54	
Irving	8	17	Precommercially thinned plots
M-301	10	47	Fire-origin stands on fresh to moist sandy loam (Wile 1964)
M-206	7	27	Thinned balsam fir stands
NBpsp	39	110	Balsam fir-dominated plots from the New Brunswick provincial permanent sample plot program (Porter *et al.* 2001).

Table 2. Summary statistics for data to develop the SDMD for balsam fir (127 plots with 584 measurements)

Attribute	Sample size	Mean (range)
Density (stems/ha)	584	3783 (457 – 44543)
Top height (m)	580	12.2 (1.7 – 19.6)
Basal area (m^2/ha)	584	25.3 (0.0 – 62.5)
Age (years)	430	21 (1 – 42)
Site index (m @ age 50)	430	20.6 (11.6 – 24.7)
Gross total volume (m^3/ha)	254	139 (0 – 391)

through the data[5]. A principal component line minimizes the sum of the squared perpendicular distances between the observations and the line, and has the property that the principal component line of Y on X is the same as the principal component line of X on Y. Principal component regression is often used when the same equation is used to predict Y from X or X from Y. Because there is interest in estimating the maximum mean tree volume for a given density, as well as estimating the density corresponding to a given mean tree volume, principal component regression was used to produce the following fitted equation:

$$\ln(\hat{volume}) = 8.49701 - 1.39954\ln(density) \quad [2]$$

where

ln = natural logarithm

$\hat{volume}$ = predicted mean tree gross total volume (m^3/tree)

$density$ = stems/ha

The first principal component accounted for 98% of the variation in the logarithm transformed volume. Assuming a normal distribution, the 95% confidence interval for the slope was (-1.56, -1.28), which includes the theoretical slope of -3/2. The estimated slope was held constant and a new intercept calculated analytically such that the largest residual was zero. This recalculation of the intercept is the principal component equivalent of the corrected ordinary least squares procedures described by Zhang *et al.* (2005).

$$\ln(\hat{volume}) = 9.20151 - 1.39954\ln(density) \quad [3]$$

The maximum size density line and the data are given in Fig. 1. Most of the mortality occurs once stands cross the 0.55 relative density index (RDI) line. The RDI is the ratio of the actual density to the maximum density (from eq. [3]) for the same mean tree volume (Drew and Flewelling 1979).

$$RDI = \frac{\text{actual density}}{\text{maximum density}} = \frac{\text{actual density}}{\exp\left(\frac{\ln(\text{actual volume}) - 9.20151}{-1.39954}\right)} \quad [4]$$

where

RDI = relative density index

$density$ = stems/ha

[5]All analyses were conducted using SAS®, version 8.2.

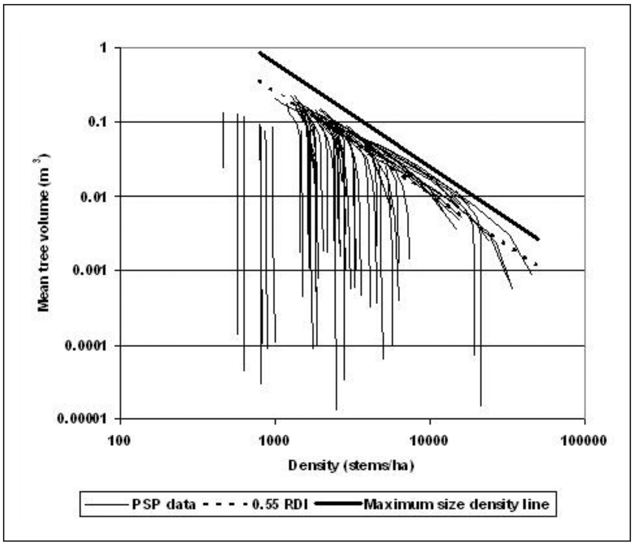


Fig.1. PSP data are plotted, along with the maximum size density line. The 0.55 relative density index line is shown.

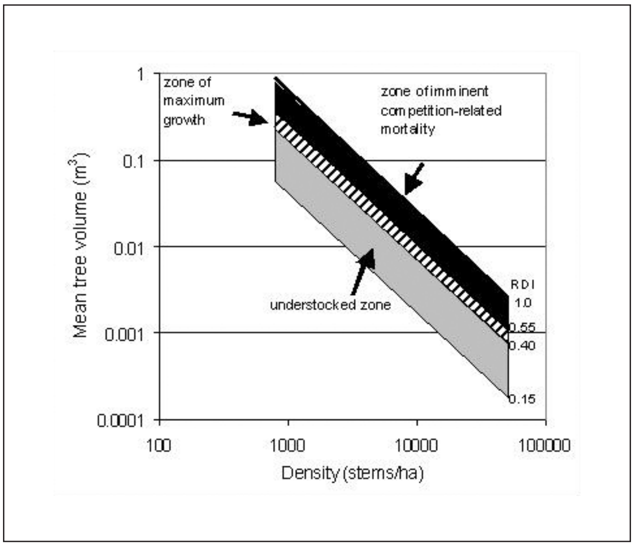


Fig.2. Stand development zones for understocked stands (RDI < 0.15) and for upper and lower bounds of maximum growth (RDIs of 0.55 and 0.40, respectively).

Techniques for fitting by stand development zone (Fig. 2) are not well developed, but a common assumption is that zone boundaries are parallel to the maximum size-density line. The zone of maximum growth should correspond to conditions of the maximum periodic annual increment (PAI) of stand volume (m^3/ha/yr). In this study, the PAI of volume was defined as the increase in stand volume between successive measurements divided by the number of years between the measurements or the average annual stand volume growth between two measurements (Fig. 3). The maximum growth was defined as a PAI > 12 m^3/ha/yr. This threshold was somewhat arbitrary and several other thresholds were also considered. A principal component line was fit through the 12 observations where PAI was >12 m^3/ha/yr.

$$\ln(\hat{volume}) = 7.86900 - 1.392810\ln(density) \quad [5]$$

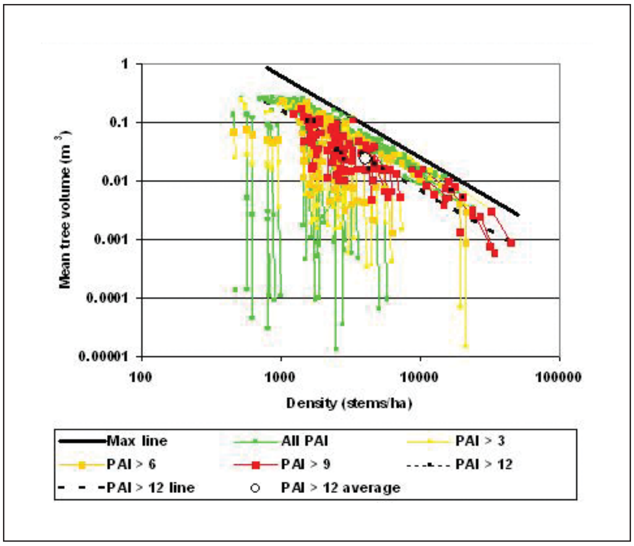


Fig.3. Mean tree volume vs. density by periodic annual increment (PAI) class (m^3/ha/yr).

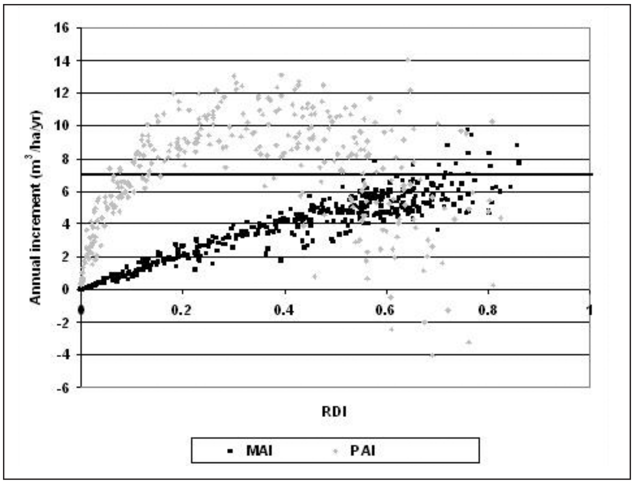


Fig.4. RDI vs. mean annual increment (MAI) and periodic annual increment (PAI) for volume (m^3/ha/yr) including a reference line for 7 m^3/ha/yr.

The principal component fit accounted for 97% of the variation and the slope was very close to –1.39954, the slope of the maximum size density line in eq. [3]. The arithmetic average of the log-transformed density and the log-transformed mean tree volume for observations where the PAI was >12 m^3/ha/yr is given in Fig. 3 and corresponds to an RDI of 0.402. Thresholds of 9, 10, and 11 m^3/ha/yr were also examined corresponding to 116, 74, and 36 observations, respectively. The principal component corresponding to maximum growth was relatively insensitive to whether maximum growth was defined using a threshold or 9, 10, 11, or 12 m^3/ha/yr. The interpretation of eq. [5] is that, as a stand approaches the line, the average PAI increases. Once a stand goes above the line, the average PAI decreases.

Fig. 4 shows the mean annual increment (MAI) of stand volume, as well as the PAI. The MAI increases to a maximum of approximately 7 m^3/ha/yr. It was assumed that stands with PAIs >7 m^3/ha/yr were in the zone of maximum growth because they exceeded the general MAI.

A zone parallel to the maximum size density line that included 50% of the observations where PAI exceeded 7 m^3/ha/yr was constructed so that 25% of the observations were below the zone and 25% above. In a similar manner, a zone was constructed that included 80% of the observations (corresponding to the 10% and 90% percentiles). The resulting zones (Fig. 5) indicate the likelihood that the growth rate of a stand is increasing. Stands within the dark-gray zone are very likely to be growing above the average growth rate. Stands within the lower light-gray zone are growing well, but are potentially understocked, whereas those within the upper light-gray zone are growing well, but are potentially slowing down because of density-related mortality.

The dark-gray zone is considered the zone of optimum growth; the lower bound corresponds to an RDI of 0.23, and the upper bound to an RDI of 0.52. This zone is considerably wider than the traditional 0.40 and 0.55 RDI lines. This wider zone appears to be supported by the high PAIs shown in Fig. 4, which covered a broad range of RDIs.

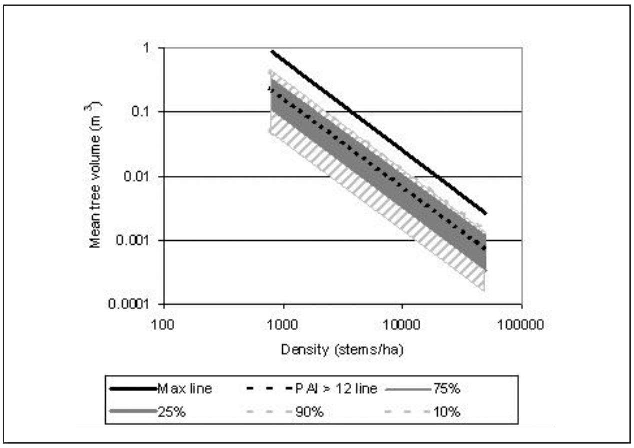


Fig.5. Maximum size density line and the line for periodic increment annual increment (PAI) >12 m^3/ha/yr are plotted with the 10, 25, 75, and 90% percentiles of the PAI line.

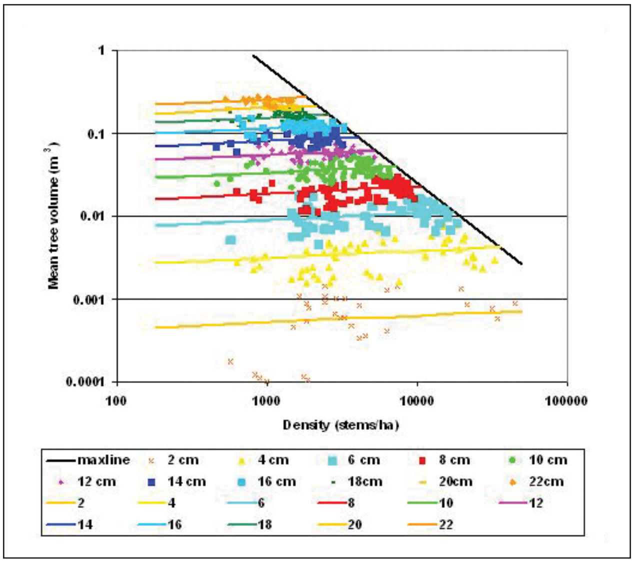


Fig.6. The maximum size density line and the dbh_q isolines are plotted with the data.

Diameter at breast height isolines

The relationship between the logarithm of mean tree volume and the logarithm of density appeared to be linear and parallel for different quadratic mean dbh (dbh_q) values (Fig. 6) where dbh_q is the diameter of the tree of arithmetic mean basal area. In effect, the dbh_q acts as a scalar, increasing the intercept of the relationship between ln(volume) and ln(density). Therefore, least squares regression was used to fit the following equation representing dbh_q isolines:

$$
[6] \quad \begin{aligned}
&\ln(\widehat{\boldsymbol{volume}}) = -9.94018 + 2.59022\ln(\boldsymbol{dbh_q}) \\
&\qquad + 0.084391\ln(\boldsymbol{density}) \\
&\text{where} \\
&\ln = \text{natural logarithm} \\
&\widehat{\boldsymbol{volume}} = \text{predicted mean tree gross total volume (m}^3\text{/tree)} \\
&\boldsymbol{dbh_q} = \text{quadratic mean dbh (cm)} \\
&\boldsymbol{density} = \text{stems/ha}
\end{aligned}
$$

The equation was based on 557 observations and resulted in a mean squared error of 0.01816, an F-value of 49 500 (p value <0.0001), and an r^2 of 0.99. There were no obvious departures from the regression assumptions of a linear relationship and homogenous variance. The average prediction error based on the untransformed mean tree volume was 0.00712 m^3/tree, calculated as follows:

$$
[7] \quad \sqrt{\frac{\sum(\text{actual volume} - \text{predicted volume})^2}{n}}
$$

The data and the isolines from eq. [6] are given in Fig. 6.

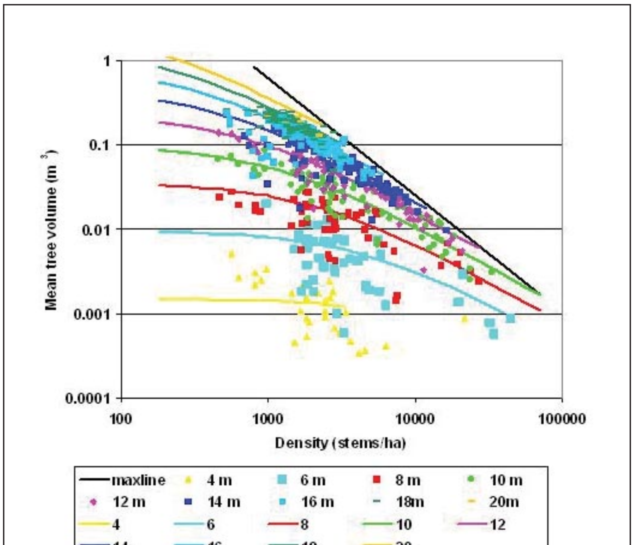


Fig.7. The height isolines from eq. (9) are plotted with the data.

Top height isolines

Drew and Flewelling (1977), citing work in the Japanese literature, presented the reciprocal law of the competition–density effect (combining their equations 29, 31, and 32) as the following:

$$
[8] \quad \begin{aligned}
&1/\boldsymbol{w} = \boldsymbol{a} \cdot \boldsymbol{ht}^{-\boldsymbol{b}}\,\boldsymbol{density} + \boldsymbol{a}'\boldsymbol{ht}^{-\boldsymbol{b}'} \\
&\text{where} \\
&\boldsymbol{w} = \text{mean plant weight} \\
&\boldsymbol{ht} = \text{mean stand height} \\
&\boldsymbol{density} = \text{stems/ha} \\
&\boldsymbol{a}, \boldsymbol{a}', \boldsymbol{b}, \boldsymbol{b}' = \text{parameters to be estimated}
\end{aligned}
$$

Substituting mean tree volume for mean plant weight and top height for mean stand height, this equation was fit to the data using non-linear least squares regression. This relationship curves downward as density increases, and the intercept of the relationship between ln(volume) and ln(density) increases as top height increases. To ensure the equation fit the observations with larger mean tree volumes, the mean tree volume was used to weight the residuals. Thiell (1971, section 6.2) provides a two-step procedure to incorporate the dependent variable as the weight in regression, using the predicted dependent variable as the weight. We used the inverse of the observed dependent variable rather than the predicted dependent variable. This complicates the estimation of confidence intervals.

$$
[9] \quad \frac{1}{\boldsymbol{volume}} = 383061\,\boldsymbol{topht}^{-4.5853} + 0.05883\,\boldsymbol{density}\ \boldsymbol{topht}^{-1.8403}
$$

The equation was based on 557 observations and resulted in a mean squared error of 0.0.0675, an F-value of 2567 (p value <0.0001), and an r^2 of 0.93. The regression assumptions appeared justified. The square root of the average prediction error based on mean tree volume (using eq. [7]) was 0.02641 m^3/tree. The height isolines from eq. [9] are plotted with the data in Fig. 7.

Mortality functions

To make the SDMD dynamic, predictions of change in density over time needed to be added to the model. Several models have been documented. Smith and Brand (1988) and Smith (1989) developed the following density estimation equation:

$$
[10] \quad \begin{aligned}
&\ln(\widehat{\boldsymbol{density}}_i) = \ln(\boldsymbol{density}_0)[1 - (1 - \boldsymbol{e}^{-(\boldsymbol{a}_{11}\boldsymbol{SI} + \boldsymbol{a}_{12}\ln(\boldsymbol{density}_0))\boldsymbol{age}})^{(\boldsymbol{a}_{21} + \boldsymbol{a}_{22}\ln(\boldsymbol{density}_0))}] \\
&\text{where} \\
&\ln = \text{natural logarithm} \\
&\boldsymbol{e} = \text{base of the natural logarithm} \\
&\widehat{\boldsymbol{density}}_i = \text{predicted stems/ha at time } \boldsymbol{i} \\
&\boldsymbol{SI} = \text{site index (m) (top height 25 years from planting)} \\
&\boldsymbol{age} = \text{stand age (years)} \\
&\boldsymbol{dbh_q} = \text{quadratic mean dbh (cm)} \\
&\boldsymbol{a}_{11}, \boldsymbol{a}_{12}, \boldsymbol{a}_{21}, \boldsymbol{a}_{22} = \text{parameters to be estimated}
\end{aligned}
$$

This model requires the specification or estimation of the initial planting density ($density_0$). Assuming managers are interested in projecting current stand conditions into the future, and may not know the initial density, requiring the specification of the initial density was felt to be too restrictive. Therefore eq. [10] was modified and $density_0$ replaced with the density at the beginning of the growth interval.

$$\ln(\hat{density}_{t+1}) = \ln(density_t)[1-(1-e^{-(a'_{11}SI+a'_{12}\ln(density_t))(age_{t+1}-age_t)})^{(a'_{21}+a'_{22}\ln(density_t))}] \quad [11]$$

Algebraic manipulation of eq. [11] and the standardization to an annual rate[6] resulted in the following prediction equation:

$$\hat{MR} = a_0[1-(1-e^{-(a''_{11}SI+a''_{12}\ln(density_t))})^{(a''_{21}+a''_{22}\ln(density_t))}] \quad [12]$$

where

$\hat{MR}$ = predicted mortality rate (stems/ha/yr)

$$a_0 \approx \frac{1}{(t+1)-t}$$

As in Smith (1989) and Smith and Brand (1988), various combinations and transformations of site index (SI), age, and density were included in eq. [12] to improve the fit. Newton and Weetman (1993) accounted for changes in stems/ha by including an ingrowth function (IGR) and a mortality function (MR):

$$\hat{IGR} = a_0 + a_1\frac{1}{H_D} + a_2\frac{1}{RDI} \text{ where } RDI < 0.5 \quad [13]$$

where

$\hat{IGR}$ = predicted annual periodic ingrowth in a 10-year period relative to the density at the beginning of the growth period (stems/ha/yr)

H_D = dominant height replaced here by top height

$$\hat{MR} = b_0 e^{b_1 RDI} \quad [14]$$

where

$\hat{MR}$ = predicted annual periodic mortality rate in a 10-year period expressed as a percentage

Combining the mortality and ingrowth curves (eq. [13] and [14] of Newton and Weetman (1993) resulted in the following equation:

$$\hat{MR} = (\phi_0 + \phi_1\frac{1}{topht} + \phi_2\frac{1}{RDI})\text{RDIflag} + \varphi_3 e^{\varphi_4 RDI} \quad [15]$$

where RDIflag = if $RDI < 0.5$, 0 otherwise

[6]As noted by a reviewer, the periodic mortality rate is not necessarily equal to the instantaneous mortality rate at any point in time. In practice, the mortality rate should be estimated for periods approximately equal to the remeasurement interval (approximately 5 years in this case).

The following coefficients were obtained using non-linear least squares.

$$\hat{MR} = (-0.0111 - 0.00236\frac{1}{topht} + 0.0000006\frac{1}{RDI})\text{RDIflag} + 0.0131e^{0.18142 RDI} \quad [16]$$

where RDIflag = if $RDI < 0.5$, 0 otherwise

The second and third terms were not statistically significant. The inclusion of the RDI flag term was felt to be somewhat arbitrary and an exponent to the second RDI term was added to improve the predictions resulting in the following equation:

$$\hat{MR} = -0.0793 + 0.2071\frac{1}{topht} - 0.00004\frac{1}{RDI} + 0.0161e^{2.1586 RDI^{0.2989}} \quad [17]$$

Only the second and third terms were statistically significant.

The modified Smith (1989) equation has the following form:

$$\hat{MR} = 0.9576[1-(1-e^{-(0.0846SI+0.0489\ln(density_t)-01154topht_t)})^{(0.0555-0.1154\ln(density_t))}] \quad [18]$$

Equations [16], [17], and [18] were compared with a regression of the following form:

$$\hat{MR} = \alpha_0 + \alpha_1 RDI + \alpha_2 density + \alpha_3 topht + \alpha_4 Dbh_q + \alpha_5 BA + \alpha_6 age \quad [19]$$

Backward selection was used to remove non-significant terms in eq. (19) using a significant level of $\alpha = 0.05$. The following equation resulted:

$$\hat{MR} = 0.00676 + 0.35509RDI - 0.00399topht + 0.00259Dbh_q - 0.00449BA \quad [20]$$

Some of the independent variables in the mortality models are highly correlated. Backward selection helped remove some of the redundant variables.

All the mortality models have similar fit statistics (Table 3), except for the modified Smith equation, which has a fairly large bias (3%). The residuals appear to be relatively homogenous with respect to the independent variables examined (Fig. 8). Site index curves (Ker and Bowling 1991) were used to predict top height.

Results

The following algorithm was used to project stand conditions.

1. Given site index, age, and an initial density, estimate top height using the site index curves.
2. Estimate mean tree volume using eq. [9].
3. Estimate dbh_q by inverting eq. [6].
4. Estimate the relative density index using eq. [4].
5. Estimate the mortality rate using eqs. [16], [17], [18], or [20].
6. Update the density and age and return to step 1.

Table 3. Equations [16], [17], [18], and [20] are compared

Attribute	Newton [16]	Newton modified [17]	Smith modified [18]	This study [20]
Sample size	400	400	278 [a]	400
Mean squared error	0.000470	0.000433	0.000239	0.00031574
Bias [b]	1.04%	0.00%	3.25%	0.00%

[a]The sample size is lower since the term age was included in the model and was not available for all measurements.

[b] $$Bias = \sum_{i=1}^{n}\left[\frac{\left(y_i - \hat{y}_i\right)}{n}\right]$$

In the results presented here, the mortality rate was estimated for a 5-year interval.

Stand trajectories for several initial starting conditions are provided in Fig. 9. The predictions are similar, except for the modified Smith model, which overestimates mortality (confirming the positive bias indicated in Table 3). Among the remaining three models, the current study has the smallest residual variation, but all models provide reasonably close estimates.

The Balsam Fir Stand Density Management Diagram

Equations [3], [6], and [9] were used to derive the SDMD for balsam fir as shown in Fig. 10. In addition, RDI lines were added, parallel to the maximum size density lines. The lines correspond to RDIs of 0.23 (lower bound for the zone of optimum growth), and 0.52 (upper bound for zone of optimum growth, and the beginning of competition-related mortality).

The maximum top height in the data was just under 20 m and the maximum age was 42 years. Projection beyond these heights and ages are extrapolations, and are not supported by the data.

Sample SDMD Use

The use of the SDMD is illustrated with a hypothetical stand with a SI of 20 m and an initial density of 5000 stems/ha (Fig. 11.). Without thinning, the stand will cross the 0.52 RDI line and out of the zone of optimum growth at around age 35 and a density of 3100 stems/ha. By age 55, the stand will have approximately 1800 stem/ha and a mean tree volume of 0.188 m^3. If the stand is thinned at age 35 to 1000 stems/ha (corresponding to an RDI of 0.32) to keep it within the zone of optimum growth, by age 55, the stand will have approximately 780 stems/ha and a mean tree volume of 0.380 m^3. Note the gross total volume/ha of the control (unthinned) stand is 336 m^3/ha at age 55 which is greater than the gross total volume/ha of the thinned stand (296 m^3/ha).

Discussion

According to Farnden (2002), SDMDs can exist at various levels of complexity depending on the amount of information presented. The simplest SDMDs contain the basic boundary lines based on the –3/2 power law for self-thinning (similar to Fig. 2). These allow the user to decide if a thinning treatment is warranted. At the second level of complexity, the addition of diameter and height isolines allows prediction of stand development through time. The height isolines are commonly based on the reciprocal law of the competition-density effect (Drew and Flewelling 1977). The third level of complexity is achieved by the addition of size-density trajectories (mortality curves) that are tied to growth models through computer software. The present balsam fir SDMD is at the third level of complexity, and follows the most common diagram format (Farnden 2002). The fourth and last level of complexity is reserved for future SDMD development, such as the addition of diameter distributions, potential wood products, and addition of multiple axes (Newton *et al.* 2004, 2005). The addition of mortality curves improves the diagram's utility and reduces errors from false assumptions by the user (Farnden 2002). Although the addition of mortality will generally improve yield prediction, the results need to be used with caution (Farnden 2002). According to Farnden (2002), the inclusion of mortality curves in an SDMD is appropriate for coarse comparisons of stand-level yields for silvicultural planning, but not for timber-supply planning.

Stand density management diagrams have been built for balsam fir forests in other regions. Bégin *et al.* (2001) determined that the self-thinning relation held constant for natural balsam fir stands of four different ecological regions from boreal forests in Quebec. The maximum density line is similar to that in Fig. 10. In contrast, Wilson *et al.* (1999) produced a red spruce–balsam fir SDMD for the northeastern forests that had significantly different maximum density lines than previously published SDMDs for boreal forests (Newton and Weetman 1994, Sturtevant *et al.* 1998). They attributed the difference to the more productive forests of the Acadian Forest Region, as opposed to those of the Boreal Forest Region (Rowe 1972). Their results suggested the importance of using SDMDs developed for specific regions and species in silvicultural planning. They also found no pattern of separate self-thinning relationships between balsam fir and spruce.

Stand density management diagram decision support software has been developed by the Ontario Ministry of Natural Resources (Woods 1998). The SDMD developed here has been incorporated into the software and is appropriate for use in New Brunswick. Users interested in using the software should contact the second author. Presently, the decision support software allows up to 50 000 stems/ha for balsam fir. However, balsam fir regeneration can be very prolific (Ker 1987, Seymour 1992): young dense stands in excess of 100 000 stems/ha are common in some parts of eastern Canada. As more data become available from research projects and growth-and-yield programs, this information should be incorporated into the existing balsam fir SDMD.

This discussion of SDMDs has ignored some important management considerations. When contemplating thinning, managers should consider the cost of thinning, the type of thinning (thinning from below, crop tree release, creation of access rows, etc.), the extent of the thinning, and the potential of the residual trees to respond to increased growing space. Residual trees with a small live crown may be prone to windthrow and delayed thinning response.

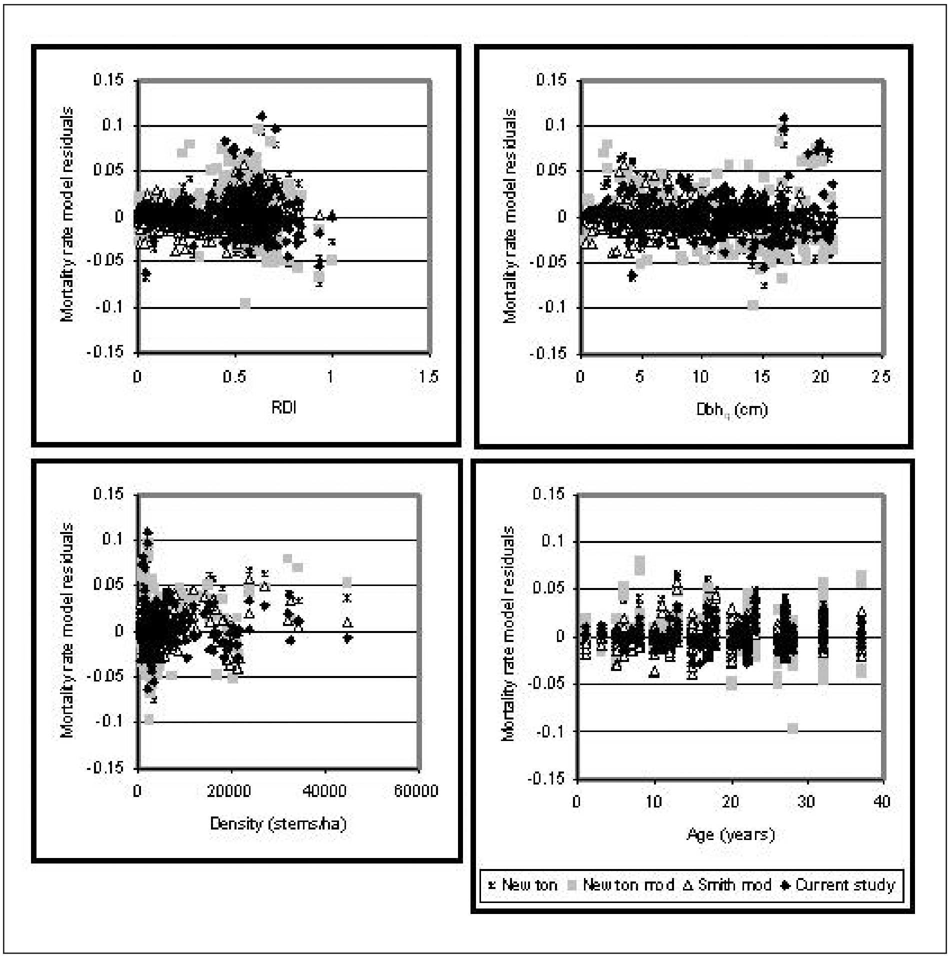


Fig. 8. The prediction errors (observed – predicted mortality rate) are plotted against various independent variables for the four mortality models. *Newton* (eq. [16]) is given by (×), *Newton modified* (eq. [17]) by (■), *Smith modified* (eq. [18]) by (△) and the *Current study* (eq. [20]) by (◆)."

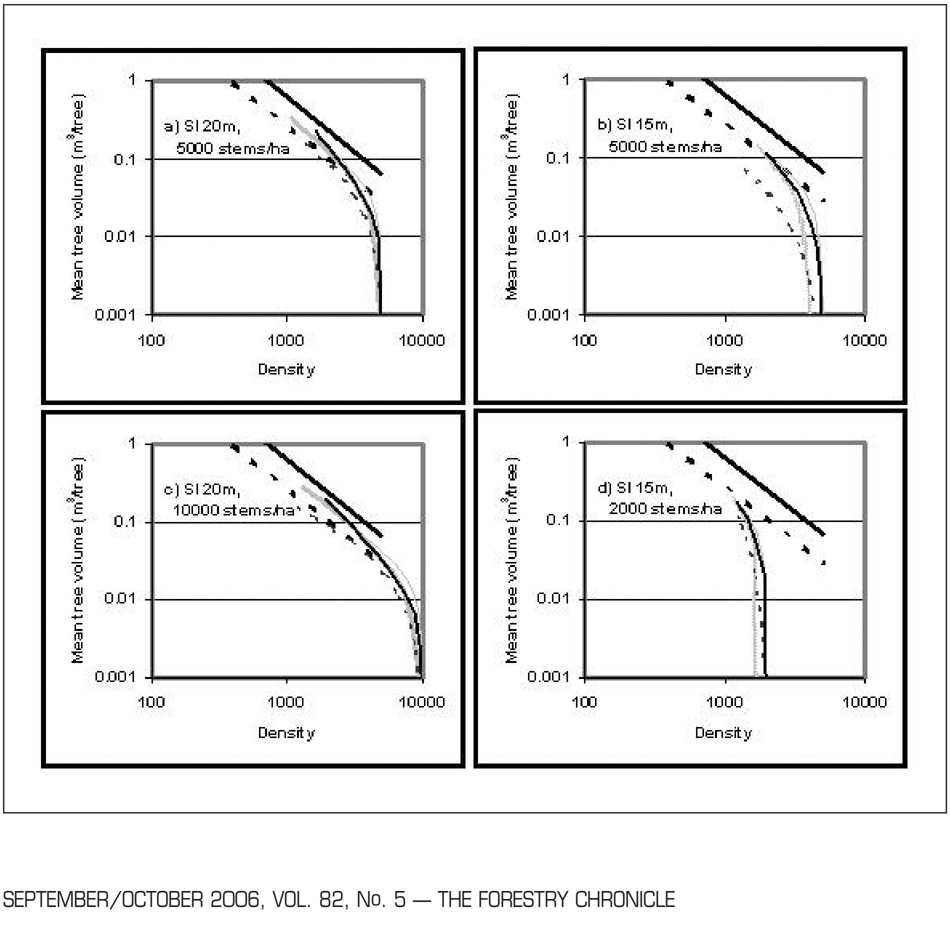


Fig.9. The stand trajectories using the various mortality functions are compared for a variety of stand conditions using the algorithm described in the text. The maximum size density line is given by (▬), the 0.52 RDI line by (- - - -), the *Newton* line by (——), the *Newton modified* line by (▬), the *Smith modified* line by (•••••), and the *Current study* line by (——). *Newton* uses equation [16], *Newton modified* uses [17], *Smith modified* uses [18], and *Current study* uses [20].

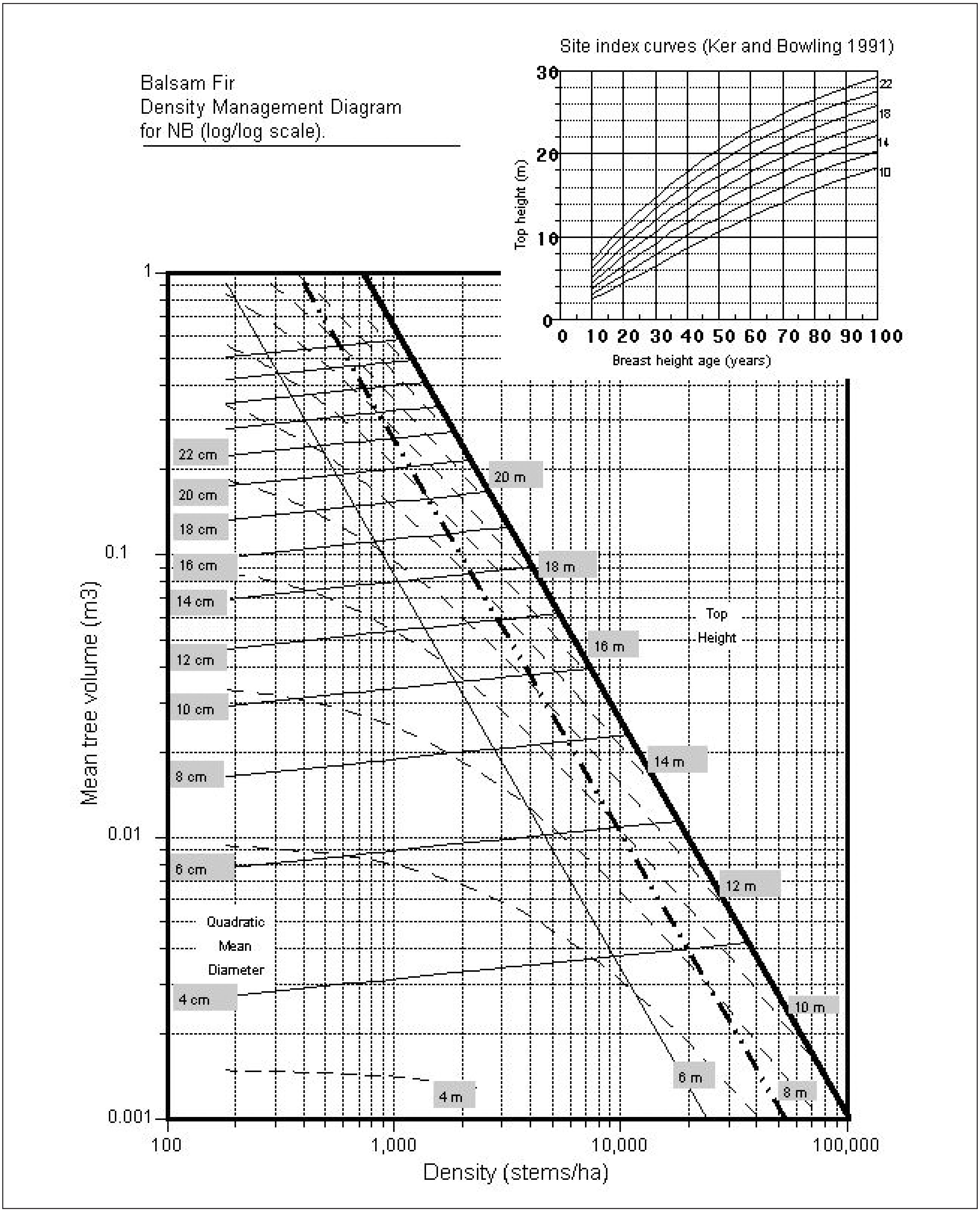


Fig.10. The SDMD for balsam fir in New Brunswick is based on equations [3], [6], and [9]. Site index curves are reprinted with the permission of Natural Resources Canada, Canadian Forest Service – Atlantic Forestry Centre.

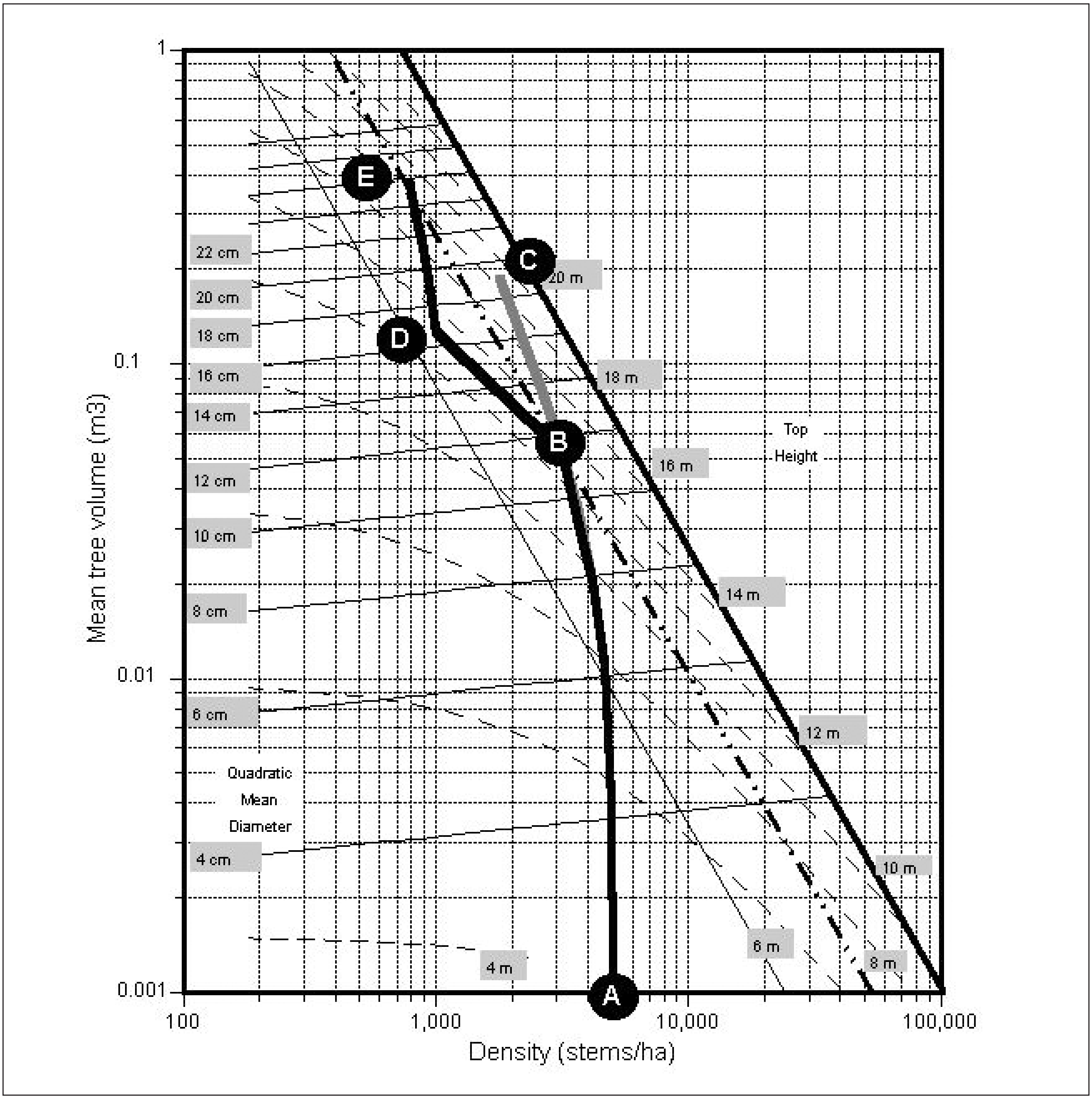


Fig.11. A hypothetical stand is plotted on the SDMD. The control stand starts with a density of 5000 stems/ha and a SI of 20 (**A**), crosses the (**B**) 0.52RDI line at age 35 and reaches a mean tree volume of 0.188 m^3 and 1800 stems/ha by age 55 (**C**). The thinned stand tracks along the control stand (**A** – **B**) up to the 0.52RDI line, at which point it is thinned from 3100 to 1000 stems/ha (**D**) and reaches a mean tree volume of 0.380 m^3 and 780 stems/ha by age 55 (**E**).

Conclusion

Stand density management diagrams with height and dbh isolines provide managers with a graphical tool to aid in the timing and extent of thinning operations. The addition of site index and mortality curves allows the projection of stand conditions. These projections are facilitated by a computer-based decision support system. The SDMD can be used to identify stands requiring thinning to reduce density-related mortality and to project the resulting stand. The balsam fir SDMD presented here is based on stands with a top height <20 m and <45 years old. Future efforts will focus on including dbh distribution models and including older stands as data become available.

Ongoing market and economic changes in the forest industry of northeastern North America are causing a shift in forest management planning from maximizing merchantable fiber production to optimizing tree size for timber products. SDMDs are one tool in a suite of tools required as the forest industry in northeastern North America shifts toward value-oriented silvicultural strategies.

Acknowledgments

Data contributions are greatly appreciated from Natural Resources Canada, Canadian Forest Service; United States Department of Agriculture, Forest Service; New Brunswick Department of Natural Resources; Quebec Ministry of Natural Resources; the Fundy Model Forest; J.D. Irving, Ltd.; Southern New Brunswick Woodlot Co-operative Ltd.; and anonymous sources. We appreciate the comments and suggestions from the following reviewers: Dr. Charles Bourque, Dr. Fan-Rui Meng, Mr. Murray Woods, and an anonymous reviewer. We also thank Mrs. Anne LeBrun-Ruff for translating the abstract into French and Caroline Simpson for her editorial suggestions.

References

Archibald, D.J. and C. Bowling. 1995. Jack pine density management diagram for boreal Ontario. Ontario Ministry of Natural Resources NEST Technical Note TN-005, NWST TN-31. 20 p.

Assman, E. 1970. The principles of forest yield study. Pergamon Press, Oxford. 506 p.

Baskerville, G.L. 1965. Cleaning and thinning young fir, 1958 (Project M-443-59). Progress report. Natural Resources Canada, Canadian Forest Service – Atlantic Forestry Centre, Fredericton, New Brunswick. 18 p.

Bégin, E., J. Bégin, L. Bélanger, L.P. Rivest and S. Tremblay 2001. Balsam fir self-thinning relationship and its constancy among different ecological regions. Can. J. For. Res. 31: 950–959.

Canadian Woodlands Forum. 1998. Commercial thinning workshop. II. Does it pay? *In* Workshop Proceedings, Canadian Woodlands Forum, Atlantic Branch, August 1998. Miramichi, New Brunswick. 249 p.

Drew, T.J. and J.W. Flewelling. 1977. Some recent Japanese theories of yield-density relationships and their application to Monterey pine plantations. For. Sci. 23(4): 517–534.

Drew, T.J., and J.W. Flewelling. 1979. Stand density management: an alternative approach and its application to Douglas-fir plantations. For. Sci. 25(3): 518–532.

Ecological Classification Working Group. 2003. Our landscape heritage: the story of ecological land classification in New Brunswick. New Brunswick Department of Natural Resources and Energy, Fredericton, New Brunswick. CD-Rom. ISBN 1-55236-220-5.

Farnden, C.F. 2002. Recommendations for constructing stand density management diagrams for the Province of Alberta. Report prepared for Ministry of Sustainable Resources and Development, Alberta Land and Forest Division, Forest Management Section, Research Analysis Section. 17 p.

Frank, R.M. 1990. *Abies balsamea* (L.) Mill. Balsam fir. *In* R.M. Burns and B.H. Honkala (eds.). 1990. Silvics of North America. Volume 1. Conifers. pp. 26–35. USDA Forest Service Handbook.

Honer, T.G., M.F. Ker and I.S. Alemdag. 1983. Metric timber tables for the commercial tree species of central and eastern Canada. Natural Resources Canada, Canadian Forest Service – Atlantic Forestry Centre. Information Report M-X-140. 139 p.

Jack, S.B. and J.N. Long. 1996. Linkages between silviculture and ecology: an analysis of density management diagrams. For. Ecol. and Manag. 86: 205–220.

Ker, M. 1981. Early response of balsam fir to spacing in northwestern New Brunswick. Natural Resources Canada, Canadian Forest Service – Atlantic Forestry Centre Information Report M-X-129. 36 p.

Ker, M. 1987. Effects of spacing on balsam fir: 25-year results from the Green River spacing trials. *In* T.S. Murray and M.D. Cameron (eds). Proceedings of the Precommercial Thinning Workshop. March 19, 1987. pp. 58–75. Natural Resources Canada, Canadian Forest Service – Atlantic Forestry Centre, Fredericton, New Brunswick.

Ker, M.F. and C. Bowling. 1991. Polymorphic site index equations for four New Brunswick softwood species. Can. J. For. Res. 21: 728–732.

Mitchell, K.J. and J.W. Goudie. 1997. The emperor's new clothes. *In* C. Bamsey (ed.). 1998. Stand density management: planning and implementation. pp. 34–44. Clear Water Ltd., Edmonton, Alberta.

Newton, P.F. and I.G. Amponsah. 2005. Evaluation of Weibull-based parameter prediction equation systems for black spruce and jack pine stand types within the context of developing structural stand density management diagrams. Can. J. For. Res. 35: 2996–3010.

Newton, P.F. and G.F. Weetman. 1993. Stand density management diagrams and their development and utility in black spruce management. For. Chron. 69: 421–430.

Newton, P.F. and G.F. Weetman. 1994. Stand density management diagrams for managed black spruce stands. For. Chron. 70: 65–74.

Newton, P.F., Y. Lei and S.Y. Zhang. 2004. A parameter recovery model for estimating black spruce diameter distributions within the context of a stand density management diagram. For. Chron. 80(3): 349–358.

Newton, P.F., Y. Lei and S.Y. Zhang. 2005. Stand-level diameter distribution yield model for black spruce plantations. For. Ecol. and Manag. 98: 181–192.

Penner, M., E. Swift, R. Gagnon and J. Knox. 2004a. A revised stand density management diagram for balsam fir in New Brunswick incorporating Green River spacing trials. Fundy Model Forest Rep., Sussex, New Brunswick. 21 p.

Penner, M., E. Swift, R. Gagnon and J. Knox. 2004b. A stand density management diagram for spruce in New Brunswick. Fundy Model Forest Report, Sussex, New Brunswick. 13 p.

Piene, H. 1992. Timing of spacing operations in young balsam fir. For. Chron. 58: 93–95.

Porter, K.B., D.A. MacLean, K.P. Beaton and J. Upshall. 2001. New Brunswick Permanent Sample Plot Database (PSPDB v1.0): user's guide and analysis. Natural Resources Canada, Canadian Forest Service – Atlantic Forestry Centre. Information Report M-X-209E. 65 p.

Pothier, D. 2002. Twenty-year results of precommercial thinning in a balsam fir stand. For. Ecol. and Manag. 168: 177–186.

Ritchie, G.A. 1996. Trees of knowledge: a handbook of Maritime trees. Natural Resources Canada, Canadian Forest Service – Atlantic Forestry Centre, Fredericton, New Brunswick. 41 p.

Rowe, J.S. 1972. Forest Regions of Canada. Environment Canada, Canadian Forest Service, Ottawa, Ontario. Publication No. 1300. 172 p.

Seymour, R.S. 1992. The red spruce–balsam fir forest of Maine: evolution of silvicultural practice in response to stand development patterns and disturbances. *In* M.J. Kelty, C.D. Larson and C.D. Oliver (eds.). The ecology and silviculture of mixed species forests: a festschrift for David M. Smith. pp. 217–244. Kluwer Academic Publishers, London, UK.

Smith, D.J., and M.E. Woods. 1997. Red pine and white pine density management diagrams for Ontario. Ontario Ministry of Natural Resources, Southcentral Sciences Section Technical Report #48. 31 p.

Smith, N.J. 1989. A stand-density control diagram for western red cedar, *Thuja plicata.* For. Ecol. and Manag. 27: 235–244.

Smith, N.J. and D.G. Brand. 1988. Compatible growth models and stand density diagrams. *In* A.R. Ek, S.R. Shifley and T.E. Burk (eds.). Forest growth modelling and prediction, Volume 2. Proceedings of the IUFRO Conference Aug. 23–27, 1987. pp. 636–643. USDA North Central Forest Experiment Station General Technical Report NC-120.

Sturtevant, B.R., J.A. Bissonette and J.N. Long 1998. Stand density management diagram for mixed balsam fir–black spruce stands. Northern Journal of Applied Forestry 15: 17–22.

Thiell, H. 1971. Principles of Econometrics. John Wiley & Sons, Inc. New York. 736 p.

Varmola, M. and H. Salminen. 2004. Timing and intensity of precommercial thinning in *Pinus sylvestris* stands. Scandinavian Journal of Forest Research 19: 142–151.

Wagner, R.G., A.F. Egan, W.D. Ostrofsky and R.S. Seymour 1999. Thinning in the Maine Forest. Conference Proceedings, November 15–16, 1999. University of Maine, Cooperative Forest Research Unit, Orono, Maine. 116 p.

Westoby, M. 1984. The self thinning rule. Advances in Ecological Research 14: 167–225.

Wile, B.C. 1964. The development of balsam fir thickets Acadia (Project M-301)–Internal Progress Report. Natural Resources Canada, Canadian Forest Service – Atlantic Forestry Centre, Fredericton, New Brunswick. 18 p.

Wilson, D.S., R.S. Seymour and D.A. Maguire. 1999. Density management diagram for northeastern red spruce and balsam fir forests. Northern Journal of Applied Forestry 16: 48–56.

Woods, M.E. 1998. Density management diagrams…tools and uses. *In* C. Bamsey (ed.). Stand density management: using planning tools. pp. 27–33. Clear Water Ltd., Edmonton, AB.

Zhang, L., H. Bi, J. H. Gove and L.S. Heath. 2005. A comparison of alternative methods for estimating the self-thinning boundary line. Can. J. For. Res. 35: 1507–1415.

Zhang, S.Y. and C. Liu. 2006. Predicting lumber volume recovery of *Picea mariana* using parametric and non-parametric regression models. Scandinavian Journal of Forest Research 21: 158–166.