

# A Simple Fire-Growth Model

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Forest fire control planning sometimes requires a mathematical model of how a forest fire grows with time. Such a formula is especially necessary in problems concerning the economics of fire detection and control. Although the model presented here is not completely original in concept, it is given in a simple but flexible algebraic form not previously used as far as the author is aware.

Assume that, after an initial short period of adjustment, the fire's linear rate of spread at each point on the perimeter remains constant. This rate will vary continuously from a maximum at the head to a minimum at the rear. For simplicity, select values of this linear rate of spread for the head, flanks, and rear of the fire, and assume a uniform fuel.

Next, assume that the fire's head burns a fan-shaped area that widens as the head advances; flank spread then proceeds from the sides of the fan. Furthermore, assume that the width of the fan is such that the fire's shape remains elliptical for any combination of head and flank rates.

Refer to Figure 1, and let the following symbols apply:

- A — fire's area, an ellipse
- a — long semiaxis of ellipse
- b — short semiaxis of ellipse
- v — linear rate of spread at head
- u — linear rate of spread at flanks
- w — linear rate of spread at rear
- t — time since ignition

Then, according to the formula for the area of an ellipse,

$$A = \pi ab$$

But  $a = (v + w) t/2$   
and  $b = 2 ut/2 = ut$   
Therefore

$$A = \frac{\pi}{2} (v + w) ut^2 \quad (1)$$

This expression can be used if all required rates are known, or simplified if necessary. For example, if the fire advances at rate  $u$  at all points on its perimeter, then expression (1) reduces to

$$A = \pi u^2 t^2 \quad (2)$$

the area of a circle of radius  $ut$ . Or, suppose  $w$  is negligible and  $u = v/4$ , then

$$A = \frac{\pi}{8} v^2 t^2 \quad (3)$$

This is the area of an ellipse whose length is twice its width, and whose perimeter is about  $1\frac{1}{2}$  times that of a circle of equal area. This is the average fire shape found by Hornby (1936) in the Rocky Mountains and for which Pirsko (1961) made an alignment chart. Peet (1967) found the same 2 to 1 ratio for length and width of small fires in Western Australia.

The area supposedly burned by the head fire in Figure 1 is shown hatched. Strong winds will result in greater ratios of  $v$  to  $u$ ; at the same time, it is reasonable to assume that the stronger the wind the less will be its directional variation and the narrower the angular width of the fan-shaped head-fire pattern. The width of the fire should thus be about the same near each end, preserving the approximate elliptical shape for all ratios of length to width. The length of  $a$ , the long semiaxis in Figure 1, is plainly half the sum of  $v$  and  $w$ , multiplied by time  $t$ . The short semiaxis  $b$  is not so plainly equal to  $ut$ , which is more exactly represented by the line  $c$  in the figure. The mathematical advantage of the elliptical shape, however, makes this approximation worthwhile for practical purposes.

It is worth noting that in expression (1) the area is proportional to the square of the time since ignition. The rate of area increase at time  $t$  will be given in terms of area per unit time by the first derivative:

$$\frac{dA}{dt} = \pi (v + w) ut \quad (4)$$

This shows that the rate of area increase is not constant but increases in direct proportion to time. However, the acceleration rate at which the area increases is constant and is given by the second derivative with dimensions of area per (unit time)<sup>2</sup>:

$$\frac{d^2A}{dt^2} = \pi (v + w) u \quad (5)$$

In spite of its simplicity, expression (1) requires more information than is commonly known about fire behavior. Ideally, for a given fuel type the rates  $v$ ,  $u$ , and  $w$  might be expressed as functions of a danger index or of specific burning conditions such as fuel moisture and wind.

A fire-growth model is not quite complete without mention of the perimeter. The perimeter of an

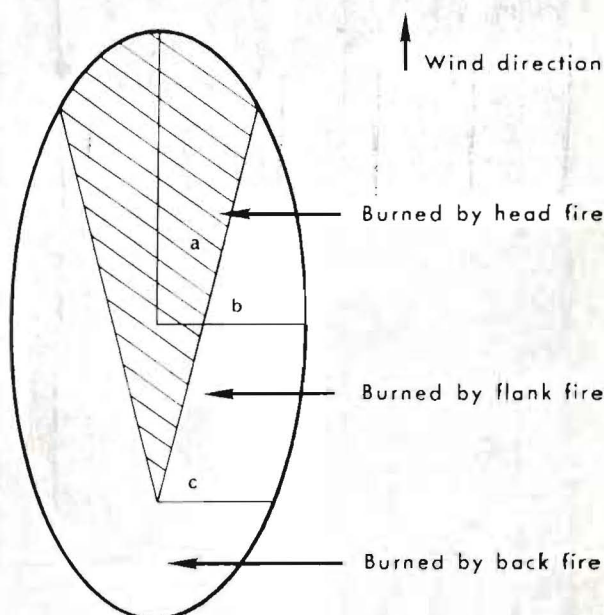


FIGURE 1. DIAGRAM OF SIMPLE FIRE GROWTH MODEL

ellipse in terms of its semiaxes is given by the rather awkward formula:

$$P = \pi (a + b) \left( 1 + \frac{M^2}{4} + \frac{M^4}{64} + \frac{M^6}{256} + \dots \right) \quad (6)$$

where  $M = \frac{a - b}{a + b}$

When  $a = 2b$ , for example, the series in  $M$  equals 1.03; it increases as the ellipse narrows, becoming 1.09 when  $a = 4b$ . For present purposes the terms in  $M^4$  and so on can be omitted, and, with the spread rates substituted, the perimeter formula becomes

$$P = \pi t \left( \frac{v + w}{2} + u \right) \left( 1 + \frac{M^2}{4} \right) \quad (7)$$

The rate of perimeter increase with time is constant and equals

$$\frac{dP}{dt} = \pi \left( \frac{v + w}{2} + u \right) \left( 1 + \frac{M^2}{4} \right) \quad (8)$$

When  $a = b$ , the series in  $M$  equals 1, and expressions (6) and (7) reduce to the formula for the circumference of a circle, which is, of course, the shape with the greatest area for a given perimeter.

## References

- HORNBY, L. G. 1936. Fire control planning in the Northern Rocky Mountain Region. US Forest Serv., Northern Rocky Mountain Forest and Range Exp. Sta., Progr. Rep. 1.
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