A stand density management diagram for spruce–balsam fir mixtures in New Brunswick

by Edwin Swift[1], Margaret Penner[2], Rolland Gagnon[3] and Jason Knox[4]

ABSTRACT

Balsam fir (*Abies balsamea* (L.) Mill.), red spruce (*Picea rubens* Sarg.), black spruce (*P. mariana* (Mill.) BSP), and white spruce (*P. glauca* (Moench) Voss) often form mixed stands throughout northeastern North America. After harvesting operations or natural disturbances, the resulting natural regeneration may require thinning prescriptions to achieve the desired future stand structure and associated forest products. Stand density management diagrams (SDMDs) can assist the forest manager in examining potential yield implications of stand density management decisions. Data from New Brunswick, Nova Scotia, and Quebec indicate a single SDMD is not appropriate for mixtures of balsam fir and spruce (red and black). The maximum size density line is flatter for mixtures than for pure species stands and the quadratic mean diameter isolines are affected by the species composition. The top height isolines are independent of species composition. The results indicate the SDMD for spruce–balsam fir mixtures needs to be dynamic, incorporating the species proportions. The SDMD has been incorporated into software that prompts the user for the balsam fir fraction and generates the appropriate SDMD.

Key words: Acadian Forest Region, eastern species mixtures, thinning decisions

RÉSUMÉ

Le sapin baumier (*Abies balsamea* (L.) Mill.), l'épinette rouge (*Picea rubens* Sarg.), l'épinette noire, (*P. mariana* (Mill.) BSP) et l'épinette blanche (*P. glauca* (Moench) Voss) constituent souvent des peuplements mélangés dans le nord-est de l'Amérique du Nord. À la suite de l'exploitation ou d'une perturbation naturelle, la régénération naturelle résultante peut nécessiter des prescriptions d'éclaircie afin d'atteindre la structure désirée de peuplement dans le futur ainsi que pour obtenir les produits forestiers qui y sont associés. Les schémas d'aménagement de la densité des peuplements (SADP) peuvent aider un gestionnaire forestier en examinant les implications potentielles au niveau du rendement suite à des décisions d'aménagement de la densité du peuplement. Les données en provenance du Nouveau-Brunswick, de la Nouvelle-Écosse et du Québec indiquent qu'un schéma unique n'est pas approprié pour les peuplements mélangés de sapin baumier et d'épinette (rouge et noire). La courbe de densité maximale est plus plate dans le cas des peuplements mélangés que dans le cas des peuplements purs et les isolignes de la moyenne quadratique des diamètres dépendent de la composition en espèces. Les isolignes de la hauteur maximale sont indépendantes de la composition en espèces. Les résultats indiquent que le SADP pour les peuplements mélangés de sapin et d'épinette doit être dynamique et tenir compte des proportions des espèces. Le SADP fait partie d'un logiciel qui souligne à l'utilisateur la proportion de sapin baumier et génère le schéma adéquat.

Mots clés : Région forestière acadienne, peuplements mélangés de l'Est, décisions d'éclaircie

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Introduction

The eastern spruce (*Picea* spp.)–balsam fir (*Abies balsamea* (L.) Mill.) forests of northeastern North America form a transition zone between the boreal forests of the north and the hardwood forests of the south (Blum *et al.* 1983). The main softwood species of this forest type are red spruce (*Picea rubens* Sarg.), black spruce (*P. mariana* (Mill.) BSP), white spruce (*P. glauca* (Moench) Voss), and balsam fir, which form both pure and mixed stands with many associated species (Eyre 1980, Ritchie 1996, Ecological Classification Working Group 2003). Rowe (1972) was the first to refer to this softwood forest type as part of the Acadian Forest Region. The prolific regeneration associated with these stands after forest harvesting or natural disturbances often produces very high densities (Seymour 1992, Brissette 1996). Precommercial thinning operations are often conducted to reduce intraspecific competition and increase the merchantable volume growth rates by accelerating diameter increment on individual tree stems, thereby decreasing harvest rotation ages (Ker 1981, 1987; Piene 1982; Seymour 1992; Pothier 2002). Although thinning reduces the final harvest volume of the stand, the increased merchantable volume may compensate for this reduction (Assman 1970, Mitchell and Goudie 1998, Varmola and Salminen 2004). Commercial thinning operations in these precommercially thinned stands are also increasing across eastern North America (Canadian Woodlands Forum 1998, Wagner *et al.* 1999).

Stand density management diagrams (SDMDs) are designed to assist managers in making density management decisions, specifically regarding the timing and intensity of thinning treatments (Woods 1999, Farnden 2002). These diagrams are based on the theory of self-thinning and the relationships between average diameter, top height, density, and volume for even-aged, pure species stands (Westoby 1984, Jack and Long 1996). Stand density management diagrams have become an important management tool for even-aged stands in many regions across North America (Drew and Flewelling 1979, Smith 1989, Dean and Jokela 1992, Newton and Weetman 1994, Archibald and Bowling 1995, Newton 1997, Wilson *et al.* 1999).

A cooperative research project was initiated in 1999 through the Fundy Model Forest to develop and produce SDMDs for the eastern spruce–balsam fir mixtures throughout the Acadian Forest Region (Swift 2003). Pure species balsam fir and spruce SDMDs have been developed for stands of the Acadian Forest Region (Penner *et al.* 2004, 2006). Recently, the balsam fir SDMD was modified to incorporate mortality functions (Penner *et al.* 2006). The objective of this study is to test the appropriateness of a single SDMD for the spruce–balsam fir mixed forest type. Other SDMDs have been developed for spruce–fir mixtures (Wilson *et al.* 1999, Sturtevant *et al.* 1998), but the effect of differences in proportion of species on the SDMD relationships has not been explicitly examined.

Data

The data used to develop the SDMD are plotted in Fig. 1 and described in Table 1. They consist of permanent sample plots (PSPs) from New Brunswick (Porter *et al.* 2001), Nova Scotia, and Quebec, and from precommercially thinned stands in southern New Brunswick, from the Penobscot Experimental Forest (PEF) in Maine, and from various studies and experiments in New Brunswick. Measurements were included if the combined basal area in spruce and balsam fir was at least 80% of the total basal area. Plots were classified into the following three species groups:

- "fir" if the balsam fir basal area was at least 80% of the spruce–fir basal area,
- "spruce" if the spruce basal area was at least 80% of the spruce–fir basal area, and
- "spruce–fir" for the rest of the plots.

The data were compiled using a standard format, with a few exceptions, as noted in the description of the Quebec and New Brunswick PSP data. The Quebec data were provided in compiled format. The New Brunswick PSP data set was provided with computed height–diameter curves and these were used in the data compilation. The rest of the data were provided as tree lists with diameter at breast height (dbh) measurements and a sub-sample of heights. As heights were not available for all trees, missing heights were predicted from species and dbh using locally calibrated height–dbh curves. For example, height–dbh curves were estimated by species, plot, and dbh measurements when sufficient height data were available. If sufficient height–dbh pairs were not available, plots were pooled and a height–dbh equation was derived by species and measurement. If sufficient data were still not available, a height–dbh equation was arrived at by species and data set. This procedure generally occurred only for minor species. The curve form used for the height–dbh curves was the Weibull function (Peng *et al.* 2001).

$$[1] \quad ht = 1.3 + \beta_0 \cdot (1 - e^{-\beta_1 \cdot dbh^{\beta_2}})$$

Height was available for all trees for some of the younger stands, but dbh was not available for the smaller trees (generally trees shorter than 3 m). For these trees, dbh was estimated as a linear function of height by species:

$$[2] \quad dbh = \beta_1 \cdot ht$$

Plot data were then compiled to per hectare values. Density was defined as the number of stems per hectare of trees taller than 1.3 m. In Fig. 1 there is evidence that some of the young stands (high densities, low mean tree volumes) are increasing in density as trees surpass the 1.3 m height threshold.

Top height was defined as the average height of the 100 trees of largest diameter per hectare. For a plot area of 0.04 ha, top height was calculated as the average height of the four thickest trees (unless some indication of defect was noted). Individual tree volumes were estimated based on Honer *et al.* (1983; eq. 14 for total volume). A local volume table for the PEF was used to estimate volumes in that data set.

Analysis

Maximum size density line

The Smith and Woods (1997) approach was used to fit the maximum size density line. The natural logarithm (ln) of density was put in classes 0.2 units wide (Fig. 2). Within each ln-density class, the observation with the largest mean tree volume was used to estimate the maximum size density line. Equation 3 was fit by species group using least squares regression:

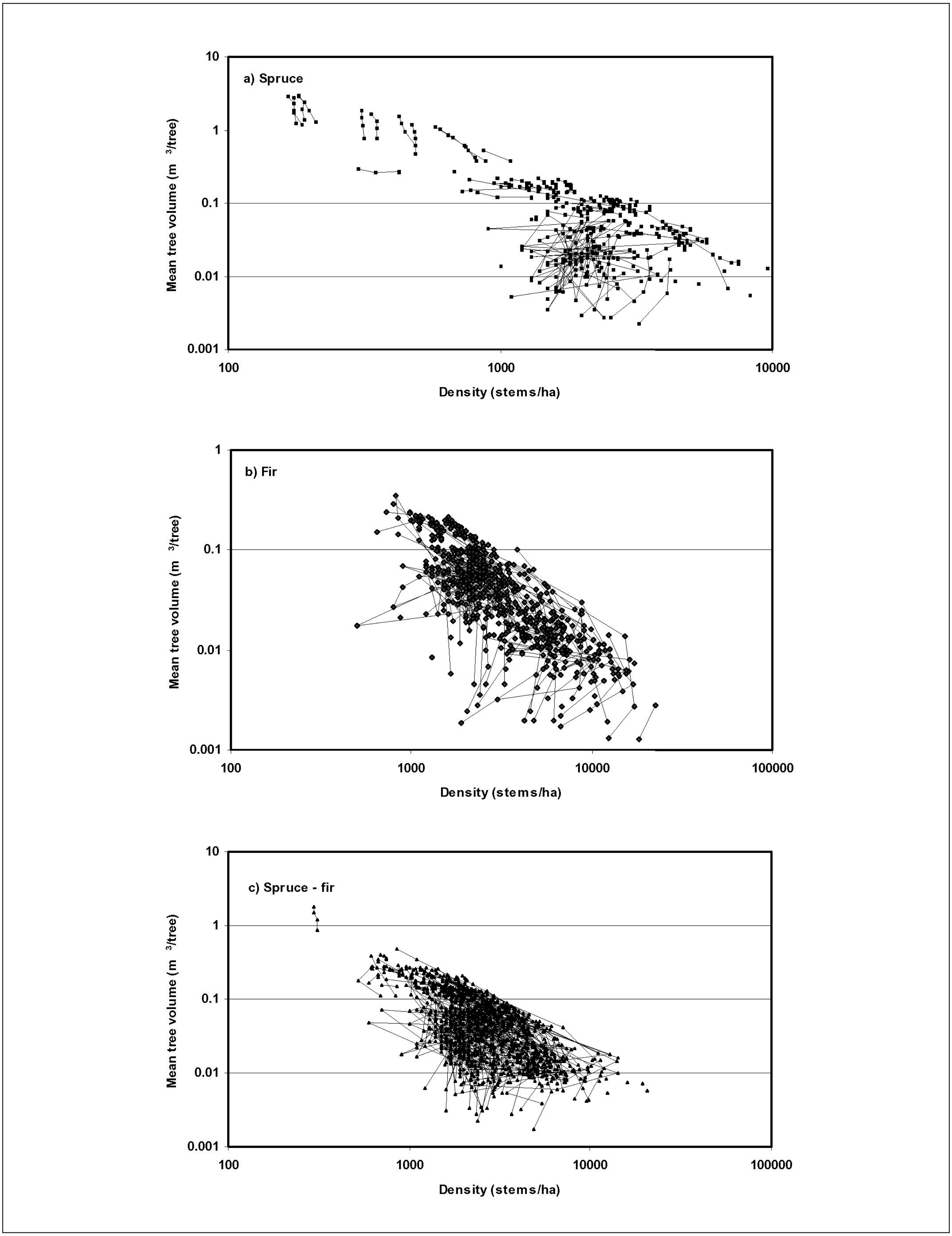

Fig. 1. The data used in developing the spruce–balsam fir SDMD are plotted by species group (Sp = spruce, Spfir = spruce–fir, fir = balsam fir). Re-measurements of the same PSP are joined by solid lines. Density increases over time are due to in-growth of trees: trees surpassing the 1.3 m height threshold.

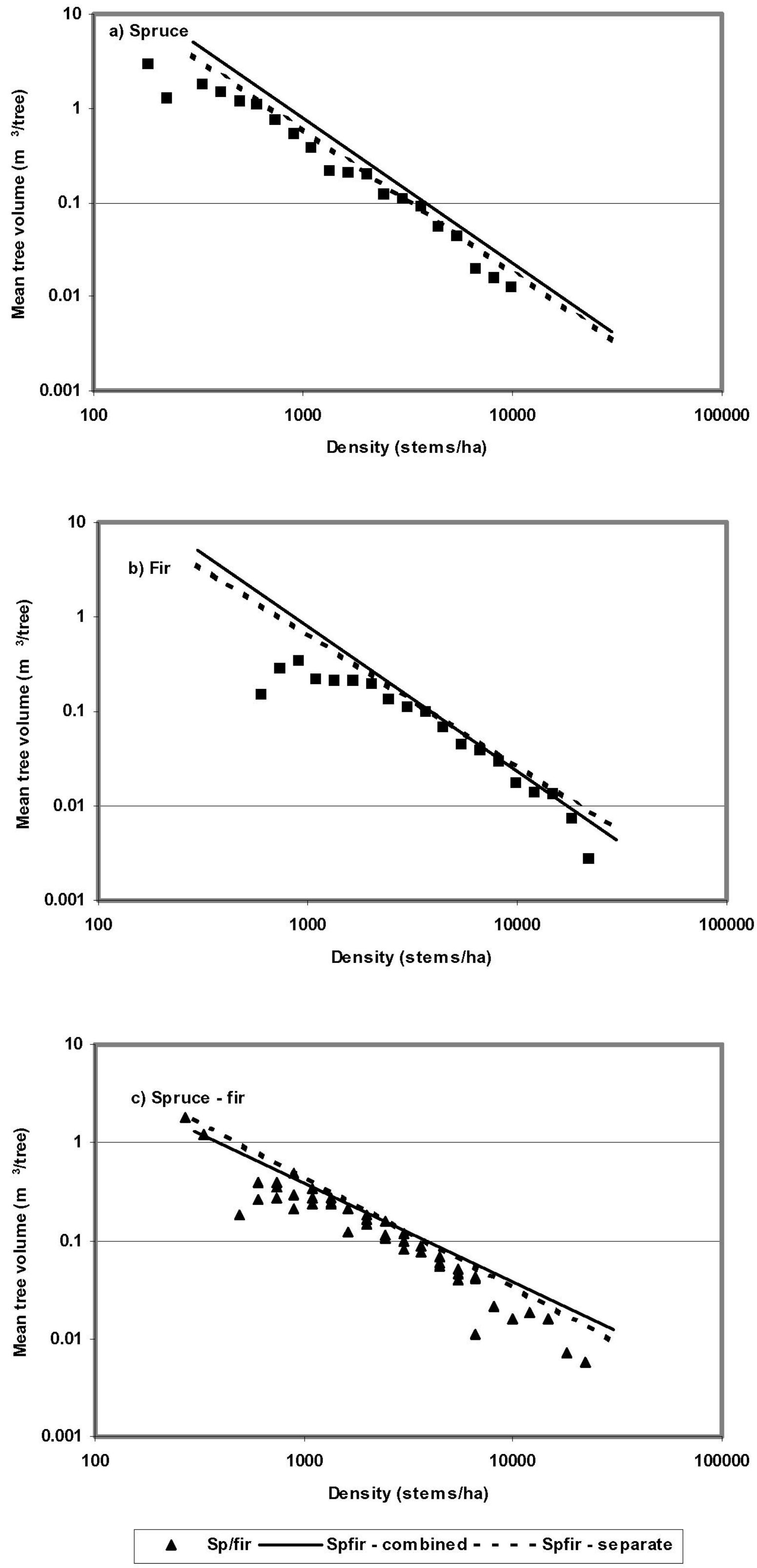

Fig. 2. The data used for fitting the maximum size density line are plotted along with eq. 3 (dashed lines) and 4 (solid lines) by species group.

Table 1. A summary of the data sets used to develop the spruce–balsam fir SDMD; n = number of plot measurements

	Fir		Spruce		Spruce/fir	
Attribute	n	Mean (range)	n	Mean (range)	n	Mean (range)
Age (years)	116	69 (42–115)	297	83 (26–151)	203	72 (28–143)
Density (stems/ha)	909	4196 (50 –22486)	472	2296(166–9612)	1017	3160 (295–20475)
Top height (m)	680	11.6 (3.4–19.5)	465[a]	14.7 (4.5–39.1)	791	2.6 (4.5–36.6)
Basal area (m^2/ha)	909	25.2 (1.5–61.7)	472	25.1 (2.6–57.9)	1017	24.6 (2.5–60.3)
Volume (m^3/ha)	909	119 (3.5–386)	472	165 (5–647)	1017	126 (5–532)
Quadratic dbh (cm)	909	10.0 (2.8–25.0)	472	13.9 (4.1–54.3)	1017	11.0 (3.7–42.8)
Skewness of dbh distribution	412	0.262 (-2.1 to 3.2)	453	0.393 (-1.3 to 3.7)	574	0.539 (-4.9 to 3.8)

[a]The dominant height was missing for five plots in the Quebec data.

$$\ln(volume) = \alpha - \beta \cdot \ln(density) \quad [3]$$

Volume is the mean tree gross total volume (m^3 per tree) and *density* is the total number of stems per hectare. The slope of the line was held constant, and the intercept increased until all the observations were on or below the maximum size density line. The magnitude of the coefficients in eq. 3 appeared to decrease for mixed species stands relative to pure spruce or balsam stands. Therefore, an alternative form of eq. 3 was fit where the coefficients were expanded to include a term for the degree of departure from a pure species condition (*mixfrac*). Again, the intercept was increased until all the observations were on or below the maximum size density line. The resulting equations are presented in Table 2, and compared with other studies in Table 3.

$$[4] \quad \begin{aligned} &\ln(\boldsymbol{volume}) = \alpha_0 + \alpha_1 \cdot \boldsymbol{mixfrac} \\ &- (\beta_0 + \beta_1 \cdot \boldsymbol{mixfrac}) \cdot \ln(\boldsymbol{density}) \\ &\text{where} \\ &\boldsymbol{mixfrac} = 0.5 - \boldsymbol{ABS}(\boldsymbol{firfrac} - 0.5) \\ &\boldsymbol{firfrac} = \frac{\boldsymbol{fir\ basal\ area}}{\boldsymbol{total\ basal\ area}} \end{aligned}$$

At younger ages, mixed spruce–fir stands have higher potential mean tree volumes than those of spruce- or fir-dominated stands for the same density. As the stands age, this trend is reversed so that spruce- and fir-dominated stands have higher potential mean tree volumes than mixed stands for the same density. The maximum size density lines cross at approximately 4000 stems/ha. Below 4000 stems/ha, mixed stands potentially have higher mean tree volumes compared with a pure spruce or fir stand of the same density.

In previous studies (Penner *et al.* 2004, 2006), a principal component approach was used to develop maximum size density lines. The principal component approach can be used to fit lines (in two dimensions) and planes (in three dimensions). The data used in the development of the SDMDs can be considered three-dimensional, with axes of density, mean tree volume, and balsam fir fraction (Smith 1996). A plane was considered to be too restrictive, and thus a regression approach was used to fit a sort of twisted plane. The statistical significance of the *mixfrac* × *density* term indicates the twisted plane is a significant improvement over a simple plane.

Dbh_q isolines

When plotted, the relationship between ln(mean tree volume) and ln(density) appeared linear for a given dbh_q and parallel as dbh_q changed. In effect, the dbh_q acts as a scalar, increasing the intercept of the relationship between ln(volume) and ln(density). Therefore, the following equation was fit for the dbh isolines using least squares regression:

$$[5] \quad \ln(\boldsymbol{volume}) = \beta_0 + \beta_1 \cdot \ln(\boldsymbol{dbh_q}) + \beta_2 \cdot \ln(\boldsymbol{density})$$

The $\beta_0 + \beta_1 \ln(dbh_q)$ term acts as the intercept between ln(volume) and ln(density). Equation 5 was fit by species group. The magnitude of the coefficients in eq. 5 was inversely proportional to the balsam fir fraction. Each of the coefficients in eq. 5 was expanded to include the balsam fir fraction, resulting in the following equation:

$$[6] \quad \begin{aligned} &\ln(\boldsymbol{volume}) = \beta_{01} + \beta_{02} \cdot \boldsymbol{firfrac} \\ &+ (\beta_{11} + \beta_{12} \cdot \boldsymbol{firfrac}) \cdot \ln(\boldsymbol{dbh_q}) \\ &+ (\beta_{21} + \beta_{22} \cdot \boldsymbol{firfrac}) \cdot \ln(\boldsymbol{density}) \end{aligned}$$

The average prediction error based on the untransformed volume was calculated as follows:

$$\sqrt{\frac{\sum(\text{actual volume} - \text{predicted volume})^2}{\boldsymbol{n}}}$$

The results are given in Table 4 and the data are plotted in Fig. 3.

Top height isolines

The data used to fit the dbh isolines were also used to fit the top height isolines. The relationship between ln(volume) and ln(density) was not parallel between height classes. The height appeared to affect the slope of the relationship. Equation 7 was developed by species group for the top height isolines by using non-linear least squares regression techniques. This relationship curves downward as density

Table 2. The results of fitting eq. 3 and 4.

Species group	Parameter	Estimate	n	Standard error	$p(ABS(t_{n-1,0.025}) > t_{obs})$	r^2
Spruce	b	–1.51939	16	0.06714	<0.0001	0.9734
Spruce–fir	b	–1.13087	49	0.05628	<0.0001	0.8957
Balsam fir	b	–1.40398	18	0.12201	<0.0001	0.8922
All	α_0	9.90170[a]	83	0.67020	<0.0001	0.9182
	α_1	–8.80539	–	2.28913	0.0002	–
	β_0	1.06069	–	0.28548	0.0004	–
	β_1	–1.53829	–	0.08305	<0.0001	–

[a]Replaced with 10.406 as described in the analysis section.

Table 3. Results of fitting eq. 3 and 4 are compared with those in the literature; volume = mean tree gross total volume (m^3) and density = stems/ha.

Species group	Equation	Source
Spruce	Ln(Volume) = 9.99664–1.51939 ? ln (density)	This study, eq. 3
Spruce–fir	Ln(Volume) = 7.02022 - 1.13087 ? ln(density)	This study, eq. 3
Balsam fir	Ln(Volume) = 9.29237 - 1.40398 ? ln(density)	This study, eq. 3
All	Ln(Volume) = 10.4060 - 8.80539 ? mixfrac - (1.53829 - 1.06069¥mixfrac) ¥ ln(density)	This study, eq. 4
Spruce	Ln(Volume) = 10.4060 - 1.53829 ? ln(density)	This study, eq. 4
Spruce–fir	Ln(Volume) = 6.0033 - 1.00795 ? ln(density)	This study, eq. 4
Balsam fir	Ln(Volume) = 10.4060 - 1.53829 ? ln(density)	This study, eq. 4
Black spruce	Ln(Volume) = 10.8014 - 1.618 ? ln(density)[a]	Newton and Smith (1990)
Black spruce (upland)	Ln(Volume) = 9.8735 - 1.552 ? ln(density)[a]	Newton (2006)
Black spruce (lowland)	Ln(Volume) = 10.2074 - 1.562 ? ln(density)[a]	Newton (2006)
Spruce–fir	Ln(Volume) = 7.9470 - 1.2149 ? ln(density)[b]	Wilson *et al.* (1999)

[a]Reformulated to natural logarithm form.
[b]Reformulated to natural logarithm form with metric variables.

Table 4. The results of fitting eq. 5 and 6.

Species group	Parameter	Estimate	Standard error	$p(ABS(t_{n-1,0.025}) > t_{obs})$	n	r^2	Prediction error
Spruce	β_0	-12.3083	0.14134	<0.0001	472	0.991	0.02626
	β_1	3.0370	0.01934	<0.0001	–	–	–
	β_2	0.2573	0.01340	<0.0001	–	–	–
Spruce–fir	β_0	-10.9689	0.09280	<0.0001	1017	0.988	0.01506
	β_1	2.8402	0.01303	<0.0001	–	–	–
	β_2	0.1343	0.00878	<0.0001	–	–	–
Balsam fir	β_0	-10.3672	0.08940	<0.0001	909	0.992	0.00607
	β_1	2.7418	0.01285	<0.0001	–	–	–
	β_2	0.08599	0.00803	<0.0001	–	–	–
All	β_{01}	-12.2673	0.11225	<0.0001	2398	0.990	0.01645
	β_{02}	2.0900	0.17371	<0.0001	–	–	–
	β_{11}	3.0458	0.01536	<0.0001	–	–	–
	β_{12}	- 0.3411	0.02439	<0.0001	–	–	–
	β_{21}	0.2473	0.01064	<0.0001	–	–	–
	β_{22}	- 0.1764	0.01605	<0.0001	–	–	–

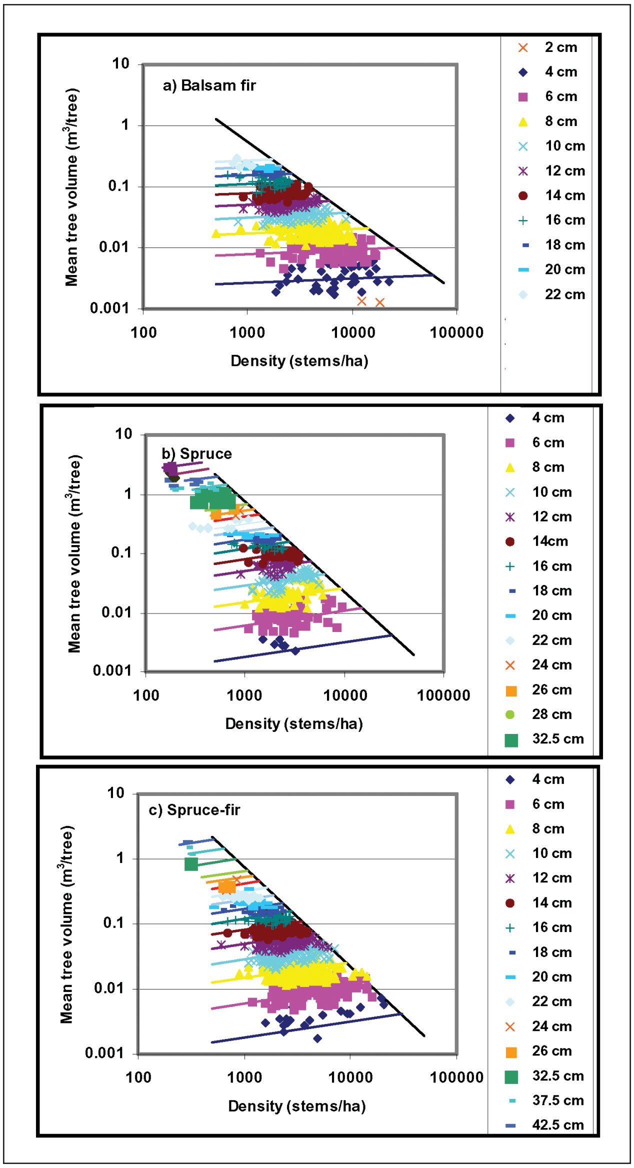

Fig. 3. The maximum size density line and the dbhq isolines from eq. 6 are plotted through the actual data by 2-cm diameter classes.

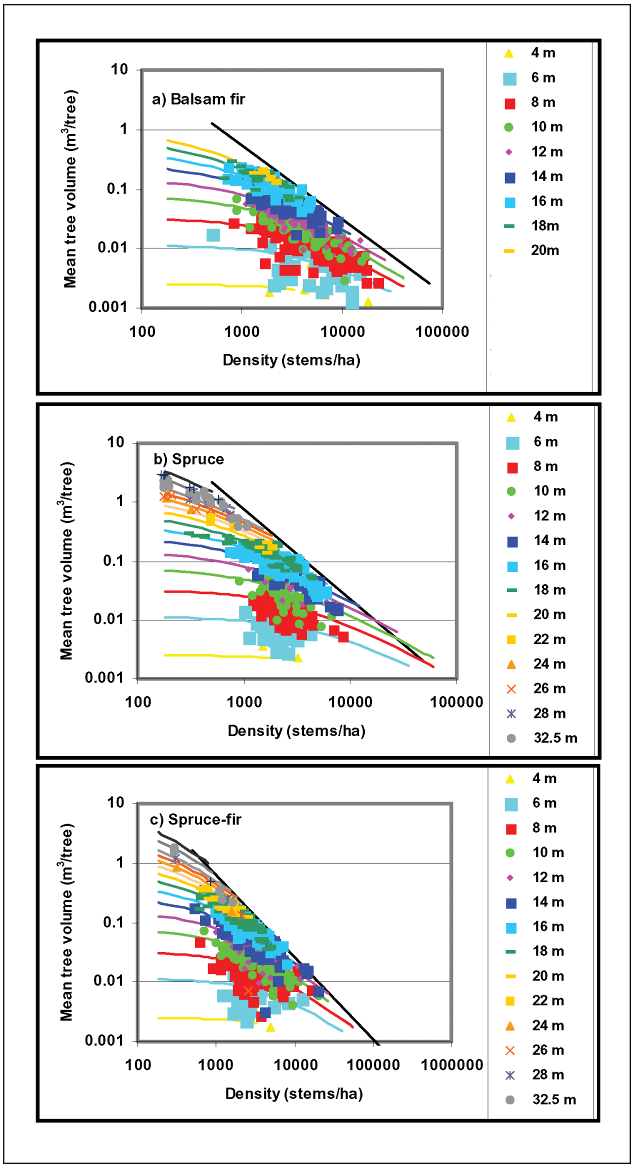

Fig. 4. The top height isolines from eq. 7 are plotted through the actual data by 2-m height classes.

increases, and the slope of the relationship between ln(volume) and ln(density) increases as top height increases:

$$[7] \quad \frac{1}{\boldsymbol{volume}} = \beta_0 \cdot \boldsymbol{topht}^{\beta_1} + \beta_2 \cdot \boldsymbol{density} \cdot \boldsymbol{topht}^{\beta_3}$$

Observations with smaller mean tree volumes get relatively larger weights than observations with larger mean tree volumes. To ensure the equation fit the observations with larger mean tree volumes, the mean tree volume was used to weight the residuals.

The coefficients in eq. 7 were expanded to include the effect of the balsam fir fractions as follows:

$$[8] \quad \frac{1}{\boldsymbol{volume}} = (\beta_{00} + \beta_{01} \cdot \boldsymbol{firfrac}) \cdot \boldsymbol{topht}^{(\beta_{10} + \beta_{11} \cdot \boldsymbol{firfrac})} + (\beta_{20} + \beta_{21} \cdot \boldsymbol{firfrac}) \cdot \boldsymbol{density} \cdot \boldsymbol{topht}^{(\beta_{30} + \beta_{31} \cdot \boldsymbol{firfrac})}$$

No coefficients associated with the balsam fir fraction were statistically significant, so these were removed (Table 5). This resulted in an equation with the same form as eq. 7 for all species combined. There was no statistical difference between fitting eq. 7 by species group or all species combined.

The relationship between top height, density, and mean tree volume is not strong. For instance, there is a great deal of overlap between some of the top height classes in Fig. 4.

Table 5. Results of fitting eq. 7 by species group and all species combined.

Species group	Parameter	Estimate	Standard error	$p(ABS(t_{n-1,0.025}) > t_{obs})$	*n*	r^2	Prediction error
Spruce	β_0	110030	35936	0.0023	466	0.914	0.3399
	β_1	–4.0406	0.1934	<0.0001			
	β_2	0.9702	0.2397	<0.0001			
	β_3	–1.9549	0.0928	<0.0001			
Spruce–fir	β_0	383171	NA		465	0.991	0.0331
	β_1	–4.6112	0.0267	<0.0001			
	β_2	0.2012	0.0460	<0.0001			
	β_3	–1.3960	NA				
Balsam fir	β_0	38260	12396	0.0021	680	0.870	0.0157
	β_1	–3.5449	0.1698	<0.0001			
	β_2	0.2622	0.0592	<0.0001			
	β_3	–1.5437	0.1020	<0.0001			
All	β_0	66665	9732	<0.0001	1940	0.871	0.0408
	β_1	–3.6970	0.0768	<0.0001			
	β_2	0.1824	0.1824	<0.0001			
	β_3	–1.4029	-1.4025	<0.0001			

NA – Not available as convergence criteria not met.

Spruce–balsam fir SDMDs

Equations 4, 6, and 7 (all species combined) were used to derive the SDMD for spruce–balsam fir mixtures. The SDMD is dynamic—the maximum size density line as well as the top height and dbh_q isolines change, depending on the mixture of fir and spruce. The SDMD presented in Fig. 5 corresponds to a 50:50 mix (by basal area) of spruce and fir. The diagram includes relative density lines (RDI lines), parallel to the maximum size density line. The lines correspond to RDIs of 0.15 (approximate crown closure), 0.40 (lower bound for the zone of optimum growth), and 0.55 (upper bound for zone of optimum growth, coinciding with the beginning of competition-related mortality). These RDI values were taken from Smith and Woods (1997) and not derived from the actual data from this study.

Discussion and Conclusion

The spruce–balsam fir SDMD developed here is dynamic, varying with the mixture of spruce and balsam fir. The analysis indicates that, at younger ages, spruce–balsam fir mixtures can maintain a higher mean tree volume for the same density than pure spruce or pure balsam fir stands. At older ages, the reverse is true. Pure stands can maintain a higher mean tree volume for the same density compared with mixtures at older ages. In discussing Reineke's stand density index (SDI) for species mixtures, Shaw (2006) described three possibilities. The maximum SDI for mixtures may be a linear interpolation of the SDI for the pure species SDI, weighted by the species proportion; the maximum SDI may be higher; or it could be lower. In this study, we observed all three behaviours.

At younger ages, mixtures are more productive than pure spruce or pure balsam fir, likely because of differences in light tolerance between the species. DeBell *et al.* (1997) found mixtures of Eucalyptus (*Eucalyptus saligna* Sm.) and Albizia (*Albizia falcataria* (L.) Fosberg) were more productive than single-species plantations. In a mixture of three species, Kohyama (1992) observed a maximum size density line similar to those observed in pure species stands. Chen at al. (2003) found the effect of one species on the productivity of another is specific to species, site, and development stage. Balsam fir is considered very shade tolerant, more so than black spruce (Burns and Honkala 1990). Adding a more shade-tolerant species under a canopy of a less shade-tolerant species should increase the total photosynthetic capacity of the site (Kelty 1992).

At older ages, spruce–balsam fir mixtures are less productive than pure species stands. Balsam fir is a short-lived species, susceptible to various heart-rot fungi as well as periodic infestations of spruce budworm (*Choristoneura fumiferana*), leading to a pathological rotation of 40–70 years, depending on site quality (Seymour 1992), but is very shade tolerant and regenerates well in shade. As spruce–balsam fir stands age, the balsam fir starts to die either because of budworm attack or rot-induced windthrow. This releases small advanced regeneration, mainly of balsam fir (Seymour 1992). Older mixtures may have lower mean tree volumes than pure stands because the older larger balsam fir stems are replaced by smaller, regenerating stems.

The pure species maximum size density lines observed in this study are close to those reported for black spruce in Ontario (Newton 2006) and Newfoundland (Newton and Smith 1990) and close the to theoretical value -1.5. There is evidence the maximum size density line for spruce–fir mixtures has a lower slope than the maximum size density line for either pure spruce or pure fir stands. Newton (2006) studied pure jack pine (*Pinus banksiana* Lamb.) and black spruce

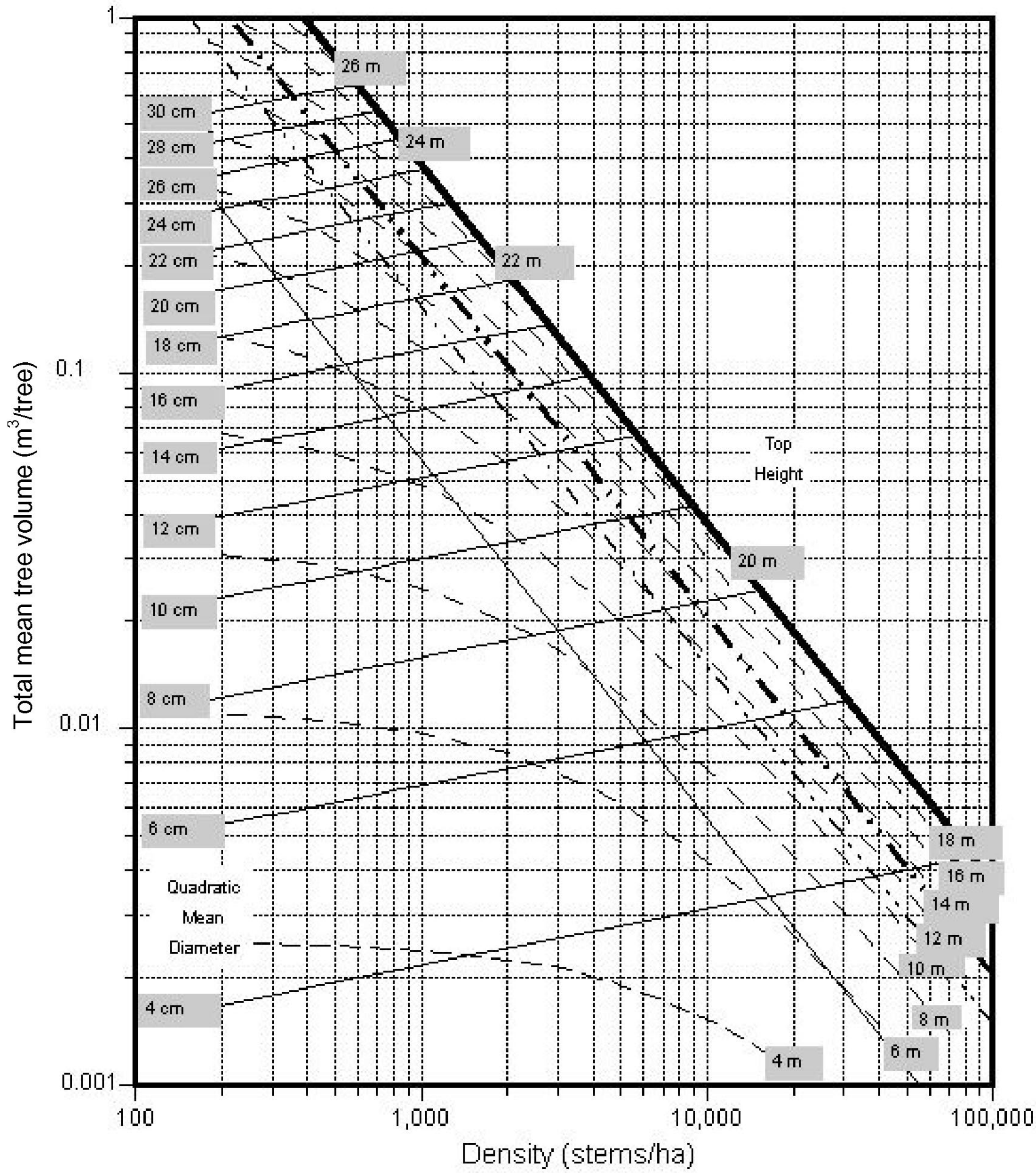

Fig. 5. The SDMD for spruce–balsam fir mixtures (50:50 mixes by basal area) in New Brunswick.

stands, as well as mixtures of the two species. He observed a similar pattern. The maximum size density line for spruce–pine mixtures had a lower slope than the theoretical value of -1.5 and was lower than that for either pure spruce or pure pine stands. Turnblom and Burk (2000) found that red pine (*Pinus resinosa* Ait.) stands with higher initial densities exhibited lower maximum size density lines than stands with lower densities. This does not explain the lower line observed here for mixtures, because the pure balsam fir stands had the highest initial densities.

Sterba and Monserud (1993) found the steepness of the maximum size density was related to the stand structure as measured by the skewedness of the dbh distribution. The more skewed the distribution, the flatter the slope. This appears to be supported by the results of this study. The spruce–fir mixtures generally have a more skewed dbh distribution (average skewedness of 0.54) compared with pure spruce (0.39) and pure fir (0.26). There is a very minor shift toward more spruce over time in the spruce–fir mixtures, with an average yearly increase in basal area of spruce of 0.1%.

Most SDMDs produced to date have been for single species stands, rather than mixed-species stands (Farnden 2002). Bi-species SDMDs have been created for spruce and balsam fir forests in other regions. Wilson *et al.* (1999) produced a red spruce–balsam fir SDMD for the northeastern forests of Maine that had a significantly higher maximum size density line than the SDMD for managed black spruce created by Newton and Weetman (1994) or the spruce–fir SDMD by Sturtevant *et al.* (1998). They attributed the difference to the more productive forests of the Acadian Forest Region compared with those of the Boreal Forest Region (Rowe 1972). Their results suggested the importance in silvicultural planning of using SDMDs developed for specific regions and species. They also found no pattern of separate self-thinning relationships between balsam fir and spruce. The spruce was primarily composed of red spruce, but the data sets also contained a minor component of white and black spruce. Their SDMD does not separate spruce and balsam fir into two separate diagrams, as do the diagrams for this study.

A first approximation diagram of an SDMD for mixed balsam fir–black spruce stands was produced for western Newfoundland by Sturtevant *et al.* (1998). As with the bi-species SDMD produced in Maine, this diagram does not vary with species proportions. In addition, the authors caution that the use of the SDMD should be restricted to the mixed-species stands in western Newfoundland from which it was developed. A comparison between maximum density lines resulting from our project and the other two studies may provide more information on whether intercept and slope components of an SDMD remain constant for a species over its native range (Tang *et al.* 1995).

A sample eastern spruce–balsam fir SDMD, corresponding to a 50:50 spruce–balsam fir mixture, presented in Fig. 5 provides forest resource managers in New Brunswick and other jurisdictions with a new management tool for making stand density management decisions. As with other SDMDs produced as cooperative research projects through the Fundy Model Forest, this diagram is included in the SDMD decision support software developed by the Ontario Ministry of Natural Resources (Woods 1999). The software prompts the user for the fir fraction and then produces a custom SDMD for that fraction. This provides resource managers with a simple and time-efficient method for using the complex diagrams. Illustrations on the use of SDMDs are presented in Archibald and Bowling (1995) and Smith and Woods (1997).

Depending on the amount of information presented, SDMDs can exist at various levels of complexity (Farnden 2002). Basic boundary lines that allow the user to decide whether a thinning treatment is warranted are the simplest form of SDMDs. The addition of volume, diameter, and height isolines allows prediction of stand development through time in the absence of mortality; these form the next or second level of complexity. The present spruce–balsam fir SDMDs are at this level of complexity. Also, most formats of SDMDs produced in North America, including these diagrams, are based primarily on a pair of reciprocal equations (Farnden 2002). These equations describe the competition-density and yield effects, -3/2 power law for self-thinning (Newton 1997). Future work will focus on adding size–density trajectories (mortality curves) to the present spruce–balsam fir SDMD. The addition of mortality curves improves the diagram's utility, and reduces errors caused by false assumptions by the user (Farnden 2002).

When adequate data become available, separate SDMDs should be produced for red and black spruce in conjunction with balsam fir, but they should be tested to determine whether substantial differences exist between them and the equations in this paper.

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