Forest Research Branch

AN INTRODUCTION TO POINT SAMPLING

by

C. L. Kirby

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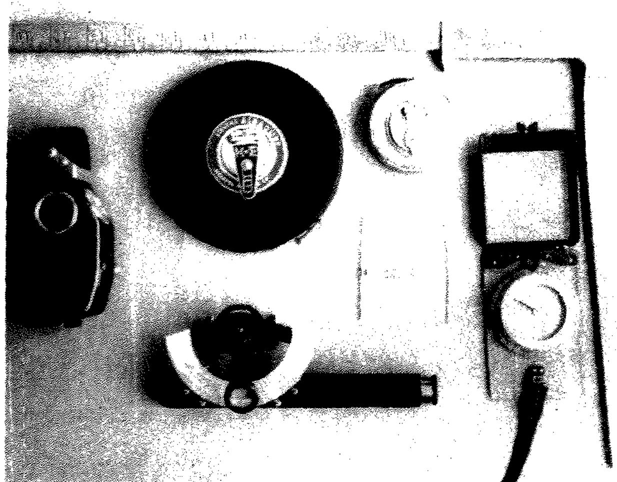
Point



Sampling

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INTRODUCTION

A revolution in forest sampling procedures is taking place. The new method which is causing this revolution has many names, but for the purpose of this thesis it will be referred to as point sampling. Whether the instrument be a stick relascope or a wedge prism, its basic use is to estimate basal area per acre from a sampling point without a fixed plot area. Point sampling methods may be used to determine volume, growth, stem taper, and silvicultural treatment. Although point sampling is a radical departure from plots with a definite area, its use has become widespread. The method is simpler than standard plot procedures and it saves a substantial amount of field and computation time, reduces personal errors and provides a better sample of the larger trees. In fact, much of the forest inventory in the United States and Canada is now done by this method.

The method, as applied to forestry, was first proposed by Bitterlich (1949), an Austrian forestmeister. Keen (1950) of England showed how volume and basal area per acre could be determined. Grosenbaugh (1952, 1955, 1957, 1958), an American forester, was the first to bring the idea to North America and he has further extended the application of point sampling.

INSTRUMENTS

A wide variety of instruments for point sampling are on the market, ranging in price from two dollars to one hundred dollars. They are all angle gauges, and trees that are larger than the angle defined by the angle gauge are selected as sample trees at the sample point in the forest (see Figure 1). Only the two most promising instruments - the "Spiegel-Relaskop" and the wedge prism - will be discussed in this section.

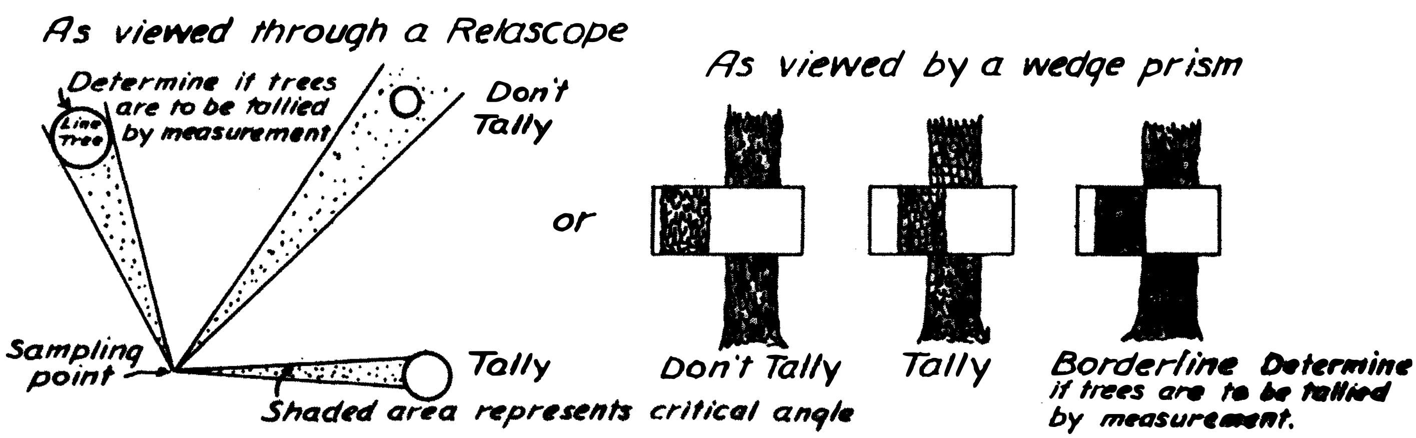


FIGURE 1: VIEWING OF TREES THROUGH STICK RELASCOPE AND PRISM

The best instrument is, of course, the one built for the job, and it is called the "Spiegel-Relaskop". This instrument permits different critical angles to be used for various timber types and automatically adjusts for slope, which may cause a discrepancy if not accounted for. It can also be used as a hypsometer and a rangefinder, but its main use and advantage is as an angle gauge. Wedge prisms and abney levels can accomplish with just as much accuracy most of the other uses of the "Spiegel-Relaskop".

As an angle gauge, the manufactured instrument is very good and its automatic slope correction is highly desirable. The main disadvantages are its price (nearly one hundred dollars) and the poor definition of trees in s tands when the light intensity is low or when sighting through the instrument into the sun.

A simple wedge prism described by Bruce (1955) may be sufficient where the terrain is relatively flat. A prism is a piece of glass ground to form a wedge. Prisms are calibrated in diopters and one diopter is defined as giving a right-angled deflection of one unit per one hundred units of distance. A prism of 3.03 diopters displaces one unit in 33 and has a basal area factor of 10.

Since: 1 unit of displacement in 100 = 1 diopter, a 3.03 diopter lens will therefore displace 1 unit in $\frac{100}{3.03}$ = 33 units.

Bower et al (1959) found that horizontal rotation of a prism can easily change the resultant strength of a wedge prism. They found that by mounting a prism on a piece of lucite or other plastic (see Figure 2)

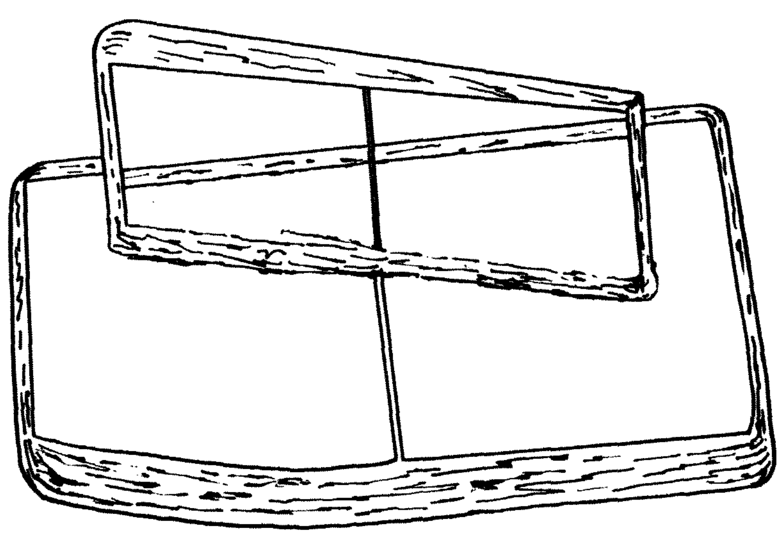


FIGURE 2: A PRISM MOUNTED ON LUCITE WITH SIGHTING LINE.

an exact basal area factor may be obtained. It may also be possible to have a prism so mounted that rotation of the prism on the lucite base could give different basal area factors. In extremely cold climates the glass prism usually separates from the lucite base at the bond line due to different coefficients of contraction.

The use of a prism or relascope and a tape to check doubtful trees on slopes, or those hidden by brush and other obstacles in the line of sight, is probably the most accurate method for point sampling. Regardless of what angle gauge is used, the instrument is only a guide, and for best results borderline trees are checked by measurement to determine whether they are to be selected for a particular point sample. For relatively flat country where horizontal distances can usually be measured, a hundred-foot steel tape with special graduations on one side may be used. For a basal area factor of 10, one would simply find maximum distances on the tape at which various diameters could be included in the point sample.

For example:

6" = 2.75' x 6" = 16.50. At 16.50' - 6" would be marked on the opposite side of the tape.

7" = 2.75' x 7" = 19.25'. At 19.25' = 7" would be marked on the opposite side of the tape.

With a tape graduated in diameters it would then not be necessary to perform this multiplication to compute the maximum plot radius for specific diameters each time a borderline tree is encountered. A manufactured tape for this purpose is now on the market for approximately $50.00.

For trees on a slope, the slope angle and tree diameter should be measured. The maximum slope distance that a tree may be out from the plot centre would be: $\frac{\text{(Tree diameter) (Plot radius factor)}}{\text{(Cosine of slope angle)}}$

A convenient computer for the above equation is presented by Stage (1959). (See Figure 3).

When measurements along the slope are made to check borderline trees and no corrections for horizontal distances are made, the following correction factors are applied (Grosenbaugh, 1955):

Slopes of 10.0 - 17.4 percent correction factor 1.01

Slopes of 17.4 - 22.5 percent correction factor 1.02

Slopes of 22.5 - 26.7 percent correction factor 1.03

The basal area, volume and/or number of trees calculated from unadjusted angle gauge tallies are then multiplied by slope correction factors. Slopes of 17 percent or less only introduce an error of one percent. Without correction for slope the estimates of point sampling would be biased on the conservative side.

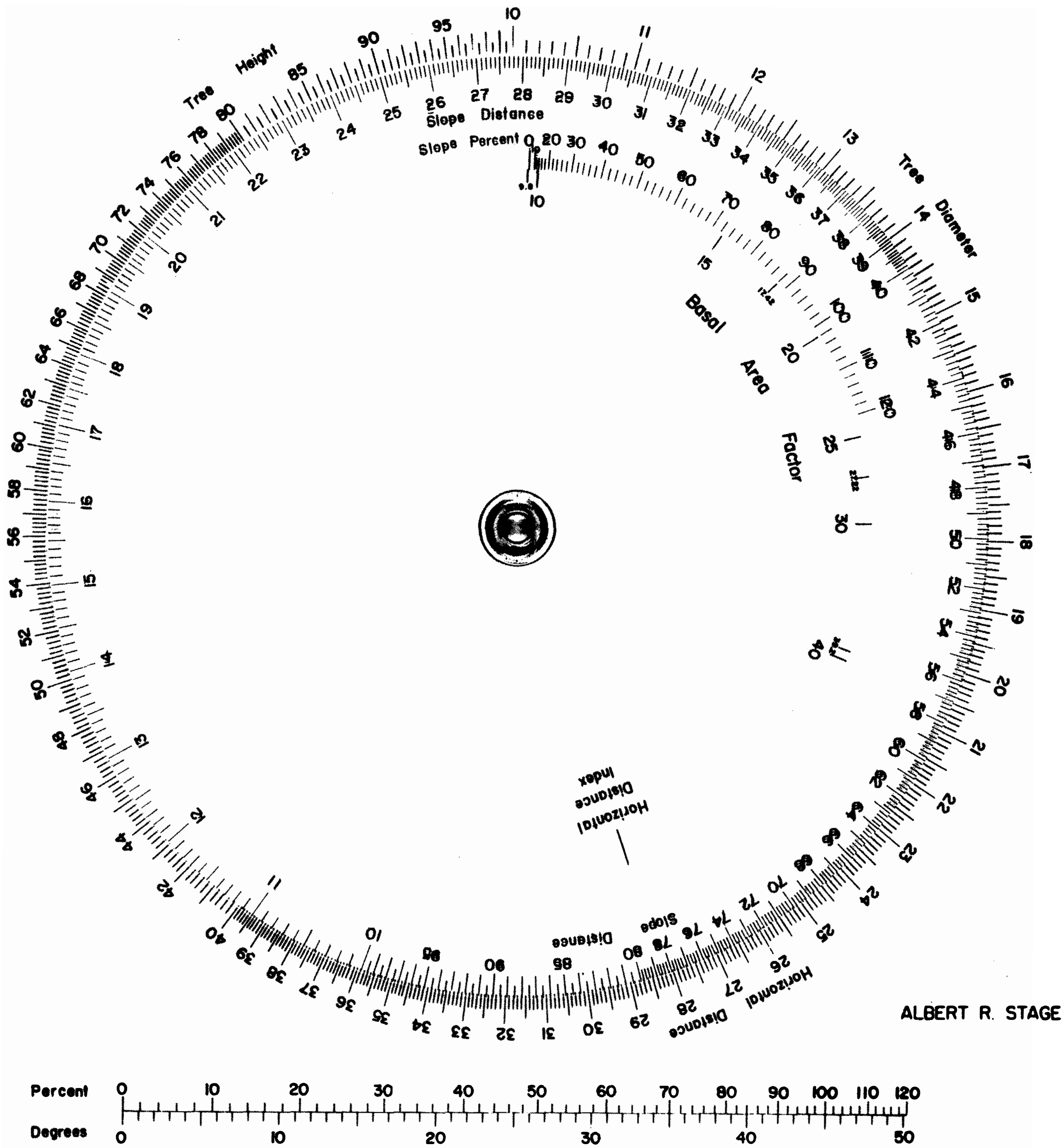
A wedge prism can be obtained at any place where eye glasses are ground for approximately two dollars. The main disadvantages are:

(1) No slope correction

(2) Confusion of trees to be counted in dense stands.

If proper care in the use of the prism is taken, it can give results as reliable as more expensive instruments.

No matter what angle gauge is used, the same procedure is valid. In the field trees are simply counted as in or out, using the angle gauge. If the angle gauge has a basal area factor of 10, then 10 times the number



TO CHECK MARGINAL TREES IN VARIABLE PLOTS — **Example**

1. Measure: tree diameter in inches — 14.0"
   slope percent — 30%
2. Basal area factor of prism being used: — 20
3. Set SLOPE PERCENT opposite BASAL AREA FACTOR on computer.
4. Opposite TREE DIAMETER on outer scale, read LIMITING SLOPE DISTANCE on inner scale. — 28.41'
5. Compare the LIMITING SLOPE DISTANCE with MEASURED SLOPE DISTANCE from prism to b.h. center of tree to determine whether tree is "in" or "out".

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TO CONVERT SLOPE TO HORIZONTAL DISTANCE: — **Example**

1. Measure: slope percent — 30%
   slope distance — 50'
2. Set SLOPE PERCENT opposite HORIZONTAL DISTANCE INDEX
3. Opposite SLOPE DISTANCE read HORIZONTAL DISTANCE = — 47.89'

TO COMPUTE TREE HEIGHTS:

1. Measure: percent slope to tip of tree — 90%
   percent slope to base of tree — 15%
   slope distance from Abney to tree base — 90'
2. Convert SLOPE DISTANCE to HORIZONTAL DISTANCE = — 89.0'
3. Use outer two scales as an ordinary slide rule to multiply:
   a. PERCENT SLOPE TO TIP by HORIZONTAL DISTANCE 90 x 89.0 = 80.1'
   b. PERCENT SLOPE TO BASE by HORIZONTAL DISTANCE 15 x 89.0 = 13.4'
4. Add the two products in (3) if observer is uphill (subtract if downhill) to obtain TREE HEIGHT = — 93.5'

FIGURE 3: TREE HEIGHT, MARGINAL TREE, AND HORIZONTAL DISTANCE COMPUTER

of trees counted within the point sample will give the basal area per acre.

HOW THE METHOD WORKS

The method is most easily explained with a relascope constructed from a rod with a blade. Any angle gauge simply insures that there be a certain ratio of tree diameter (d) to plot radius (R) (see Figure 4). The basal area factor is 10 when the blade width is one inch and the rod length is 33 inches. To maintain this ratio of tree diameter to plot radius one could just as well go out and measure the distance to each tree, and in some areas where point sampling has been very successful the distance out to any doubtful tree is measured (Grosenbaugh and Stover, 1957). For a 10 basal area factor, 2.75 times the tree diameter in inches gives the maximum plot radius in feet; e.g. a 12-inch tree must be within a plot radius of 2.75 x 12 inches = 33 feet. For determining the factor:

$$\text{Plot radius factor} = \frac{\text{Maximum plot radius in feet for a given tree diameter}}{\text{Tree diameter in inches}}$$

$$\text{i.e.} \quad \frac{33 \text{ feet}}{12 \text{ inches}} = 2.75$$

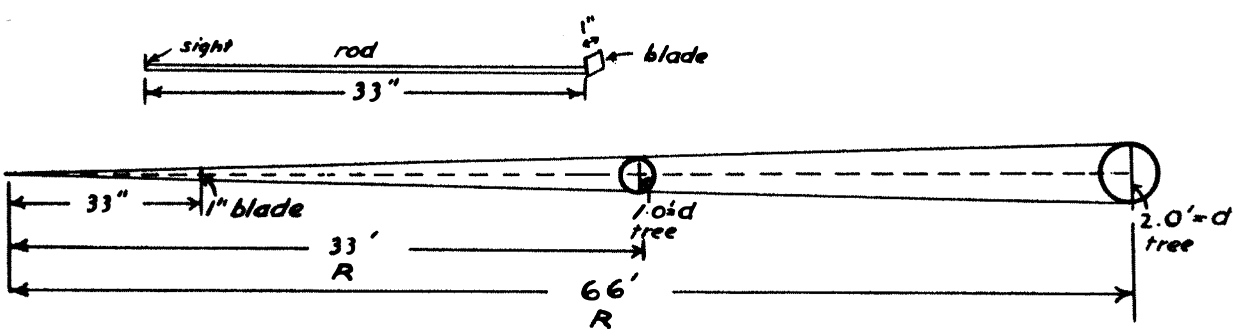
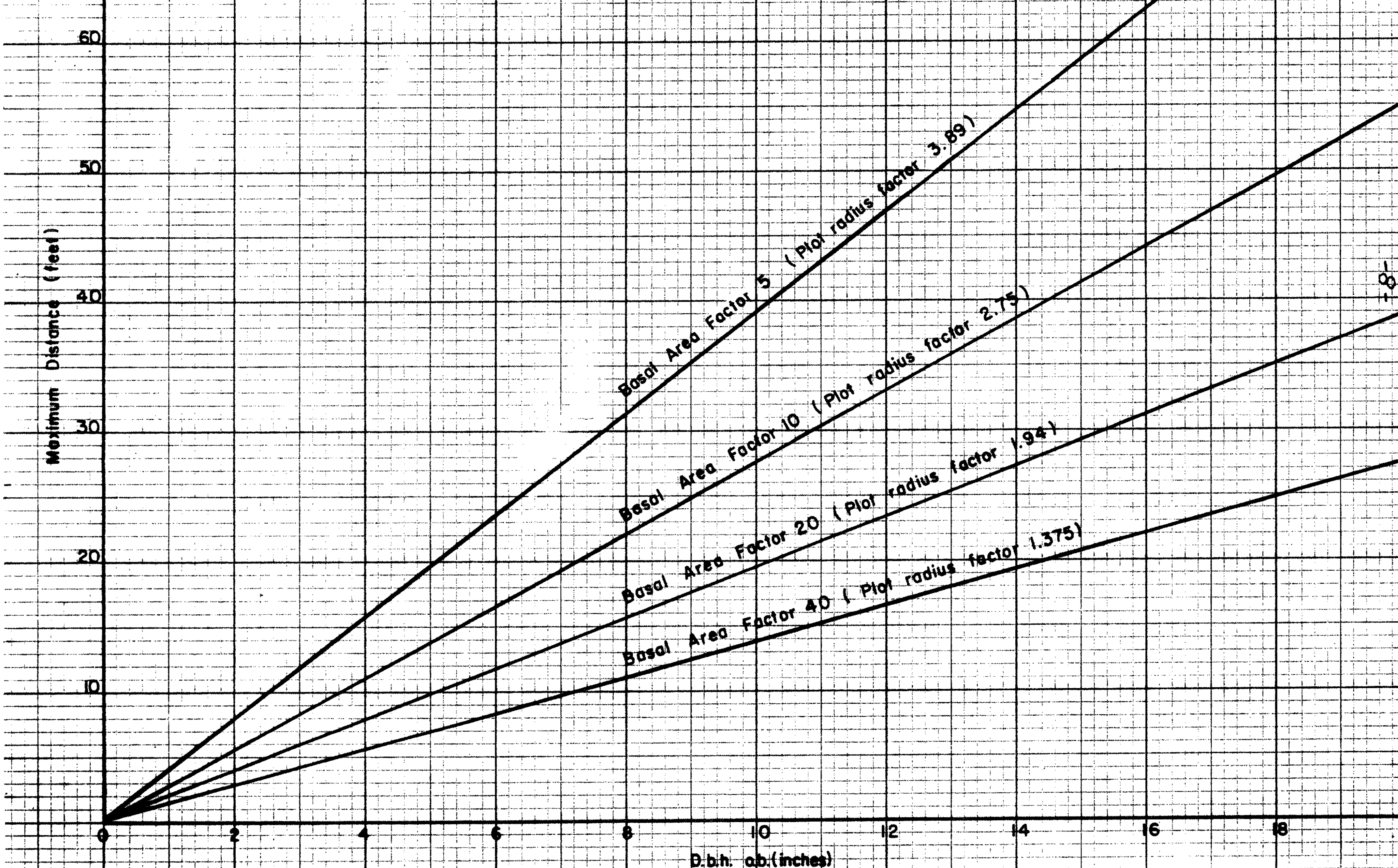


FIGURE 4: A STICK RELASCOPE

Figure 5 presents the maximum plot radius over tree diameter relationships for basal area factors of 5, 10, 20 and 40.

FIGURE 5: MAXIMUM PLOT RADII FOR VARIOUS BASAL AREA FACTORS AND DIAMETERS



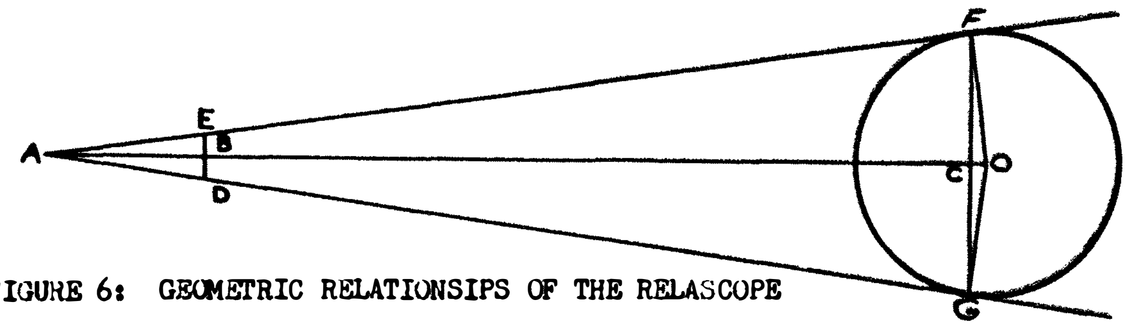


FIGURE 6: GEOMETRIC RELATIONSIPS OF THE RELASCOPE

The mathematics involved in determining the basal area factors and plot radius factors are derived from the following relationships:

From Figure 6:

(1) AB:AC = DE:FG

The substitution of FG for tree diameter introduces a small error of 0.01 percent for a relascope with a 1:50 ratio. Keen (1950) presents the following mathematical relationships. In a complete enumeration sweep of 360 degrees with radius AC in which all stems of diameter FG will be counted, the sampling area has a diameter of 2AC. (Plot radius will vary with diameter). (See Figure 5).

(2) The sample area = $\pi\ (2AC)^2$

(3) The stem basal area is $\pi\ (FG)^2$

(4) $\frac{\text{Stem basal area}}{\text{Sample area}} = \frac{FG^2}{4AC^2}$

From equation (1) $\frac{FG}{AC} = \frac{DE}{AB}$

(5) Then $\frac{FG^2}{4(AC)^2} = \frac{DE^2}{4(AB)^2}$

Let DE = 1 inch and AB = 33 inches:

Stem Basal Area = $\frac{(1)^2}{4(33)^2} = \frac{1}{4{,}356}$

For converting to a convenient area basis:

$\frac{\text{Stem Basal Area}}{\text{Sample Area}} = \frac{\text{Stem Basal Area Per Acre}}{\text{1 Acre}}$

Stem basal area per acre of one tree sampled = $\frac{(1)^2}{4(33)^2}$ x 43,560 = 10 sq. ft.

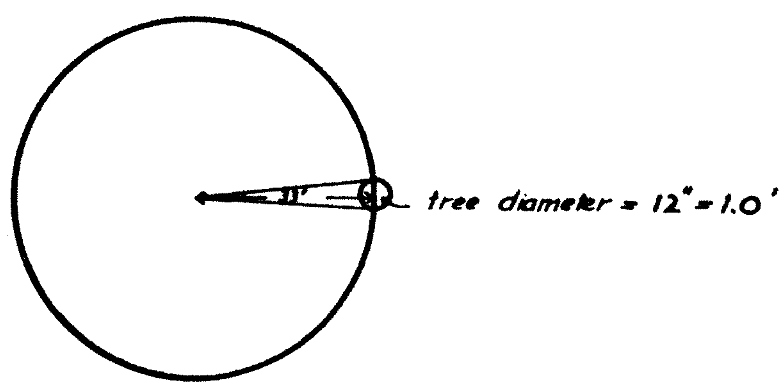
Similarly, each tree sampled in a relascope sweep is equal to 10 square feet of basal area when an angle gauge with a displacement of 1 in 33 units is used. If n trees are counted then (n)(10) equals the total stand basal area per acre.

For determining the basal area factor of a stick relascope the same relationships hold (see Equation 5) and the following formula is used:

$$\text{Basal area factor} = \frac{(\text{blade width})^2}{4(\text{rod length})} \times 43{,}560$$

Afanasiev (1955) explained how point sampling determines basal area as follows (this explanation in my opinion is much easier to understand than Keen's (1950) geometric explanation or Grosenbaugh's (1952) (1958) probability explanation):

(1) When using a relascope with a basal area factor of 10, suppose only one 12-inch tree were counted in one complete sweep with a relascope. One could infer that there is only one 12-inch tree within a circle with a radius of 33 feet.

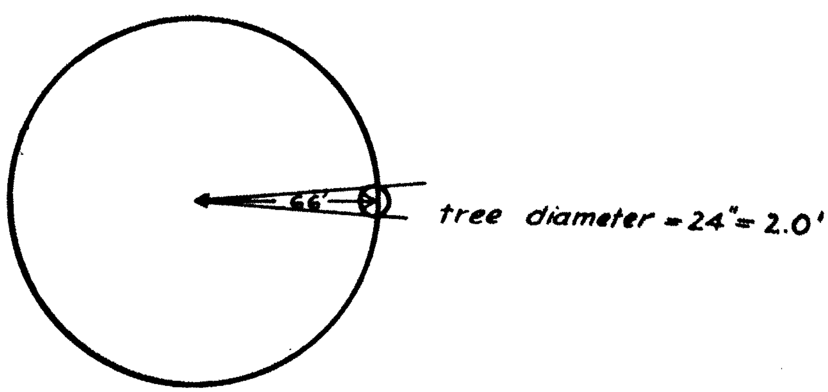


The plot area of a circle with a radius of 33 feet is πr^2 = 3.146 x $(33)^2$ = 3,426 square feet =.078 acres.

There are 43,560 square feet in an acre. Therefore, there are $\frac{43,560}{3,426}$ = 12.73 trees per acre.

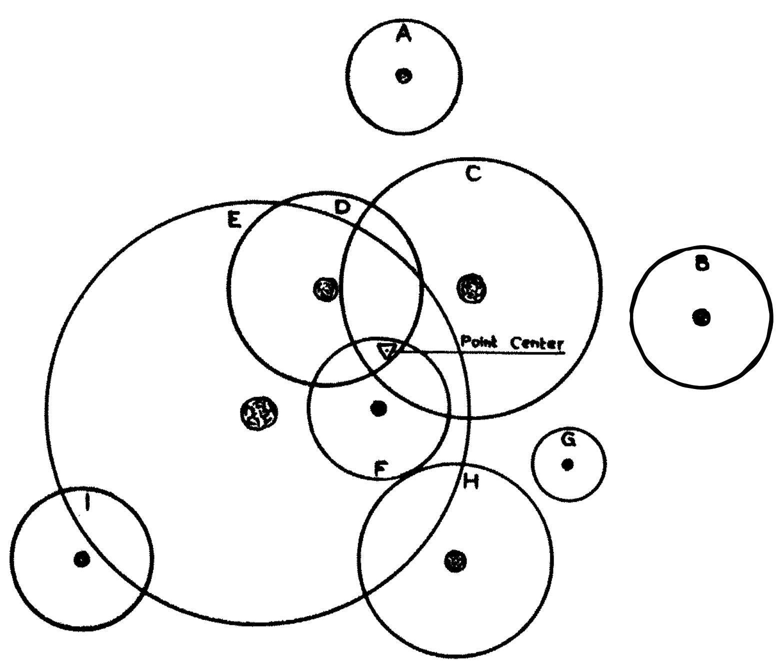
The basal area of a 12-inch tree = .785 square feet. Therefore, the basal area per acre = .7854 x 12.73 = 10.00 square feet per acre. To obtain the basal area per acre take the number of trees counted, which in this particular case is one tree, and multiply by the basal area factor of 10 and you get basal area per acre of 1 x 10 = 10 square feet per acre.

(2) Similarly, a 24-inch tree would be within a circle with a radius of 66 feet. A plot with a radius of 66 feet has 13,685 square feet = .314 acres. There would then be in 43,560 square feet (1 acre) 3.18 trees per acre. The basal area of a 24-inch tree = 3.1416 square feet. The basal area per acre = 3.18 x 3.1416 = 10.0 square feet per acre.



Essentially this explanation is a ratio when the tree selected is out the maximum allowable distance as defined by the angle gauge. The ratio is the basal area of the tree selected and the area of a circle whose radius in feet when a basal area factor of 10 is used is 2.75 x d.b.h.o.b. in inches. This is 1 square foot of tree basal per 4,356 square feet or 10 square feet of basal area per acre. Therefore, the number of trees counted in a point sampling sweep when multiplied by 10 gives the basal area in square feet on a per acre basis.

When trees are not out the maximum allowable distance from the sampling point Grosenbaugh's (1952) probability approach is best to explain the theory of point sampling. Here the most important premise of point sampling is emphasized, and that is that sampling is proportional to basal area. To explain his probability approach all trees are encircled with rings whose radius in feet are 2.75 x d.b.h. in inches, when an angle gauge with a basal area factor of 10 is used.



Those trees whose rings encircle the sampling point are selected for that point sample, and the probability that any one tree being sampled is thus proportional to its basal area. Therefore, the sampling is concentrated on the larger trees.

The probability of each tree being sampled in a given forest tract may be expressed as follows:

$$\frac{\text{Area of circle around a tree of given d.b.h.}}{\text{Area of forest tract}}$$

Let the diameter of the circle around a tree of given d.b.h. = K (d.b.h.) in inches, and the area of the circle in square feet around this tree = K^2 (B.A. of tree)

L.A. = area of the forest stand

Therefore, the probability of each tree being sampled

$$\frac{K^2\ (\text{B.A.})}{43{,}560} \div \text{L.A.} = \frac{K^2\ (\text{B.A.})}{43{,}560(\text{L.A.})}$$

Population totals for any desired variable can then be estimated by summing individual sample values of the desirable variable (X), which may be volume, tree frequency, height, etc., divided by individual probabilities of being drawn. The following example illustrates this point:

If M trees are selected in a single random point sample, and if some variable quantity X such as volume or height, etc., is measured with each tree as well as the basal area, the estimate of X is then

$$\left(\frac{43{,}560\ (\text{L.A.})}{K^2} \right) \left(\sum^{M} \frac{X}{\text{B.A.}} \right)$$

The per acre estimate is the above divided by L.A. $\left(\frac{43{,}560}{K^2} \right) \left(\sum^{M} \frac{X}{\text{B.A.}} \right)$

which in effect is the basal area factor multiplied by a summation of

$\frac{X}{\text{B.A.}}$ of all trees selected in a point sample.

For illustrative purposes the computation of volume estimates from point samples using volume-basal area ratios may be as follows. The local volume-basal area ratios may be determined from an appropriate standard volume table or formula by a conventional method using a height-diameter curve, or the local ratios may possibly be determined more accurately by the scaling of fallen trees, or the measurement of standing trees with a magnifying optical dendrometer. Measurement of diameter and merchantable length of all tree samples in a survey would no doubt give the most accurate estimate of volume. Volume-basal area ratios for white spruce may be as follows:

D.B.H.	Local Volume (Bd. Ft.)	Board Foot Volume/Basal Area Ratios
12	114	146
14	176	165
16	251	180
18	335	190

To calculate the volume on three point samples where the following tally was obtained, the procedure would be:

D.B.H. (Inches)	(N) Number of trees tallied on 3 Point Samples.	(R) Ratio	NR
12	2	146	292
14	4	165	660
16	6	180	1080
18	2	190	380
			2412

Board Foot Volume per acre $\frac{2412 \times 10}{3} = 8040$

Stand tables may also be calculated from the diameter tally.

BASAL AREA FACTORS

A wide difference of opinion exists as to the best basal area factor in point sampling. Ker and Smith (1957) suggest that a basal area factor of 20 for west coast timber would ;pear to provide an optimum balance between cost and accuracy. Carow (1953), in an investigation to find the best angle gauge size in sampling spruce, pine and aspen types ranging in diameter from 6 to 14 inches, and in timber types of varying density, found an angle gauge with a basal area factor of 4.4 to be most satisfactory; it gave the best standard error of the mean considering time to establish the point sample. He tried angle gauges with basal area factors of 2.2, 4.4 and 8.7. No allowance for travelling time was made. On the other hand, the most popular basal area factor appears to be 10, which has been favored by Grosenbaugh.

On many surveys in this province point sampling with a basal area factor of 10 is resulting in many samples having a zero tally. This appears to be due to the heterogeneity of our forest types which contain stand openings and patches of trees below measurable size and to the low density in some types. From a statistical point of view having zero tallies is most undesirable and indicates that the sample size is probably too small.

There are at least two alternatives that could improve sampling techniques when zero tallies occur. One is to decrease the basal area factor to 5. From the author's field experience in this province this leads to an increase in the possibility of errors. On the average, the number of trees selected on each point sample, using a basal area factor of 5 in stands with a basal area per acre of 100 to 150 square feet, is from 20 to 30--which is just double the number when a basal area factor of 10 is used. With 20 to 30 trees on a point sample it become difficult to remember which trees were selected unless they are plainly marked. Also, the distance out to qualifying trees is increased as the basal area factor becomes smaller. For example, the plot radius for a basal area factor of 10 is 2.75, and for a basal area factor of 5 it is 3.89. Therefore, a 10-inch tree can be out from the sampling centre up to 27.5 feet for a basal area factor of 10, or up to 38.9 feet for a basal area factor of 5. With trees qualifying at greater distances from the sampling centre difficulties of determining which trees fall within the prescribed angle are increased, and the method becomes more tedious.

These objections to a basal area factor of 5 may be only psychological because in the final analysis, for a given accuracy determination with various basal area factors, the number of trees counted may be fairly close. But none the less, from production studies of workers it has been shown that it is important to have goals that are easily attained at short intervals. This is especially true in forestry where sampling crews are working in adverse weather conditions and with bothersome insects such as mosquitoes. Therefore, the more routine and simple a field procedure can be made, the better.

An alternative to reducing the basal area factor may be the use of cluster sampling. Then it may be found desirable to use a basal area factor greater than 10 and instead of varying the basal area factor, vary the number of point samples in a cluster to suit the sampling design and type of timber being sampled. It could possibly be that a basal area factor of 18.9 with an even plot radius factor of 2 to facilitate the checking of live trees is desirable when cluster sampling is used. With a basal area factor of 18.9, 4 to 8 trees would usually be counted on each point sample in stands with approximately 80 to 160 square feet of basal area per acre. There should be no difficulties here in remembering which trees have been selected, and the possibilities for a better distribution of the sample increased. The average results in each cluster could possibly represent a sampling unit of weight 1 for statistical purposes. The judgment of cluster sampling with a basal area factor of nearly 20 should not only be based on statistical analysis but on the basis of human behavior with a certain field technique.

A summary of general points regarding the selection of a basal area factor is as follows:

1. The basal area factor most appropriate will vary with the size of trees with homogeneity of the area to be sampled and the sampling design.

2. The basal area factor should be large enough to avoid too much difficulty in seeing the trees that fall within the prescribed angle.

3. In some cases there may be more convenience in having an odd basal area factor and an even plot radius factor to facilitate the checking of borderline trees.

NUMBER AND DISTRIBUTION OF POINT SAMPLES

The amount of sampling required is usually limited by economic considerations and as such is a managerial decision guided by statistical

TABLE 1. A SMALL SAMPLE CRUISE TO HELP DETERMINE THE NUMBER OF SAMPLES REQUIRED FOR A GIVEN ACCURACY

Point Sample	Estimated Cords Per Acre
1	15
2	20
3	5
4	15
5	17
6	10
7	13
8	14
9	16
10	25
Sum	150

Mean cords per acre $\frac{150}{10}$ = 15.0 cords

$\Sigma\ d^2 = 260$

Standard deviation = $\sqrt{\frac{260}{n-1}}$ = 5.4 cords

Standard error of the mean = $\sigma\ m = \sqrt{\frac{\sigma^2}{N}} = \sqrt{\frac{5.4^2}{10}}$ = 1.7 cords

Coefficient of variation = $\frac{5.4}{15.0}$ x 100 = 36.0 percent analysis.

1. Number of Points or Plots Required

The number of point samples or plots required for a certain degree of accuracy when sampling an infinite population is determined as follows:

$$N = \left(\frac{\sigma}{\sigma m} \right)^2$$

where N = required number of sampling units for a certain precision;

σ = estimate of variation between plots

σm = required precision of extimated mean.

For example, suppose a prism with a basal area factor of 10 is used. A small sample of 10 point tallies is taken to gain some idea of the variation in a stand, using a defined sampling procedure.

It is estimated that there are 15.0 cords per acre. The standard deviation shows the variation in the point samples (which reflect the variation in the timber and sample size). Sixty-eight percent of the plots sampled will be within 5.4 cords of the mean volume of 15.0 cords. The standard error of the mean gives a measure of the accuracy of the estimated mean. That is, there is approximately a 65 per cent chance of capturing the true mean between the limits of 15 ± 1.7 cords, and there would be approximately a 95 per cent chance of capturing the true mean between the limits of 15 ± 3.4 cords. To be more exact in calculating confidence limits, "t" corrections for small samples should be made. For example, for a p.(.95) the confidence limits are actually 15 ± = (1.7) (2.26) = 3.88. If the desired precision is ± 10 per cent within one standard deviation (p.(.65) approximately) the standard error of the mean is:

$$\sigma m = \frac{10}{100} \text{ of } 15 = 1.5$$

The number of point samples required will then be:

$$N = \left(\frac{\sigma}{\sigma m}\right)^2 = \left(\frac{5.4}{1.5}\right)^2 = 13$$

If the desired precision were ± 5 per cent within one standard deviation (p.(.65) approximately) the standard error of the mean is:

$$m = \frac{5}{100} \text{ of } 15 = .75; \text{ so that}$$

$$N = \frac{5.4}{.75}^2 = 52 \text{ point samples required.}$$

Table 2 prepared by Grosenbaugh (1952) gives the number of sampling units required for various degrees of precision with a probability of approximately .65 (corrections of "t" should be made in this table if confidence intervals are to be stated), when sampling an infinite population with a known coefficient of variation. The table is based on random sampling formulae and, should a systematic sampling design be used, results of slightly higher accuracy may be expected. Grosenbaugh (1952) suggests that those wishing to estimate the error of a systematic sample more precisely should refer to De Lury[1].

---

1 De Lury, D.B. Values and Integrals of the Orthogonal Polynomials up to N = 26, University of Toronto Press, Toronto, Ontario. 1950.

TABLE 2. NUMBER OF SAMPLES TO BE TAKEN FROM INFINITE POPULATIONS

Coefficient of Variation (Percent)	Specified Percent Limit for Standard Error			
	± 1½ Percent	± 5 Percent	± 10 Percent	± 20 Percent
	N			
5	12	1	1	1
10	45	4	1	1
15	100	9	3	1
20	178	16	4	1
25	278	25	7	2
30	400	36	9	3
35	545	49	13	4
40	712	64	16	4
45	900	81	21	6
50	1,112	100	25	7
55	1,345	121	31	8
60	1,600	144	36	9
65	1,878	169	43	11
70	2,178	196	49	13
75	2,500	225	57	15
80	2,845	256	64	16
85	3,212	289	73	19
90	3,600	324	81	21
95	4,012	361	91	23
100	4,445	400	100	25
125	6,945	625	157	40
150	10,000	900	225	57
175	13,612	1,225	307	77
200	17,778	1,600	400	100

Shiue (1960) questions De Lury's assumption of constant variance in a forest area and regards systematic sampling as being of questionable value in that it may over-estimate the sample variance. He suggests that exact estimates of sample variance are possible when multiple random starts are taken. The systematic sample then represents a cluster sample with several random starts.

The foregoing appears to solve the problem of the number of point samples required when sampling infinite populations. In forestry, however, we are usually dealing with finite populations which require different statistical treatment. Table 3 prepared by Meyer (1949) is based on a formula for finite populations developed by Girard and Gevorkientz (1939). The formula is as follows:

$$n = \frac{Nt^2C^2}{Na^2 + t^2C^2}$$

where: n = number of plots required

C = coefficient of variation for tract

N = maximum number of plots that could be put in the known area = $\frac{\text{total acreage}}{\text{plot size acres}}$

a = maximum acceptable sampling error

t = factor of probability (2 for a factor of 95 percent).

Meyer's (1949) Table 3 is based on the assumption that acre plots are used and he states that a greatly exaggerated value of the coefficient of variation would be obtained by using smaller plots.

From Meyer's Table 3 it can be seen that a one percent cruise may give results of satisfactory accuracy for estimates on 10,000 acres

TABLE 3. STANDARD ERROR OF ESTIMATED VOLUME FOR DIFFERENT INTENSITITES OF CRUISE, FOR VARIOUS SIZE FORESTS, AND FOR DIFFERENT VALUES OF COEFFICIENT OF VARIATION

Size of Tract or Forest	Cruising Percent					
	1%	2%	5%	10%	20%	30%
	Standard error in percent of volume					
	(i) for coefficient of variation of 15 percent[1]					
50	21.1	14.8	9.2	6.4	4.2	3.2
100	14.9	10.5	6.5	4.5	3.0	2.3
200	10.6	7.4	4.6	3.2	2.1	1.6
500	6.7	4.7	2.9	2.0	1.3	1.0
1,000	4.7	3.3	2.0	1.4	1.0	0.7
2,000	3.3	2.3	1.5	1.0	0.7	0.5
5,000	2.1	1.5	0.9	0.6	0.4	0.3
10,000	1.5	1.0	0.7	0.4	0.3	0.2
	(ii) for coefficient of variation of 30 percent[1]					
50	42.2	29.7	18.5	12.7	8.5	6.5
100	29.9	21.0	13.1	9.0	6.0	4.6
200	21.1	14.9	9.2	6.4	4.2	3.2
500	13.3	9.4	5.8	4.0	2.7	2.0
1,000	9.4	6.6	4.1	2.8	1.9	1.4
2,000	6.7	4.7	2.9	2.0	1.3	1.0
5,000	4.2	3.0	1.8	1.3	0.9	0.6
10,000	3.0	2.1	1.3	0.9	0.6	0.5
	(iii) for coefficient of variation of 60 percent[1]					
50	84.4	59.4	37.0	25.4	17.0	13.0
100	59.8	42.0	26.2	18.0	12.0	9.2
200	42.2	29.7	18.5	12.7	8.5	6.5
500	26.7	18.8	11.7	8.0	5.4	4.0
1,000	18.9	13.3	8.3	5.7	3.8	2.9
2,000	13.3	9.4	5.8	4.0	2.7	2.0
5,000	8.4	5.9	3.7	2.5	1.7	1.3
10,000	6.0	4.2	2.6	1.8	1.2	0.9

1 Coefficient of variation is here defined as the standard error of observations of volume measured on one acre (observation of weight one) expressed in percent of volume per acre. For stands of high uniformity and normal density this coefficient may be as low as 15 percent, provided an efficient cruising design is applied which will eliminate as much as possible the effect of visible stand stratification. A coefficient of 30 percent corresponds to average conditions of forest cruising. Values of 60 percent are characteristic of irregular stands with a patchy distribution of volume. Estimates of the actual value of the coefficient of variation are best determined from a trial cruise.

but would be entirely inadequate for 1,000 acres. Although this table is not applicable to point sampling, it may be used as a guide.

2. Sample Allocation

The allocation of sampling units within a timber survey area will depend upon the purpose for which the survey is designed. One example is broad repeated inventory giving overall volume and drain estimates for large areas. Here, a grid of mechanically spaced, permanently marked sampling points with no stratification of the sample would probably be used. On the other hand, an operational cruise using forest type maps would probably require a higher intensity of cruise with a concentration of sampling in the merchantable and near merchantable types. Arbitrary decisions on the required accuracy for various strata may be made. A combination of ground and photo estimation could be desirable. Another approach that is used to gain the most accurate estimate of volume from a given number of sampling units is called optimum allocation (Hasel, 1950), (Osborne, 1951).

$$n_i = \frac{n(P_i s_i)}{P_i s_i}$$

A_i = acres in the ith class

P_i = proportion of total area in A_i

s_i = standard deviation usually based on volume

n = total number of sample units to be apportioned

n_i = number of sample units that should be taken in the ith class.

A concrete example of using the preceding formula is given in the Forestry Handbook for British Columbia (1959) where, for example, a forest area of 3,700 acres is stratified into 9 classes as follows:

Class	Area (A) (acres)	Proportion of Total Area (p)	Units (S.D.)	p x S.D.
a	620	0.175	7.07	1.237
c	290	0.080	4.30	0.344
e	316	0.085	4.21	0.358
g	518	0.140	6.30	0.883
i	340	0.090	5.51	0.495
k	280	0.075	6.23	0.467
m	1,070	0.285	8.77	2.250
o	66	0.015	1.33	.020
q	20	0.055	4.88	.268
Total (Σ)	3,700	1.000		6.322

Then for the "i^{th}" term, assuming the total number of sampling units equals 100: $n^{"i"} = \frac{100\ (.495)}{6.322} = 7.8$ sampling units.

This formula can only be used where some idea of the variation in each class is known.

The distribution of sampling using the above formula may also be apportioned on a value basis, thus concentrating the sampling on the more valuable areas.

Although stratified sampling by various cover-types is generally used in Canada, there are some valid reasons why only minor photo stratification separating forest from non-forest may be desirable. This is especially true where short rotations are possible. The reasons for using only minor photo stratification are as follows:

1. The gain in efficiency of sampling design may be small due to:
    a. small scale and old photography;
    b. many aged and patchy stands with a high volume variance within cover-types;
    c. inability to recognize stands with above average cull from aerial photographs.

2. Growth and drain may be of more importance than current inventory. What would be optimum allocation now might be very poor allocation ten to fifteen years hence.

3. Point sampling gives weight to large trees (sampling proportional to basal area): therefore, it tends to give optimum allocation of trees.

PREDICTION OF GROWTH

1. Permanent Samples

Grosenbaugh (1958) describes a method in which only those trees contained in the initial tally are dealt with. The determination of survivor growth, mortality and cut for a single growth period is based on observations taken on the same trees at the beginning and at the end of the period. In order to prepare a point sample for a second growth period newly qualifying trees will have to be measured. Distinctions in newly qualifying trees between ingrowth (those trees that were below measurable diameter) and larger trees that because of increased size are included in the point sample will have to be made for any detailed analysis of growth. Grosenbaugh proposes two methods for obtaining ingrowth; the coring of newly qualifying trees to determine whether they had a measurable d.b.h. at the beginning of the past growth period, or alternatively the establishment of a small permanent plot. Coring will obviously not be necessary on all trees and probably will only be required on a few trees, or eliminated if the estimate of the ingrowth is based on an assumed growth rate. Another complicating factor is that some trees through mortality will not be included in any tally (Sayn-Wittgenstein; 1961, 1962).

From the above discussion it would appear that the use of permanent point samples for a "continuous forest inventory" system, such as advocated by Cal Stott for permanent sample plots has some serious limitations.

2. Single Examination Sampling Points

In single examination point samples, increment cores extracted from trees selected by point sampling may also be used to determine growth (Spurr, 1952; Stage 1958-b). Sampling of growth is thus concentrated on trees contributing most to the growth of the stand rather than tree frequenc , as trees selected by point sampling are proportional to their basal area. A ratio of future gross basal area increment to present basal area increment may be determined as follows. This is based on the premise that future growth is predictable from past performance. Climatic cycles and various disturbances (i.e. cutting) may invalidate this premise. Ratio of basal area increment to basal area per acre is:

$$\text{Ratio} = 1 - \frac{\sum_{i}^{n}\left(\frac{\text{past diameter inside bark of sample trees}}{\text{present diameter inside bark of sample trees}}\right)^2}{\text{Number of trees measured}}$$

$$\text{Gross basal area increment per acre per year} = \frac{\text{B.A./acre x Ratio}}{\text{No. of years in period}}$$

This method of predicting growth is relatively untested and needs further study. Spurr (1952), with a similar formula, thinks it holds some promise. Some corrections for mortality would also be necessary. This method should at least indicate a relative index of growth potential, and as such be a useful tool for forest management.

SILVICULTURAL PRESCRIPTION

Grosenbaugh (1955 and 1958) presents a new approach to apply point sampling to silvicultural prescription. Methods of conducting silvicultural prescription surveys and record keeping are described by him. His suggestions are ideal for areas where stand improvement work is contemplated. As he states himself, "Compliance with management-plan volume or area regulation can be ensured with minimum sacrifice of growing stock and maximum attention to regeneration needs". He suggests that an angle gauge with a basal area factor of 10 be used to make a tally by crown and vigor classes. From this tally, some idea of the trees that should be cut is determined. From silvicultural knowledge, the residual basal area left to gain desired regeneration or increased growth is decided upon. Then an angle gauge with a larger basal area factor, say a basal area factor of 50,(possibly corresponding to B.A. that should be left) is used in actual field marking to determine which clumps should be loosened and the number of trees that can be removed for cutting. Whether marking can be made this mechanical is subject to some debate, because in the final analysis the marking of trees for cutting is an art which will depend upon many variable stand and operating conditions. It does, however, allow some objective guides to be determined for tree marking, namely a quick estimate of basal area before and after marking. Judgment based on experience and spacing factors may accomplish the same purpose.

ESTIMATION OF AVERAGE STAND HEIGHT

Point sampling methods offer an excellent opportunity to determine an unbiased estimate of average stand height. This is done by averaging the heights of trees selected in point sampling. Kendall and Sayn-Wittgenstein (1959) in a comparison of different measures of determining average stand height showed the following results for arithmetic mean height of all trees on a fifth-acre plot, average height weighted by basal area and average height of trees included in a relascope count:

Species	Diam. Range	Trees per Acre	Arithmetic Mean Ht. (1/5-Acre Plots)	Average Stand Ht. Weighted by B.A. (1/5-Acre Plots)	Average Height of Trees in a Relascope Count
wS	3-6	1900	21.6	24.6	24.6
wS	2-6	1900	21.6	24.6	25.3
rP	7-12	370	53.3	53.5	52.5
rP	5-10	775	52.5	53.5	52.7
rP wP	3-17	390	74.3	77.9	77.0
rP wP	6-16	390	74.3	77.9	79.3
rP wP	8-16	220	67.0	82.3	81.1

The average height of trees in a relascope count in this test agrees most closely with the average height of a stand weighted by basal area - which should be a most useful statistic for volume determinations.

As the estimation of tree height is a time-consuming procedure, further study may show the most efficient sampling designs for volume determinations to measure only heights on a few of the trees selected in point sampling. Height measurements may be limited to trees of average basal area or to co-dominants. Point sampling methods would then only assist in making an unbiased selection of trees.

SOURCES OF ERRORS

Point sampling, because of relatively simple procedures in the field, may be carelessly done. This can lead to serious errors. Therefore, it is absolutely essential for good results to observe the following precautions:

1. Calibrate the angle-gauge properly.

2. Locate point samples in an unbiased fashion - mechanically or randomly. The timber margins should have a representative chance of being sampled.

3. Use an angle gauge large enough to avoid brush bias (inability to count trees as in or out due to density of stand or undergrowth) but small enough to give a representative sample of the timber.

4. Corrections for slopes over 10 percent should be made, or better still, use an instrument that compensates for slope.

5. Check borderline trees with a tape. In this regard, it is essential that a definite plot centre be established and used for sighting or measuring purposes and the angle gauge be held consistently over the sampling centre.

SUMMARY AND CONCLUSION

Point sampling has proven to be an acceptable method for estimating basal areaand volume, but it is still in the developmental stage and procedures are changing. In this thesis, a rview of some of the more pertinent literature on point sampling has been made, and the theoretical background set down. With data that were readily available, methods were developed and tests done to make point sampling most useful in the Province of Saskatchewan. Because of the time it takes to train personnel and revise computation procedures, some of the methods suggested here will take time to be adopted in Saskatchewan. Further tests will continue to alter point sampling methods.

A summary of the more important conclusions to this thesis is as follows:

1. A basal area factor of 10 appears to be unsatisfactory for some conditions in this province. Tests on other basal area factors should be made. (A basal area factor of nearly 20, with a plot radius factor of exactly 2, would perhaps be more convenient and be most efficient with cluster sampling).

2. The use of a factor that is divided into average stand height to determine pulpwood volume in cubic feet or cords per unit of basal area appears to be an excellent method, especially for rapid field calculations. A factor for black spruce and a tally sheet has been developed for use in Saskatchewan, (Mimeo 64-A-12).

3. The determining of average stand height using trees selected in point sampling appears to be entirely satisfactory, although it has not been tested in this province.

4. A cumulative volume tally sheet has been developed from which rapid estimates in board feet for various species in Saskatchewan can be made.

5. Growth estimates may be made with point sampling, but mortality and ingrowth are complicating factors that are yet to be dealt with, especially for permanent point samples with periodic remeasurement. However, some assessment of current and future growth can be made from the examination of increment cores taken from trees selected by point sampling, and at least some relative criteria of growth can be established.

6. Some guides as to sampling intensity and expected accuracy are given, but more study is required, and the whole system of sampling design in general should be studied further. This, of course, has to be fitted to a framework where the objectives of the survey are clearly defined.

7. Point and plot sampling with regression were not tested in Saskatchewan, but the method for double sampling with regression is set down.

The few tests done in Saskatchewan add to other evidence in support of point sampling as a desirable method for many forest inventory and management purposes.

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