Dynamic programming: a tool for financial analysis of stand management regimes

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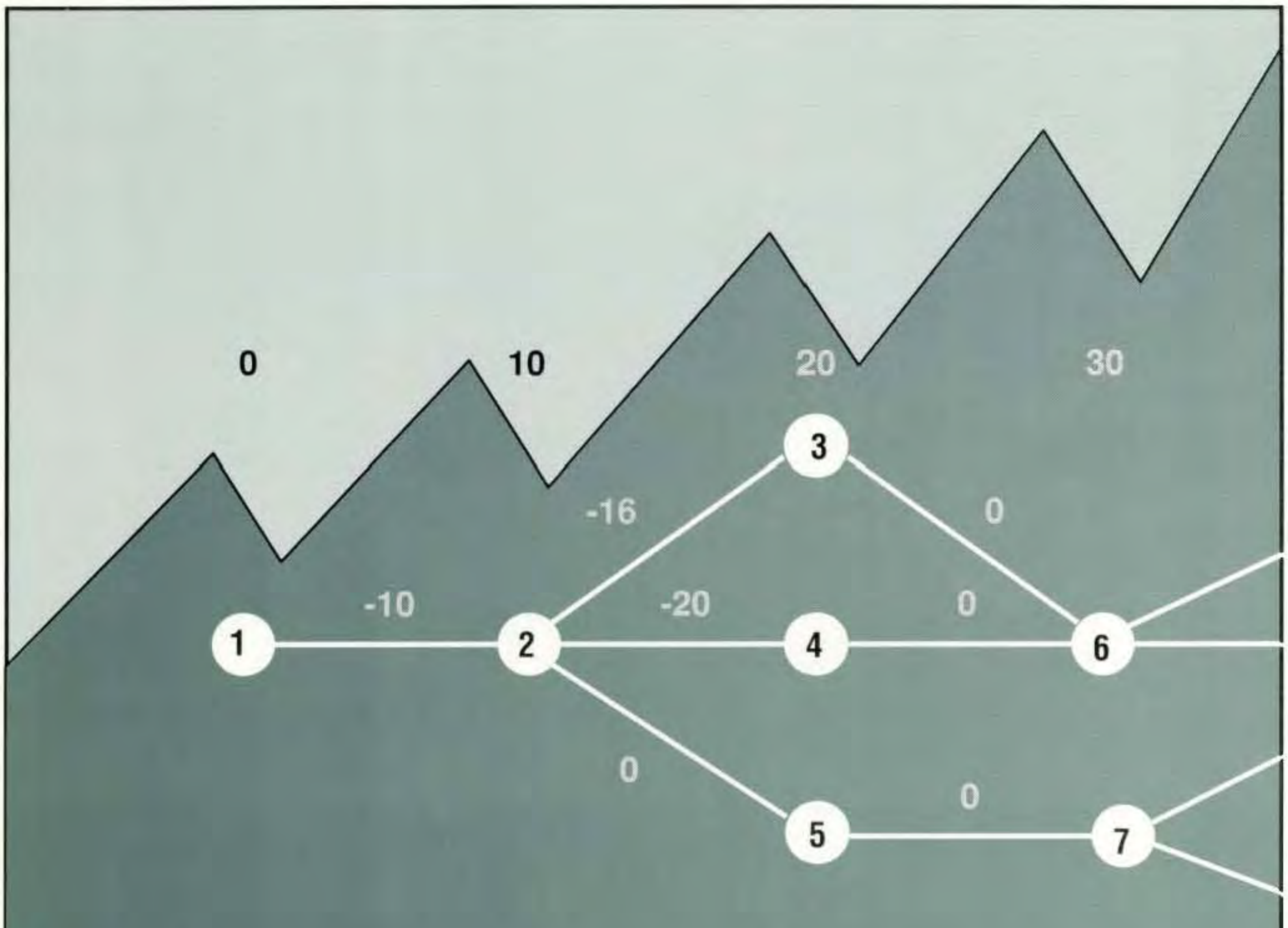
William A. White



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by
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Canadian Forest Service
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Abstract

Dynamic programming is used to determine the optimal management regime for a stand of coastal Douglas-fir. Aspects of stand management considered include initial planting density, the timing and intensity of precommercial thinning, fertilization timing, and the optimal rotation length. Tree growth is simulated using the Tree and Stand Simulator (TASS) while costs and revenue functions reflect conditions on the British Columbia coast.

The results obtained show forest management fails to increase stand values given the 1985 costs and revenues used in this study. It becomes profitable if one assumes increased real prices in the future, if firms are given mature timber today in expectation of increased future volumes, or if a low discount rate is used.

Résumé

La programmation dynamique est utilisée pour déterminer le régime sylvicole optimal d'un peuplement côtier de douglas taxifoliés. Parmi les aspects de l'aménagement du peuplement examiné, mentionnons la densité initiale de plantation, la période et l'intensité de l'éclaircie précommerciale, la période de fertilisation et la révolution optimale. La croissance des arbres est simulée à l'aide du simulateur de développement des arbres et des peuplements (TASS), tandis que les coûts et les revenus reflètent les conditions prévalant sur la côte de la Colombie-Britannique.

Les résultats obtenus montrent que l'aménagement forestier n'a pas réussi à faire augmenter la valeur des peuplements au regard des coûts et des revenus de 1985 utilisés dans la présente étude. L'aménagement forestier devient rentable si on suppose une augmentation des prix réels à l'avenir, si les entreprises se voient attribuer des peuplements mûrs aujourd'hui en prévision de volumes futurs plus élevés ou si un faible taux d'actualisation est utilisé.

Introduction

The forest sector has long been the dominant component of British Columbia's economy. It consistently accounts for nearly half of the total manufacturing shipments in the province and close to 60% of provincial exports. Approximately 85 000 persons or over 6% of British Columbia workers are directly employed in the forest sector and about twice this number are indirectly dependent on the forest sector for their livelihood in service industries such as transportation, capital repair and construction, material and supply (Canadian Forestry Service 1983). In non-metropolitan areas outside the lower mainland and the southern part of Vancouver Island there are many communities and regions which are dependent on the forest resource as their major or only economic base (White *et al.* 1986).

The British Columbia forest sector was based and has been maintained on an endowment of high-quality timber. However, the finite nature of the old-growth forest has become evident. The severe impact that decreased harvests could have on the provincial economy has led to a significant increase in silvicultural activity in the province and a keen interest in second-growth forests and the impact of forest management practices on them.

The primary objective of this report is to demonstrate how dynamic programming can be used to determine forest management regimes which will bring about the highest financial return from a forest stand. This work will attempt to synthesize some important elements of the literature into an approach which will be applicable to British Columbia. A stand optimization model is developed in a deterministic setting. From the model it will be shown how optimal regimes can be obtained using dynamic programming, which has emerged as a powerful approach to stand level problems (Brodie and Haight 1985; White 1989). Full information about stand growth and unlimited funds to carry out the work necessary to achieve optimality are assumed.

The stand management problem

Initially, the manager is faced with a bare plot of land on which he wishes to establish a stand of Douglas-fir (*Pseudotsuga menziesii* (Mirb.) Franco)1. The manager wishes to maximize the financial (as opposed to volume) return from the stand over an infinite number of timber crops. It is assumed that forestry is the best use for the site and that Douglas-fir is the best species for the site. Non-timber values are not considered. See Hartman (1976) and Calish *et al.* (1978) for how these might be used.

The lengthy period of optimization can be justified in either of two ways. It can be assumed that the Crown is the manager and is interested in maximizing returns to the public over all generations or that a private land owner plans to leave the land as a bequest and wants to leave the site in a state that all future generations may benefit from equally. To meet this objective the manager must make a number of decisions on how the stand is to be managed. Should the site be left to regenerate naturally, or should seedlings be planted? If seedlings are planted, how many? Should the stand be thinned or fertilized? If so, when and to what degree? When and how should the stand be harvested?

The dynamic programming approach

Dynamic programming is essentially an optimization approach that simplifies complex problems by transforming them into a sequence of smaller simpler problems (Bradley *et al.* 1977). For an introduction to dynamic programming see Hillier and Lieberman (1980), Wagner (1975) or Nemhauser (1966). An introduction to forestry applications of dynamic programming can be found in Dykstra (1984) and White (1988) while the existing literature in the field is reviewed by Brodie and Haight (1985) and White (1989).

Although not all problems are suitable for solution by dynamic programming, problems of forest stand management are (White 1989). White (1989) discusses in some detail the general treatment of forest management problems with dynamic programming, and reviews key assumptions, the form of a suitable recursive function, the properties of state variables, and the effects of the principle of optimality.

Growth, cost and revenue assumptions

In this report, growth, cost, and revenue assumptions that reflect the situation in coastal British Columbia are used in a case study. The purpose of this section is to outline those assumptions.

Growth

Dynamic programming stand optimization models have been driven by two basic growth mechanisms. The early work relied on yield tables (Amidon and Akin 1968; Martell 1980; Brodie *et al.* 1978) while the later work (Kao 1980; Brodie and Kao 1979; Riitters *et al.* 1982) has taken advantage of stand projection models such as DFIT (Bruce *et al.* 1977), DFSIM (Curtis *et al.* 1981) and PPINE (Hann 1980).

The model chosen for this study is TASS (Mitchell 1975).

TASS is a single-tree, distance-dependent simulator (Munro 1974; Mitchell 1980). In particular, TASS is a crown stand model. It simulates the development of the crown and the bole of individual trees in considerable detail. The sum (or average in the case of dbh and height) of the tree statistics make up the stand statistics. The center of activity in TASS is the crown. It responds to growing space constraints through death in the lower branches. This recession of the crown ceases when competitors are removed through mortality or thinning creating additional space for crown expansion. External influences on the tree such as pest attack, pruning, or fertilization are transmitted through the crown.

The most compelling reason for using TASS was that it is a model based primarily on British Columbia data and thus could best simulate growth in coastal British Columbia. Also, it is the growth model used by the British Columbia Ministry of Forests. This will make this study more practical than if another model was used. Finally, it has not been used in previous optimization studies of this type. Comparison of Douglas-fir yields generated from TASS and DFSIM have shown that these independently developed models using different methods and data have produced similar results. TASS stands tend to have less mortality and therefore grow more volume but smaller trees (Mitchell 1986). The overall result is that stands grown by TASS result in lower financial returns than stands grown with DFSIM.

TASS allows for many management options and environmental factors to be considered. Those included in this report are initial density, juvenile spacing and fertilization. Stands may be thinned by removing a certain basal area, a certain volume, a percentage of trees, particular individual trees, or strips of trees. Trees can be removed starting with those of smallest diameter (thinning from below) or the largest diameter (thinning from above). As well, limits may be set on what diameter trees should be left or removed. While TASS has the valuable feature of keeping track of diameter distributions, it was decided to remain consistent with previous work (notably Kao 1980) and remove trees by specifying a number of trees. In this study, thinning will commence with the smallest trees.

Only one site quality will be used in this study. Since low (poor) sites do not receive much attention from managers and top quality sites are rare, the site quality chosen is in the good to medium range. This is appropriate since forest management in the province is concentrated on good and medium sites.

The method used by Kao (1980) with respect to fertilization will be followed in this report. Kao utilized the work of Turnbull and Peterson (1976a, b) where response to fertilization is a function of age, basal area, number of trees and site. These are the state variables employed both by Kao and in this study, which makes the adaptation easy. The version of TASS used in this work did not contain its own predictive response to fertilization.

Costs

A vital component of stand optimization studies are the assumptions with regard to cost. Cost components used here include site preparation, planting, juvenile spacing, fertilization, and harvesting.

Site preparation costs

Following harvest, a site must be prepared for prompt regeneration. Site preparation includes activities such as surveys, slash burning and falling residual stems. The average cost for preparing land for planting in 1984-85 was $172 per hectare for the province as a whole and between $151 and $162 per hectare in coastal regions (British Columbia Ministry of Forests 1985). Site preparation costs can also be estimated from the Vancouver Stumpage Appraisal Guide (Cooney 1981; Heaps 1985).

Because of the narrow range of site preparation values included in the British Columbia Ministry of Forests annual report and its relative simplicity, a roughly weighted average of costs in the coastal regions will be used in this study, viz., $155 per ha. This figure will be used on all rotations except the first; it is assumed that the decision maker is starting with bare land suitable for planting.

Planting costs

Two levels of initial spacings will be considered in this study—550 and 1100 seedlings per ha. Dense stands tend to increase in volume quickly in the early years but this advantage dissipates due to high mortality. The surviving trees tend to have considerable vigour as they are sorted by natural selection. Less dense stands do not have this advantage but are less expensive to establish and the average diameter of the trees is greater than for more dense stands.

In most of the previous dynamic programming studies, planting costs have been treated as a simple fixed value as only one initial stocking density was considered. Site specific considerations, such as slope, were ignored. Exceptions to this were Lembersky and Johnson (1975) and Hann *et al.* (1983) where a variety of initial densities were considered. In these papers equations were presented which correlated planting cost with initial density. Lembersky and Johnson used an equation developed from Buongiorno and Teeguarden (1973). Specifically:

Planting Cost per hectare = \$52.5 + \$.14 p

where p is the number of trees planted and costs are converted to 1985 Canadian dollars. Hann *et al.* present separate costs for four different planting levels. When these points are joined they can be expressed by the equation:

Planting Cost per hectare = \$112.09 + \$.14 p.

A third source for planting cost information was a survey of planting contracts by Fraser and Howard (1987). Their survey of twenty-seven contracts across B.C. revealed the following relationship between planting cost and initial density:

Planting Cost per hectare = \$57.92 + \$.14 p.

The weighted average of costs from the British Columbia Ministry of Forests Annual Report (1985) and the first year of the Canada-British Columbia Forest Resources Development Agreement (FRDA) are \$293 and \$227 respectively. By calculating the weighted average of these two figures, it will be assumed that the cost of planting 1100 trees (exclusive of seedling cost) in a coastal stand is \$236. This means that the equation used to determine planting costs at various densities will be:

Planting Cost per hectare = \$82 + \$.14 p.

Thinning costs

Juvenile spacing, or precommercial thinning as it is also known, removes inferior trees from dense plantations between 10 and 15 years old. This leaves room for accelerated growth in the biggest, healthiest, best formed trees. Trees that are felled are left on the ground to decay and return nutrients to the soil.

The cost of precommercial thinning is influenced by various factors. A cost function for precommercial thinning would increase with the age and initial density of the stand and with the intensity of the thinning. Site-specific factors such as slope and density of undergrowth also influence cost. Most previous dynamic programming work has ignored the variables affecting the cost of precommercial thinning and has treated it as an average or fixed cost since they contained a single case of precommercial thinning. However, using a fixed cost may lead to some biases in the results when an array of precommercial thinning options are being considered. The potentially more expensive options of heavier thinnings, postponed thinnings, and thinning of stands with larger average diameter become more attractive. Ideally the cost function for precommercial thinning would include terms for initial density, intensity of thinning, and other variables. A search of the literature failed to uncover such a function for the British Columbia coast.

The costs of precommercial thinning used in this study are given in Table 1. While no specific function is used to generate these costs, they are based on average precommercial thinning costs in the British Columbia Ministry of Forests Annual Report (1985) (\$600/ha) and on the costs of precommercial thinning in coastal regions during the first year of the Canada-British Columbia Forest Resources Development Agreement (FRDA) (\$700/ha). (The difference can be explained by the fact that more of the thinnings under FRDA occurred in the Prince Rupert region where thinning tended to be more expensive than in the Vancouver region.) Berg (1970) noted a steep increase in the time required to fell larger trees during precommercial thinning even when the diameter differences are small; this is reflected in the costs shown in Table 1.

Table 1. Assumed precommercial thinning costs

Intensity of Thinning	Average Diameter Small (<8 cm)	Medium (>8 x <13 cm)	Large (>13 cm)
Light	325	450	625
Medium	450	600	700
Heavy	575	650	800

Fertilization costs

Fertilizer is applied to forest stands to increase the diameter, height, gross volume, and merchantable volume. Through improved yields obtained by fertilization, it is hoped that crop rotations can be shortened. Funds from FRDA have permitted fertilization to be carried out over large areas during the last few years. Private firms have also increased the use of fertilization ("BCFP launches program to fertilize forests" by J. Twigg, Vancouver Sun, November 17, 1986). Fertilizer is normally applied by helicopter. The 1985 Annual Report of the British

Columbia Ministry of Forests gave the average cost of fertilization as \$227/ha. According to the 1986 Vancouver Sun article cited above, the expected cost of work to be carried out in that year was \$160/ha; this figure will be used in this study.

Harvesting costs
The cost of harvesting a forest stand is the sum of the cost of many operational phases: (i) roads must be constructed and maintained (development costs); (ii) the trees must be felled and bucked, yarded, and loaded onto trucks (tree-to-truck costs); and (iii) the logs must be hauled to the mill or to dump sites in preparation for barging or towing (hauling costs). Other costs associated with logging include crew transportation, engineering, and scaling costs as well as administration and overhead.

Models exist which estimate logging costs as a function of parameters such as species harvested, stand diameter, volume per hectare, slope, terrain, and obstacles. Cooney (1981), Morrison *et al.* (1985) and Williams (1987) have produced models of recovery costs for the British Columbia coast based on the Coast Lumber Appraisal Manual. This is the manual used by the British Columbia Ministry of Forests to estimate logging costs on the British Columbia coast for the purpose of calculating stumpage under a residual system.

The cost of developing a site for logging can be expressed as a function of side slope, road construction costs, number and size of bridges and culverts, haul distance for ballast, and the volume of timber per ha (Williams 1987). With the exception of volume of timber, all these factors will be fixed to reflect an average stand. The average development cost on the coast in 1985 was \$3600/ha. Development costs per cubic metre will be viewed as a decreasing function of volume. So as the volume in a stand increases, development costs per m^3 will decrease.

Tree-to-truck costs will be estimated using the model contained in Williams (1987). It estimates tree-to-truck costs per cubic metre as a function of species, average stand diameter, volume per ha, slope, terrain, and logging system. Slope and terrain will be fixed[1] along with species (Douglas-fir) and the breakdown of the logging system. The logging system is broken down to reflect the proportion of high-lead and grapple yarding system used on the coast. Average stand diameter and volume per ha become the TASS outputs which drive this phase of the model. As stand diameter or volume per ha increase, the tree-to-truck costs per cubic metre decrease.

Hauling costs per cubic metre are a function of haul distance, road conditions, and load size. To these costs must be added water transportation costs which averaged \$7.88/m^3 in 1985. Total transportation and crew related costs will be assumed to be a fixed \$15/m^3. Future work will consider a range of logging costs. Other costs such as scaling, administration, overhead and allowance for profit and risk will also be treated as fixed costs.

Discount rate
The question of the proper discount rate to be used in the context of forestry investments has enjoyed a long debate in the literature. Recent reviews of the subject by Fraser (1985) and Heaps (1985) support a range of real discount rates between 5% and 10%; Heaps favours the lower figure because although there are substantial risks inherent in the growth of a forest stand (failure, pests, fire, etc.), these become negligible across many stands. Heaps and Pratt (1989) suggest a range of between 3% and 7%. Five percent will be used in this study for the base case, while other rates will be used at times for comparison. A higher discount rate would affect the results by favouring shorter rotations.

Revenues
The revenue function in this report is based on the work of R. Gasson and D. Williams (A method for estimating the value of timber inventories. Unpublished Report. Forest Economics and Policy Project, University of British Columbia, Vancouver, 1986). The value of a stand is a function of the species contained in the stand and the average volume of the logs produced. Log volume is a function of mean diameter and is adjusted for yarding losses and current utilization standards, and is net of "logs left due to defect, size, or breakage" (Page 2 of the report by Gasson and Williams cited above). The authors also estimate the expected distribution of log grades from each species and calculate the relative value of those grades from information on the Vancouver Log Market. The prices used in the study are average log market prices over the years 1980 to 1985. From this information a single value can be calculated which represents the value per cubic metre obtained based on the average stand diameter and the species present in the stand. This method is

[1] Slope will be assumed at the 1985 coastal average of 46% while the terrain will be assumed to be even. The average slope figure is high but reflects the sites that will have to be replanted.

consistent with other recent dynamic programming studies in that it reflects the benefits accruing to harvesting logs of larger diameter. This long-range benefit of thinning can thus be accounted for.

Results

In a deterministic model all factors influencing the solution of the model are known and constant. Specifically, the stand grows in a predictable fashion. There are no shocks in the form of periods of exceptionally good (or bad) growing conditions, random pest attacks, or fires. When a stand is thinned or fertilized, it responds in a consistent and predictable manner. Prices are assumed constant as are the costs of silvicultural activities and timber harvesting. The discount rate is also known and constant.

Assumptions must also be made with regard to the method of solution of the model: factors such as the stage interval and the various thinning and basal area intervals must be specified.

The role of the stage interval in a forestry-dynamic programming problem is considered by Kao and Brodie (1979) and to a lesser extent by Roise (1986). Reducing the stage interval from 10 years to 1 year increased the soil expectation value of the stand by about 1% according to Kao and Brodie. Roise increased the soil expectation value by 39% by reducing the stage interval from 15 years to 10 years. Since the gains obtained by reducing the interval to less than 10 years are small, an interval of 10 years is used in this study.

States will be described by age, number of trees, and basal area. White (1989) discusses the effect of various intervals on soil expectation value, computing time, and occurrence of artifact effects. Artifact effects can be minimized by using large tree intervals or by using a combination of small tree intervals and small basal area intervals. Following Kao (1980) the latter method will be used here and intervals of 25 trees and 0.5 m^2 will be used to classify the states.

In the cases involving juvenile spacing, a given number of the stems will be removed beginning with the shortest trees. Thinnings will begin at 25 trees per ha and increase up to 250 trees per ha for stands planted at 550 stems per ha, and up to 600 trees per ha for stands with an initial density of 1100 stems per ha. Two initial densities—550 and 1100 trees per ha —will be considered. Each will be run separately to avoid adding to the state space. The impact of adding 222 kg per ha of fertilizer at 30, 40, 50 and 60 years will also be evaluated.

The unconstrained deterministic models are solved by means of forward induction. Specifically, they take on the following form:

$$T(x+1,y) = \max_{y'} \{T(x,y') + R(y',y)\}$$

where y' is any node stocking level in the current stage that can reach stocking level y in the next stage.

$R(y',y)$ is the discounted revenue or cost associated with the transition from y' to y.

$T(x,y')$ is the total discounted net revenue up to the stage being considered.

$T(x+1,y)$ is the total discounted value of the optimal schedule up to $(x+1,y)$.

This recursion can be modified into more conventional mathematical programming notation. Specifically:

$$w^j = \max_{k} R(\text{k,j}) + w^k$$

A state k is being chosen from all states which lead to state j in the next stage. That k is chosen so that the value of being in that state (w^k) combined with the return from making the transition from k to j provides with the maximum possible value (w^j) of being in state j.

A small modified example (Figure 1) extracted from the solution to the problem with an initial stand density of 550 trees per ha will help the reader visualize how a dynamic program is solved by forward induction and what makes it more efficient than simulation.

Given the network form of this problem it can be solved using a simple shortest route recursion such as:

$$Y_i = \max_{j} \{R_{ij} + y_j\}$$

where the values of Y_i and Y_j are the values being at nodes i and j respectively and the R_{ij} are the costs of traversing the arcs.

The main point of interest in the example occurs when the decision maker decides how to get to state 6. The figure shows that it is less expensive to pass through state 3 than state 4 so this is the optimal path. This implies that the path 1-2-4-6 need never be considered again. That is, the level of precommercial thinning at age 10 that caused the transition from state 2 to state 4 is eliminated and we do not have to look at any events beyond age 30 along that path. This highlights the benefit of dynamic programming over simulation. Simulation would continue to use that path for comparison of regime values while dynamic programming has saved time and effort by eliminating this path sooner.

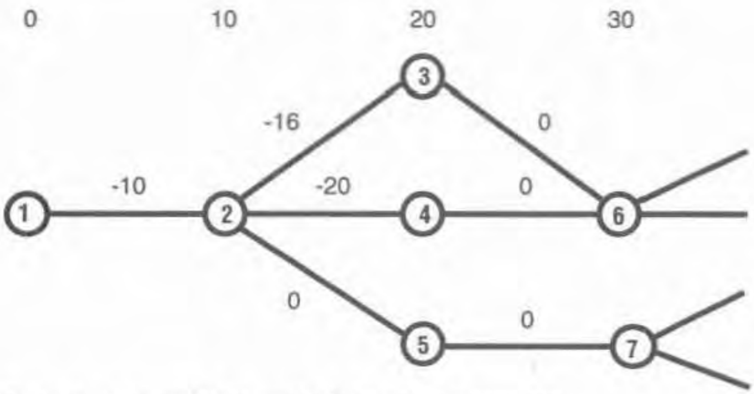

Figure 1. A simplified example of dynamic programming

The results are presented in two ways—one reflecting a single rotation on a site, and the other reflecting an infinite series of rotations. The first will be the Net Present Value (*NPV*) return while the second is the soil expectation (*SE*) value. These can be calculated using the following formulae:

$$NPV_r = \frac{Y_r + T_b\,(1+i)^{r-b} - C_c\,(1+i)^{r-c}}{(1+i)^r}$$

$$SE_r = \frac{Y_r + T_b\,(1+i)^{r-b} - C_c\,(1+i)^{r-c}}{(1+i)^r - 1}$$

where, Y = net yield at rotation age
T = net value of commercial thinnings
b = ages of commercial thinnings
C = cost of treatments (including planting, spacing and fertilization)
c = ages at which cost is incurred
r = length of rotation
i = interest rate

A simpler way to obtain the *SE* value once the *NPV* has been calculated follows:

$$SE_r = \frac{NPV_r\,(1+i)^r}{(1+i)^r - 1}$$

Since i and r are known the calculation simply becomes NPV multiplied by a single number.

While the above equations define *NPV* and *SE* they do not reflect how they are obtained in a dynamic programming setting. At each stage there is a node which contains the maximum *NPV* for that rotation (r) if harvest were to take place at that age. The *SE* value is obtained by taking the *NPV* value and adding to it an infinite series of similar rotations. This can be expressed as:

$$SE_r = NPV_r\,(1 + a^r + a^{2r} + \ldots)$$

This reduces to:

$$SE_r = \frac{NPV_r}{1-a^r}$$

So the maximum value can be found according to:

$$SE^* = \max_{r = 10, 20\ldots} \frac{NPV}{1-a^r}$$

and a is the discount rate.

Using the 5% discount rate selected for this case study, a plot of bare land of high medium quality devoted to timber growing could not provide a positive return on investment given average logging conditions in 1985. Furthermore, over one or an infinite number of rotations (Table 2) losses could be minimized by planting a stand and then ignoring it until harvest[2]. The following paragraphs investigate the impact each of the decision variables had on stand growth and the optimal level of each of these variables. An effort is also made to reconcile the fact that many dollars are invested in silviculture each year when returns appear to be negative.

Stands planted at the lower density (550 trees per ha) tended to grow faster, mature sooner (i.e., the stands had shorter rotations) and produce plots of trees of greater average diameter. The higher returns were beyond those which could be explained by the lower cost of planting alone and lend credence to the views recently expressed by Smith (1986) about the benefits of wide spacing.

In general, stands which were juvenile-spaced produced larger trees, matured sooner and provided greater revenues at harvest than unthinned stands. This is consistent with the results obtained for initial

[2] TASS does not consider competition the seedlings may face from weeds and/or unwanted hardwood species. Experience in British Columbia has shown that removal of these is essential for a stand to thrive.

density. With a 5% discount rate, however, these benefits did not exceed the cost of obtaining them.

Regardless of initial density, earlier thinnings lead to greater revenues at harvest time than the same intensity of thinning at a later date. Though thinning later is more expensive in pure dollar terms and has less growth impact than earlier thinnings, when discounting is taken into effect later thinnings become the preferred alternatives in terms of *NPV* and *SE* (Table 3).

The optimal intensity of thinning also varied depending on initial density and whether harvest revenue or financial return is optimized. The optimum financial return was achieved at a lower intensity than the maximum level of harvest revenue when initial density was 1100 trees per hectare (Table 4).

Fertilization was allowed to impact the stand as described by Turnbull and Peterson (1976 a,b). Fertilized stands tended to produce more total volume, have a greater average diameter, and produce greater harvest revenue. Despite this, fertilization treatments did not pay for themselves at a 5% discount rate. Financial losses from fertilization were minimized when it was applied later (age 60) rather than earlier (age 30). Harvest values also peaked if fertilizer was applied at age 60.

The results change as we allow the discount rate to fall. A lower discount rate decreases the financial benefits accruing to later silvicultural treatments and reduces the penalty on harvest returns. Hence regimes compared under lower discount rates would differ from higher discount rate regimes in four distinct ways:

Table 2. *NPV* and *SE* values by rotation age and initial density

Rotation Age	Initial Density	Optimum Regime[1]	*NPV* $/ha	*SE* $/ha
50	550	Thin by 250 at 20	-428.15	-469.05
50	1100	-	-	-
60	550	No action	-243.09	-256.84
60	1100	-	-	-
70	550	No action	-121.47	-125.59
70	1100	Thin by 750 at 30	-381.13	-394.07
80	550	No action	-84.69	-86.46
80	1100	Thin by 750 at 20	-327.13	-333.99
90*	550	No action	-82.72	-83.76
90	1100	No action	-203.97	-206.53
100	550	No action	-97.46	-98.21
100*	1100	No action	-194.53	-196.02
110	1100	No action	-202.39	-203.34

[1]Blanks indicate trees were too small to obtain an accurate return on harvest figure.
*Optimal rotation ages by initial density

Table 3. Impact of thinning age on harvest return, *NPV* and *SE* values

Initial Density (trees/ha)	Thinning Intensity (trees removed/ha)	Stand Age at Thinning	Harvest Age[1]	Harvest Revenue[1]	*NPV* $/ha	*SE* $/ha
1100	250	10	100	6052	-389	-392
1100	250	20	100	6018	-313	-315
1100	250	30	100	5521	-269	-271
1100	500	10	90	5393	-445	-451
1100	500	20	90	5229	-340	-345
1100	500	30	90	4511	-284	-288
550	200	10	80	5635	-252	-254
550	200	20	80	5488	-225	-226
550	200	30	80	4633	-217	-218

[1]Harvest age is optimal for the regime considered.

Table 4. Optimal thinning intensities for maximizing harvest revenue, *NPV* or *SE*

Initial Density (trees/ha)	Thinning Intensity (trees removed/ha)	Stand Age at Thinning	Harvest Age	Harvest Revenue	*NPV* $/ha	*SE* $/ha
550	175	30	80	5084	-208	-212
550	175	20	80	5688	-221	-225
1100	200	30	100	5521	-269	-271
1100	600	10	100	9455	-517	-521

Table 5. Effect of the discount rate on optimal regime and returns

Initial Density trees/ha	Discount Rate %	Optimum Regime	Harvest Age	*NPV*	*SE*
1100	5	No action	100	-195	-196
1100	4	No action	100	-128	131
1100	3	No action	100	47	50
1100	2	Thin by 600 at 20 fert. at 60	100	690	800
550	5	No action	90	-83	-84
550	4	No action	90	31	32
550	3	No action	90	304	327
550	2	No action	100	1078	1251

Table 6. Optimal regimes when Allowable Cut Effect is considered

Rotation Age (years)	Optimal Regime	*ACE NPV* $/ha	*NPV* of Treated Stand Including *ACE NPV*
70	No action	2545	2092
80*	No action	2636	2340
90	No action	2315	2111
100	No action	2241	2046

ACE = Allowable Cut Effect
* = Optimal Regime

1) Overall they would be more profitable;
2) Management costs would be accepted earlier in the regime;
3) Optimal regimes are more likely to include silvicultural work; and
4) Optimal harvest age may be later.

The results shown in Table 5 confirm point one and suggest that there is merit in points three and four. A closer look at the results shows that although "no action" remained optimal, its advantage over the next best regimes was only 20% (at 550 trees per ha) at a discount rate of 2%, compared with 98% at 3%, and 147% at 5%. A similar trend exists at 1100 trees per ha. This lends credence to point three. Point four is verified by the fact that for individual regimes the optimal rotation length never fell as the discount rate fell and increased in one case. Comparison of non-optimal regimes showed that as the discount rate fell, regimes with earlier management actions moved up to a rank higher than regimes with later management actions, thus confirming the second point.

Separate runs were conducted to measure the financial impact of fertilization at discount rates below 5%. Fertilization proved to be a viable option when the discount rate fell to 2%.

The reasons for using the 5% rate were discussed earlier. The results presented here show why it is a topic of heated debate: a movement of a single percentage point can radically alter the results.

The planting of backlog reforestation sites where site rehabilitation and planting costs average $700-$800 per ha also fails to provide positive net stand values at 1985 costs and revenues even when the federal government is paying 50% of the cost.

Another factor which can influence the real cost of silvicultural treatment is known as the allowable cut effect. In brief, the allowable cut effect permits the expected increase in mature timber at harvest resulting from a silvicultural investment today to be partially recouped today. For example, a reforestation project estimated to provide 750 m^3 in 55 years could provide an additional 13.6 m^3 additional harvest from mature stands today (Fraser 1985). That is, the government would increase the annual allowable cut of firms engaging in silviculture activities, or a private firm may cut more of its stock today in anticipation of adequate supplies to meet its future needs. This approach has some limitations. Sufficient mature forests must be available over the life of the project to allocate additional timber to the firm. It has also been shown (Fraser 1985) that the allowable cut effect encourages reforestation projects over spacing and fertilization projects since these bring a greater increase in allowable cut today. Indeed, it is questionable whether spacing adds volume at all.

An increase in cut today could also be viewed as a decrease in the cost of a silviculture project. For instance, Fraser provides an example where a spacing and fertilization project provides an additional 195 m^3 at harvest time. The allowable cut effect would be 4.4 m^3 per year over the 44 years until harvest. If we assume that the additional harvest has a mean dbh of 50 cm and a net value therefore (according to our value and costs model) of about $5/$m^3$, a flow of $22 per year would accrue over the 44-year period. This would be divided between the firm and the Crown. Discounted at a rate of 5% the net present value (*NPV*) of this flow is $389 which helps offset the approximate $700 cost of the project. If this flow is taxed at a full 40% the NPV is still $233 while the net (tax benefits considered) cost of the project would be about $420. Table 6 illustrates the impact of the allowable cut effect on the optimal regime where the initial density is 1100 trees per ha and in particular on the return to silvicultural investment.

Table 7. Expected returns given various future price increases and regimes and a 4% discount rate

Regime	Annual Percentage Price Increase			
	0	1	2	3
No Planting	0	0	0	0
Plant but no management	-129	-23	84	190
Plant, thin by 600 at age 20, fertilize at age 60	-318	-123	72	268

Table 8. Expected returns from stands at risk from pest attacks at harvest

Probability of pest attack in years 70	80	90	Optimal regime[1]	Expected Return $	Loss $	Loss %
0,	0,	0	A*	2888	-	
.3,	0,	0	A	2294	594	21
0,	.3,	0	A	2738	150	5
0,	0,	.3	A	2663	225	8
.3,	.3,	0	A	2244	644	22
0,	.3,	.3	A	2595	293	10
.3,	.3,	.3	A	2166	722	25
.8,	0,	0	B**	1362	1526	53
0,	.8,	0	B	2619	269	9
0,	0,	8	A	2419	469	16
.8,	.8,	0	A	1266	1622	56
0,	.8,	.8	A	2254	634	22
.8,	.8,	.8	A	1210	1678	58
.6,	0,	0	B	1735	1153	40
0,	6,	0	A	2667	221	8
0,	0,	6	A	2516	372	13
.6,	.6,	0	A	1643	1245	43
0,	.6,	.6	A	2388	500	17
.6,	.6,	.6	A	1525	1362	47

1 This represents the optimal planned regime. If a pest attacks, this regime would not be followed but rather the stand would be harvested as explained in the text.

* Thin by 600 at 10; fert. at 60; harvest at 100.

** Thin by 600 at 20; fert. at 60; harvest at 100.

When the allowable cut effect is considered, the optimal action is still to leave the trees to grow after planting. The difference is that the additional mature timber the firms are allowed to cut makes planting the trees very profitable for the firm. The alternative of applying no silvicultural treatment provides the greatest benefits under the allowable cut effect because it provides the most volume. Note that the return from the allowable cut effect and the new stand is less than that from the allowable cut effect alone (Table 6). The newly planted stand does not provide a positive return on its own but is planted because it produces revenues for the firm today through the allowable cut effect. The promise of additional volumes of mature timber today should encourage firms to reforest stands even on Crown lands.

While planting stands on the basis of the allowable cut effect is profitable from the firm's perspective and helps the government maintain a viable forest industry in the province, the benefits from the allowable cut effect that stem from spacing and fertilization are not as clear. TASS runs show that spacing decreases or provides very small increases in the volume of planted stands at maturity though the logs are of greater size and are therefore of greater value. Fertilization consistently provides small increases in volume. For instance, a stand on a good-medium site planted at 1100 stems per ha and thinned 500 stems creates only 4 m^3 of additional volume. This does not justify the cost of the project.

Another factor which will affect stand values is the future trend of forest product prices (Table 7). Dynamic programming is a sound tool for handling these kinds of problems. In their Markov model, Lembersky and Johnson (1975) assumed a general upward trend in prices (2.7% per year) and that market fluctuations retained their present character. This is equivalent to assuming that the probability of making a transition between stages to a state characterized by a certain level of prices is the same today as it will be far into the future. This is a bold assumption which puts the problem into the category of risk rather than uncertainty where it more probably belongs (Kao 1984).

The impact of future increases in the prices of forest products can be seen quite readily from the results showing the impact of lower discount rates. Just as lower discount rates make investment in forestry and more particularly in forest management activities profitable, so projected price increases have the potential to make forestry a profitable investment.

Stochastic dynamic programming

The outcomes in the previous sections were deterministic or certain. This is known as decision making under certainty.

Moving away from certainty we approach various degrees of indeterminateness. The one closest to certainty is when we know exactly what the probability is of each of a number of outcomes for a given decision. An example of this would be if we know that there is a 1% chance that a stand in a given state will be destroyed by fire if not harvested in a given period. Technically, the decision to harvest or not harvest is known as decision making under risk, which will be the context used in this chapter.

In the case study that follows it will be assumed that the manager is risk neutral. This implies that the criterion for choosing the best outcome will be Expected Net Present Value (*ENPV*) in all cases involving risk.

The recursion described early for the deterministic model can be modified for use in stochastic dynamic programming. The form the recursion takes is that of a probabilistic shortest route problem. As well, the problem is written only in terms of states rather than states and stages. This follows convention. Finally, the problem is solved via backward recursion rather than forward recursion.

Following Wagner (1975) we can modify our previous deterministic model to a stochastic dynamic program. Furthermore, this stochastic dynamic program will be a Markov process if certain generalizations are made. First of all, a decision d in state i can result in the system moving into one of several states rather than only one state as in the deterministic case. This can be written as a conditional probability

$p(j|i,d)$

which refers to the probability that we move to state j from state i given decision d, where the decisions are spacing, fertilization and harvesting. To be a first-order Markov process, each of these conditional probabilities must embody the Markov property that the probability depends only on the existing state regardless of the history of the system. That is, no matter what decisions (thinning, fertilization) brought us to state i, the probability of going to state j is the same. We assume a forest stand will grow the same way no matter how it comes to be as it is.

Secondly, $R(j'k)$, the immediate return of going to k from j' must now be expressed as a random process to reflect the fact that it is now an expected return with the expected return varying with the management decision taken. The cost or return on any arc of this random process can be expressed as $C(j|i,d)$. This is the cost or return of going from i to j under decision d with probability $p(j|i,d)$. Hence the expected cost or return to starting in state i and making decision d is C_{id} which is:

$$C_{id} = \sum_{j} C(j|i,d)p(j|i,d)$$

j represents all states reachable from state i.

The third property noted by Wagner is the acyclic property which means that no node will be visited more than once in any turn of the system. As age is one of the state variables, this property is easily met in our forestry problem. This makes the problem non-ergodic, that is the trapping system does not allow us to return to any state once we leave it.

The stochastic recursion can now be in the general form:

$$Y_i(n) = \max_{d \text{ in } (D_i)} \left[\sum_{j=0}^{i-1} \propto p(j|i,d)Y_j(n+1)+C_{id}\right]$$

where $D(i)$ is a decision set at node i and is the 10 year discount factor.

One form of catastrophic loss is an attack by pests. It is assumed that the stand must be harvested shortly after the attack. Indeed, it is British Columbia Ministry of Forests policy to harvest "all beetle attacked timber as soon as possible so that the timber is utilized before it starts to deteriorate." (Truant 1986). If this attack is anticipated it could affect the way in which a stand is managed since managed stands tend to mature more quickly.

The pest attack case study was modeled in a simple fashion but this was still effective in showing three things:

1. The impact of anticipated pest attacks on the expected value of the stand.
2. The impact of anticipated pest attacks on the optimal management regime.
3. The returns to research from correctly predicting pest attacks and hence implementing appropriate management regimes.

The deterministic optimization model used earlier was modified and run to show the impact of the probability of a pest attack in a single 10 year period or over an extended period and the resulting optimal regimes. Returns from harvesting the infested stands were assumed to be 90% of their full values (Giles 1986) and that losses due to fungal decay are zero[3]. A very low discount rate of 1% was used to ensure that there were a number of profitable regimes to choose from. The results are displayed in Table 8. Stand value fell as the probability of pest attack increased and as the period in which the pest might attack was extended. The greatest impact occurred when the pest attack was expected later and the stand had neared or reached its potential maximum value.

The probability of pest attack had only a marginal effect on the optimal management regime. Only when the probability of attack was projected in year 70 or year 80 alone did the regime change and then only the age of thinning was modified. One would have expected the optimal harvest to fall in order to avoid the potential for loss by pests, but this failed to occur in all cases. This means that the expected increase in stand value was greater in every case than the expected loss due to pests and early harvesting. Two of the assumptions of the analysis had a significant influence on this result. First of all, the low discount rate of 1% placed only a small penalty on waiting another ten years to harvest. A larger penalty or cost brought on by a larger discount rate could have altered the results. Secondly, the stands were harvested at 90% of value if the pest attacked. Once again this is a small penalty that the results indicate a manager would be willing to bear over the probabilities and discount rate used here.

The results show that there is significant value in research that can predict when a pest is likely to attack a stand. Note for example the large differences in the expected returns between stands that have the probability of pest attack in year 70 and those that do not. While all results in the cases run here have positive NPVs, the magnitude of the differences could, under different assumptions, make the difference between the expected value of the stand being positive or negative. In terms of percentage of stand value lost, losses from stands which might be attacked in year 70 range from 21% to 58% of stand value while stands that can only be attacked later have losses ranging from 5% to 22% of stand value. The results would be even more dramatic if the probability of attack was considered at earlier ages of a stand's life.

While this analysis has been highly restricted and simplified, it has shown that dynamic programming can handle problems where risk of pest attack is a concern. Where more detailed information is available on the likelihood of attack and the expected impact of that attack on stand values, useful optimal regimes could be derived.

[3] Fungal decay can destroy a dead tree in 2 to 3 years if it is not properly stored. Where this decay can be avoided a 20-year -old dead tree can retain over 70% of its value (Giles 1986).

Summary and conclusions

Dynamic programming was shown to be a powerful algorithm for solving stand optimization problems. A stand model was created for solution using this technique. The model was based on the single-tree distance-dependent crown-based stand growth model TASS and costs and revenues which reflected 1985 conditions in coastal British Columbia. The base case solution showed that planting a stand of Douglas-fir on bare land could not provide a positive return when the discount rate was greater than 3%. Positive returns could also be obtained by assuming real price increases of 2% annually and fixed management costs.

To improve the efficiency of obtaining stand level optimal regimes, the TASS growth model should be integrated with the dynamic programming algorithm as has already been done with DFSIM (DOPT) and some other stand simulators. The stand solutions could also be improved with improved information on management costs. This is particularly true of thinning costs. A model is needed which predicts thinning costs given at least stand characteristics and density of thinning. For comparison, runs simulating natural regeneration should be compared with the results obtained here. Data on survival rates and regeneration delays would be necessary to include this option. Finally, an improved stand model should include realistic fire and pest probabilities.

The stand results obtained here should be compared with results obtained using a variety of logging conditions. The impacts of non-timber values and of using shadowed-priced labour for forest management would also be of interest.

Given an improved stand model with a more efficient means of obtaining optimal solutions, the next step would be to obtain forest level solutions. Decomposition appears to be a technique with much potential in this area (Williams 1976; Nazareth 1973). The impacts of stumpage rebates, allowable cut effects, budget constraints, and uncertain management funding would be significant contributions to the existing literature and provide a useful model to forest planners.

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