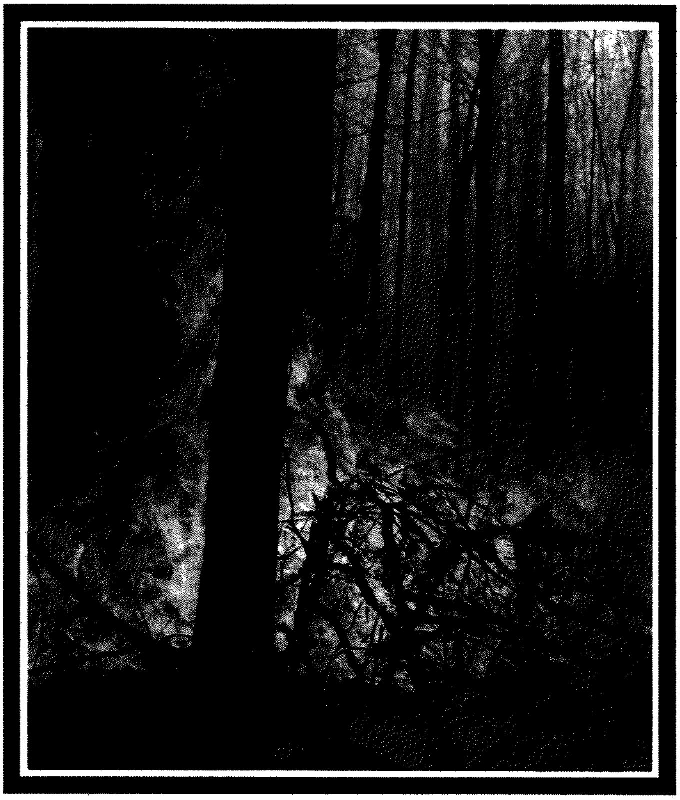
FOREST LANDSCAPE ECOLOGY PROGRAM

FOREST FRAGMENTATION AND BIODIVERSITY PROJECT

Report No. 19



MODELING THE EFFECT OF SCALE AND FIRE DISTURBANCE PATTERNS ON FOREST AGE DISTRIBUTION

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Abstract

Boychuk, D., A.H. Perera, M.T. Ter-Mikaelian, D.L. Martell and C. Li. 1995. **Modeling the effect of scale and fire disturbance patterns on forest age distribution.** Forest Fragmentation and Biodiversity Project Technical Report Series No. 19. Ontario Forest Research Institute, Ministry of Natural Resources.

Van Wagner (1978) demonstrated the important insight as to the relevance of the exponential model to tree age at both the stand and landscape levels. He showed that, under certain conditions, the probability distribution of the age of a stand subject to periodic renewal by fire is exponential. The extension of this model to the landscape-level results, also under certain conditions, in an exponential shape for the forest age distribution. Empirical studies have given partial support to this hypothesis. The results depend upon the size of the landscape in question, the patterns of fire disturbance, and changes in the disturbance regime over time and space. We attempt to provide insight into some of the fundamental factors that determine the forest age distribution. We examine whether the forest age distribution would have an exponential shape, and whether it would be stable or variable over time under different conditions. We used different spatial and temporal disturbance patterns, some of which represent correlation due to fire growth and episodes of high fire disturbance. We describe the theoretical models that we developed for this investigation, and give the results of computational tests based on hypothetical data. Our results show that, under typical boreal disturbance regimes, we should not expect to find forest age distribution stability even at very large spatial scales due to the correlation of disturbances.

Keywords: fire, disturbance, landscape ecology, exponential age distribution, scale, stability, steady state, variability.

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1. Introduction

It is important to understand and manage ecosystems at multiple temporal and spatial scales, in particular, at landscape scales. One of many ideals in landscape management planning in North America is that the landscape should resemble its "natural" condition before European settlement. In order to guide fire and landscape management activities to approach this ideal, it is necessary to characterise this natural condition.

It is well known that fire and other disturbances have played important roles in shaping Ontario's forest ecosystems. In a fire-dominated ecosystem like the boreal forest, it is difficult or impossible to reconstruct the history of the forest landscape, and characterise the natural state (e.g., Baker 1989b). This is primarily due to the cumulative impact of factors like fire protection and harvesting during this century.

Even without such difficulties, researchers use theoretical models to gain insight into the dynamics of complex systems. For a fire-disturbed ecosystem like the boreal forest, Van Wagner (1978) demonstrated the important insight as to the relevance of the exponential model to tree age at both the stand and landscape levels. He showed that under certain conditions, the probability distribution of a stand subject to periodic renewal by fire is exponential. The extension of this model to the landscape-level results, also under certain conditions, in an exponential shape for the forest age distribution. This insight is relevant when managing for naturalness if it helps us to understand the natural state of the landscape, and interpret empirical data.

Van Wagner (1978) developed a simulation model of a landscape composed of many equal-sized, homogeneous, independent cells or stands which were subject to the risk of burning each year. Regarding his model, he wrote that

> " ... there is no fundamental reason why ... individual fires could not burn more than one stand. ... Similarly, there is no reason why the burned area could not vary from year to year as long as the long-term average was constant and the time scale of the fluctuations small compared with the length of the fire cycle. Such departures from the ideal would naturally result in statistical roughness but need not disqualify the negative exponential concept."

Even casual observation confirms that there will be significant deviation from the exponential shape due to large fires (e.g., Johnson and Gutsell 1994), so we might expect the exponential model to fail at small scales, and become more appropriate at larger scales. Baker's (1989b, 1989c) analyses of Heinselman's (1973) Boundary Waters Canoe Area (BWCA) fire history data are among the most comprehensive tests of the exponential hypothesis at varying spatial scales. He found that the "shifting-mosaic steady state" (Bormann and Likens 1979) hypothesis did not hold in the BWCA. He gave several reasons including the spatial heterogeneity of disturbance regimes, and that the fires were very large and infrequent. Further modeling work by Baker (1992a, 1992b, 1993) using a spatial landscape model by Baker et al. (1991) has clearly shown that under historical conditions in the BWCA, forest age distribution did not have an exponential shape, nor did it stay in a steady state due to the large but infrequent fires. In some studies, variability in the data (e.g., Johnson and Larsen 1991) or model output (e.g., Ratz 1995) was deliberately suppressed by averaging because only average system behaviour was sought. In other studies, the variability or non-equilibrium behaviour of the system was explicitly addressed (e.g., Shugart and West 1981; Boychuk 1993, ch. 4; Swetnam 1993).

Our objectives were to analyse and develop further insight into some of the fundamental factors that determine forest age distribution in a disturbance-driven ecosystem. For this, we developed theoretical models of a fire-disturbed landscape and used them to examine forest age distribution under different conditions. In particular, we focused on the effects of spatial scale and various spatial and temporal disturbance patterns. To isolate the effect of these factors from that of landscape heterogeneity, we used a simple homogeneous landscape.

Specifically, we examined whether the forest age distribution is:

- exponential, and
- stable or variable over time.

We especially examined the how the forest age distribution varied with the:

- landscape size, and
- temporal and spatial disturbance patterns.

We examined various disturbance patterns representing:

- the one assumed in the "classical" exponential age distribution hypothesis,
- ones representing spatial dependence due to fire growth, and
- ones representing spatial and temporal dependence due to variable fire disturbances over time, i.e., infrequent high fire disturbance years.

Our further objectives were to:

- develop graphical software to illustrate these theoretical principles to landscape managers, fire managers, and researchers, and
- illustrate the theoretical impact of the introduction of fire suppression on the forest age distribution (described in Boychuk and Perera 1995).

We believe that our analysis should influence the representation of spatial and temporal dependence of disturbances in comprehensive landscape models. Note that we are not suggesting that our models in FLAP-X are the final answer for fire and landscape models and analysis. We believe that they have important features that should be incorporated, and we are undertaking research for further development.

Following a literature review in Section 2 and definitions in Section 3.1, our exposition starts with a description of the classic exponential models for both the stand level (Section 3.2) and landscape level (Section 3.3). In Section 3.4 we give a quantitative analysis of the effect of the number of cells in the classical landscape-level model. In Section 3.5, we develop a continuous analogue of the landscape-level model in which we relax the assumption of no fire growth, and prove that the exponential result still holds over time for a point. In Section 3.6 we emphasise the distinction between the stand-level and landscape-level cases despite their apparently similar algebra. In Section 4 we present our cell-based theoretical models in which we relax the assumptions of no fire growth and constant probability of burning. Thus, we progresses from the classical exponential models, through intermediate models and analyses, then to our final models in FLAP-X. In Section 5 we describe our computational results. Section 6 presents a case study example, and Section 7 is a discussion.

2. Literature Review

2.1. Introduction

We review literature in which authors (1) developed the exponential and related models at the stand and landscape levels, (2) reported substantial empirical tests of the exponential or related models, (3) presented comprehensive models of fire disturbance and forest age distribution, and (4) dealt with other factors related to this research. We do not include literature regarding component processes such as ecosystem development and succession, or fire and other disturbances.

2.2. Development of Stand- and Landscape-Level Exponential and Related Models

Johnson and Gutsell (1994) reviewed and summarised procedures for analysing and interpreting fire history and forest age distribution. Van Wagner (1978, 1983, 1986) provided valuable insight into the effects of fire and harvesting on the forest age distribution. The principal result of interest is that in theory, the age distribution approaches an exponential shape under certain conditions. Johnson and Van Wagner (1985) explicitly linked the fire interval distribution at a single-point to the age distribution across the whole landscape. Harrington and Donnelly (1978) also developed a simple model of the average age, and forest age distribution if small stands in the landscape were subjected to the expected fire disturbance rates, and burned independently of each other. They defined the annual fire probability as the mean annual burned area divided by the landscape area, and argued as follows. If the very low probability that a small stand burns in a given year is assumed to be constant over time, then the sequence of fires in the stand over time is approximately a Poisson process. The fire interarrival times are therefore approximately exponentially distributed. The probability distribution of the age of the stand (implicitly, in the long term, without the influence of the initial age) is therefore exponentially distributed. Since the landscape is assumed to be composed of many small stands that burn independently of each other, the forest age distribution (in the long term) would therefore also have an exponential shape. Van Wagner (1978) arrived at a similar conclusion using both an analytical model and a stochastic simulation model. He also reviewed case studies of actual landscapes to test the relationship, and introduced harvesting into the simulation model. Wilson (1983) analysed the problem more formally, modeling the growth and burning of a single stand as a Markov chain. He extended his results to the landscape level which was composed of many independent stands. He argued that, after a sufficient length of time, the forest age distribution would attain and maintain an exponential shape.

Johnson and Van Wagner (1985) generalised the exponential result to the Weibull distribution for the case where the probability of burning varies with the age of the stand. The exponential model is a special case where burning is independent of age. Clark (1989) developed this case as a renewal process and warned of an important problem in fire history analysis when the fire regime is non-stationary.

Boychuk (1993, ch. 3) discussed the relationship between the stand-level and landscape-level scope of these studies. He emphasised that the linkage between the single-point fire interval and forest age distribution at the landscape level depended on the assumption that the whole landscape was composed of many cells that burned independently of each other. The dependence of burning adjacent stands is, however, very apparent from observed fires and maps of the boreal forest. The effect of spatial scale has been recognised (e.g., Simard 1976, Van Wagner 1983, Wiens 1989) and studied empirically (e.g., Baker 1989b) and with models (e.g., Baker 1989c, 1993). There is a considerable amount of work on various measures of landscape pattern as a function of spatial scale (see e.g., Turner 1989, Turner et al. 1989), but our concern here is with the simpler measure of forest age distribution. Boychuk (1993, ch. 4) developed analytical models relating the aggregate proportion of a landscape burned to the sizes of the fires and the size of the landscape. He showed that with no spatial or temporal correlation of fire disturbance, the annual proportion burned converges to the expected proportion burned for a sufficiently large landscape. The implication is that this would lead to an exponentially-shaped age distribution for a sufficiently large landscape. He showed, however, that with significant spatial correlation and annual variability of fire disturbances due to weather, the annual proportion burned does not converge to the expected proportion burned, even for an arbitrarily large landscape.

2.3. Empirical Studies of Exponential or Related Models

Many studies contained empirical tests of their models or hypotheses. Here we list several studies that were substantial empirical tests of the exponential or related Weibull models. Baker (1989b) undertook a comprehensive analysis of the fire history and consequent forest age distribution of the BWCA based on data from Heinselman (1973). He described the theoretical and practical problems and complexities of defining and measuring fire histories. Suffling et al. (1982) attempted to reconstruct past age distributions and past fire disturbance rates by postulating an initial age distribution and a fire disturbance distribution, and simulating deterministic fire disturbances and landscape ageing over time. They adjusted the initial age distribution and fire disturbance distribution until the simulated final age distribution matched the current actual age distribution. As they acknowledged, they could not be certain they identified the correct initial age distribution and fire disturbance distribution. Suffling (1983, 1991, 1992) also presented tests of the exponential model fitted to age distribution data. Reed (1994) estimated the age-dependent natural fire hazard from the current age distribution of undisturbed old growth forest. Bergeron (1991) examined the fire regimes and forest age distribution on island and lakeshore sites in northwestern Québec. He found climate-driven changes in the fire frequency. Hemstrom and Franklin (1982) examined the disturbance history of Mount Rainier National Park and found the exponential distribution did not fit well due to the infrequency and large size of fires. Yarie (1981) tested exponential and Weibull distributions to estimate survivorship for forests in parts of Alaska. Johnson and Larsen (1991) fit the exponential model to age distribution data from part of the Canadian Rockies.

2.4. Landscape Models of Fire Disturbance and Forest Age Distribution

Baker (1989a) gave a comprehensive review and classification of models of landscape change, and we make no attempt to duplicate his efforts. Here, we review selected landscape models with fire disturbance that were used to study the effect of fire on forest age distribution, or that had other features related to this research.

Antonovski et al. (1992) developed a stochastic spatial simulation model of a hypothetical landscape. The model landscape was a grid of cells that represented even-aged stands. Each year, a lightning fire could ignite in each cell with a given probability. Each cell with an ignition could burn with a probability that depended on the stand age. Each cell adjacent to a burning cell could ignite and burn with a given probability that also depended upon age. The ignition and burning probabilities were adjusted until empirical distributions of area burned and current age were reproduced by the simulation model.

Baker (1989c) developed a Markov chain model to represent the fire and landscape dynamics of the BWCA. He found that given the infrequent large fires, the age distribution of the BWCA would not attain an exponential shape, nor be stable over time. Baker (1992a, 1992b, 1993) used a fire disturbance and landscape simulation model by Baker et al. (1991) to explore the impacts of a changing fire regime on landscape characteristics in the BWCA. Baker (1993) refined the analysis to multiple scales.

Green (1989) developed a simulation model of the effect of fire and other factors on landscape patterns. His model is similar to, but more comprehensive than FLAP-X although he did not analyse forest age distribution directly. Ratz (1995) developed a simulation model with two versions of stochastic fire spread: age-independent and age-dependent. He reported significant spatial and temporal variability of forest age distribution, but used combined time- and replication-average (see Section 4.2) measures of forest age distribution for analysis.

3. Theoretical Basis of the Exponential Model

3.1. Some Definitions

Johnson and Gutsell (1994) summarised terminology and the relationships between various measures of disturbance and forest age distribution for both the exponential and Weibull cases. Here, we give definitions and relationships for the exponential case.

While our research is directed at the landscape level, we also investigate the stand level due to its role in the derivation of the exponential age distribution model. **Patch, stand**, **cell**, **atom**, and **point** have been used to describe small homogeneous patches or dimensionless points for modeling purposes. The term stand is often used to describe a possibly large homogeneous patch that originated from a fire or other disturbance, but we do not use this interpretation here. **Whole forest**, **forest-level**, and **landscape** have been used to describe relatively large areas composed of many cells.

The relationships between the landscape measures of area, fraction, proportion, and percentage burned are as follows. The **proportion burned** is the **area burned** divided by the area of the landscape. **Fraction burned** is the same as proportion burned. **Percentage burned** is simply the proportion multiplied by 100%. The term **probability burned** has also been used to mean the proportion burned, but this usage is misleading. We prefer its use at the stand level to denote the probability that the entire stand burns. At the landscape level, there is a probability distribution of the area or proportion of the landscape that burns during a specified time interval such as a year.

Several measures and terms have been used to quantify or characterise fire disturbances, sometimes with alternative meanings. **Fire cycle**, **fire interval**, **fire rotation**, and **recurrence time** refer to the (expected) time between fires. These are essentially stand-level measures, but averages can be applied at the landscape level. Annual **percentage burned**, **fraction burned**, **fire frequency**, **fire hazard rate**, **fire loss rate** (and sometimes **fire probability**) refer to the proportion of the landscape burned in a year. Generally, the measures of the expected time between fires are reciprocals of the probability of burning for single stands, and the expected proportion burned for whole landscapes.

Inferences are often made between stand-level and landscape-level concepts and measures. Suppose a stand has a probability of burning each year of 0.01. The expected fire recurrence time at that stand would be $1/0.01 = 100$ years. What can we say about a homogeneous landscape composed of these types of stands (which burn independently)? We can say that there is some probability distribution of the proportion burned, and the expected value of this distribution is 0.01.

Finally, we note the potential confusion due to the two uses of the term "distribution." **Forest age distribution** is a vector of numbers giving the area or proportion of the landscape that is in each age group or age class at some specified point in time. When the forest age distribution is a random variable, it has a **probability distribution**. Table 1 shows this distinction for the scalar stand age and the vector forest age distribution. At the risk of adding to the confusion, we note that forest age distribution is similar to probability distribution of stand age. Forest age distribution can be considered to be an exhaustive sample of individual stand ages *over space*. The probability distribution of stand age can be considered to be a series of samples of stand age *over time*. Caution must be used in both cases due to statistical dependence among the observations, as we demonstrate below.

Table 1: The two uses of the term "distribution" for age and probability.

	Single Observation	Probability Distribution
Stand Level	Stand Age	Probability Distribution of Stand Age
Landscape Level	Forest Age Distribution	Probability Distribution of Forest Age Distribution

3.2. Exponential Model for Stand-Level Age Probability Distribution

In this and the following section, we present the stand-level and landscape-level exponential models. We then emphasise their difference, and progressively relax some of their assumptions in a series of new models. As we will see, although we significantly extend the classical exponential models in FLAP-X, the fundamental insights from the classical exponential models endure.

The basis of the exponential age distribution model is the single stand. Our outline of its derivation is based on Ross (1972). An exponentially distributed random variable, X, with parameter, μ, has the probability density function given by

$$f(x) = \mu e^{-\mu x}; \quad x \geq 0, \mu > 0$$

Then $E[X] = 1/\mu$ and $V[X] = 1/\mu^2$. Exponentially distributed random variables are *memoryless*. If we interpret X as the time between events, then the probability distribution of the time to the next event is independent of the time since the last event. It can also be shown that if the time since the last event is x, then the probability that the next event will occur during the following infinitesimal time increment dx is constant. In the forest fire case, this implies that the probability of burning is constant and independent of stand age.

As clarified by Johnson and Van Wagner (1985), the time between fires, T, is a different random variable than the time since the last fire, S. Suppose we are concerned with the probability distribution of the age of a cell that has an expected time between fires of $E[T] = 100$ years (i.e., $\mu = 0.01$). If we observe the age of the cell at a random time, we are sampling from the distribution of S, the time since the last fire. Due to the memoryless property of the exponential distribution, however, the distribution of S is also exponential with parameter μ.

As Johnson and Van Wagner (1985) described further, the distribution of the time from the randomly observed time to the next fire, R, is also exponentially distributed with parameter μ. This leads to the so-called "bus paradox" (Larson and Odoni 1981, p. 58-61). If the time since the last fire, S, and the time to the next fire, R, are both exponentially distributed with parameter μ, then why is the time between fires, T, not distributed as the *sum* of S and R? The resolution of the paradox lies in the way the observation is taken. It is more likely that the random observation samples a longer interarrival time.

For our present purpose, it is sufficient to accept the following aspects of the exponential model:

- If the probability of burning is constant and independent of stand age, then
    - the fire interarrival times are exponentially distributed, and
    - the age of the stand is exponentially distributed.

Note that at the stand level, the exponential model refers to the probability distribution of the age of the stand sampled at some random time.

Next, we outline the geometric distribution, which is the discrete analogue of the continuous exponential distribution. We use it for convenience, particularly because we often model disturbance events as occurring at some unspecified time within discrete years. A geometrically distributed random variable, X, with parameter, p, has the probability mass function given by (e.g., Ross 1972)

$$Pr\{X = x\} = (1 - p)^{x-1}p \; ; \quad x = 1, 2, 3, \ldots$$

Then $E[X] = 1/p$ and $V[X] = (1 - p)/p^2$. A physical interpretation is as follows. Suppose that during each year there is a probability, p, that a fire will renew a stand. Assume that the probability of renewal is independent of previous fires, i.e., constant over time. We define X as the number of years from one fire to the next, e.g., if a stand is renewed in 1995, and again in 1996, then $X = 1$. Then the equation follows because the probability of having no fires for $x - 1$ years is $(1 - p)^{x-1}$, and the probability of having a fire in the last year is p.

(There is another definition of the geometric distribution which can lead to confusion. Y is defined as the number of years *before* the next fire, e.g., if a stand is renewed in 1995, and again in 1996, then $Y = 0$, i.e., $Y = X - 1$. Then

$$Pr\{Y = y\} = (1 - p)^y p \; ; \quad y = 0, 1, 2, \ldots$$

and $E[Y] = (1-p)/p$ and $V[Y] = (1-p)/p^2$. Both versions are correct, the only difference being the way the random variables are defined.)

The geometric distribution is a good approximation to the exponential distribution for small p. Analogous to the exponential model, we are interested in the following aspects of the geometric model:

- If the probability of burning each year is constant and independent of stand age, then
    - the fire interarrival times are geometrically distributed, and
    - the age of the stand – sampled at some random point in time – is geometrically distributed.

Figure 1 shows an exponential probability density function and geometric probability mass function each with parameter 0.01.

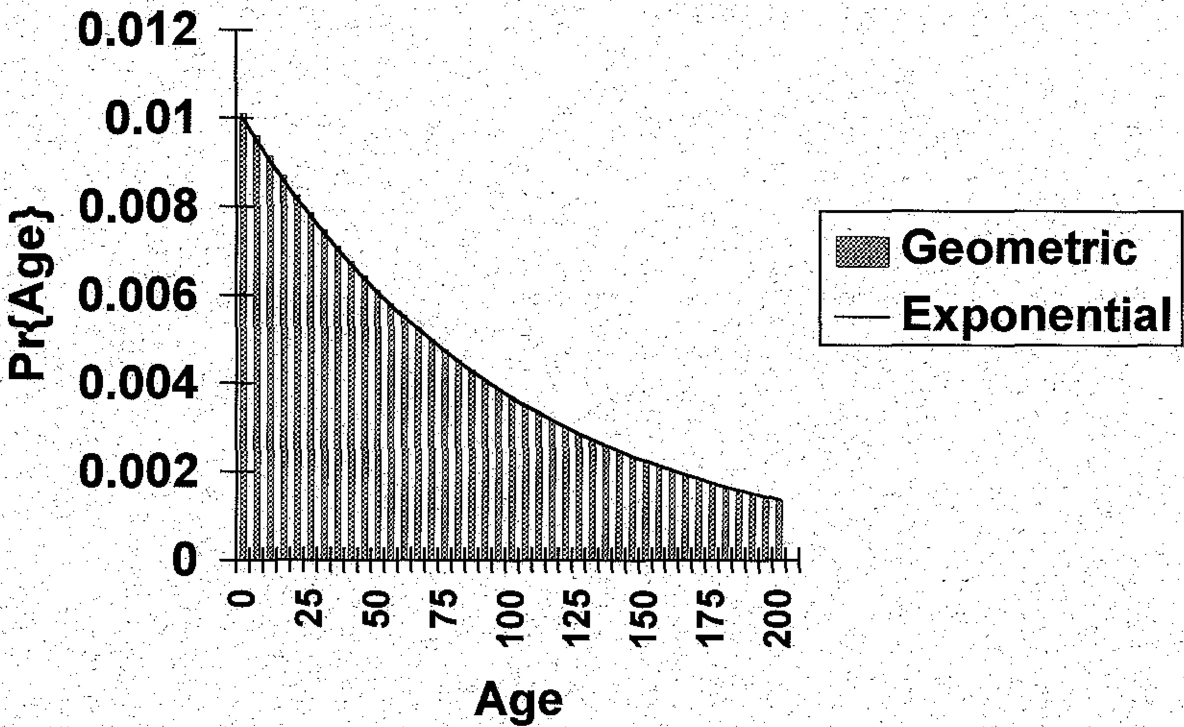

Figure 1: Comparison of geometric ($p = 0.01$) and exponential ($\mu = 0.01$) probability distributions. (Only every fifth bar of the geometric distribution is illustrated.)

3.3. Exponential Model for Landscape-Level Age Distribution: Cell-Based Models

Here, we describe the extension of the exponential model from the single stand to the landscape level. The strategy used or implied by Harrington and Donnelly (1978), Van Wagner (1978), Reed (1980), and Wilson (1983) was to assume that the landscape consisted of a collection of equal, independent single stands or cells of unspecified size, with ages discretized into one year classes. Stands were assumed to either burn entirely, or not at all in a particular year independently of each other, with the same probability of burning for all ages.

As Harrington and Donnelly (1978) and Reed (1980) argued, in the long term – when the influence of the initial conditions has vanished – the probability distribution of the age of a single stand is geometric. In the long term, the expected number of stands in the entire landscape in each age could therefore be calculated: the expected age distribution would have the shape of the geometric distribution. This expectation would be based on independent observations over time.

The *actual* age distribution at any time, however, would deviate from this: in the long term the probability distribution of the forest age distribution is approximately multinomial (Reed 1980). If the individual stands had a finite maximum age, the long-term probability distribution of the age of a single stand would be approximately geometric, and the long-term probability distribution of the forest age distribution would be exactly multinomially distributed (Reed 1980). Since each cell has an equal probability of burning each year, the distribution of the number of cells burned annually would be binomially distributed (Reed 1980, Wilson 1983). This is a special case of the multinomial distribution where there are two kinds of cells: those burned and not burned this year.

The significance of the multinomial distribution for the probability distribution of the forest age distribution can be understood intuitively as follows. Consider a landscape of n independent cells, each with an annual probability of burning of 0.01. The expected age of each cell is 100 years. In an extreme example, if the landscape had only $n = 10$ cells, a sample age distribution would likely have the 10 cells scattered widely over the wide range of possible ages. Even if the landscape had $n = 100$ cells, a sample age distribution would still have many "gaps" in the age distribution.

Most ages would have zero to four cells. As n increased, the gaps would tend to fill in, and the shape of the age distribution would approach an exponential shape. Such examples can be illustrated with FLAP-X.

Recall that at the stand level, the exponential model refers to the probability distribution of the stand age when observed at some random time. This might be thought of as a "temporal" exponential model. In contrast, at the landscape level, the exponential model refers to the shape of the forest age distribution. This might be thought of as a "spatial" exponential model. But because the forest age distribution at one point in time is not a probability distribution, we say that the forest age distribution has (or does not have) an exponential or geometric shape.

3.4. A Note on the Effect of the Number of Cells in Cell-Based Models

Simard (1976), Van Wagner (1986), Reed and Errico (1986) and Johnson and Gutsell (1994) argued that as the landscape size increases, the variance of the proportion burned should decrease. This would be due to having more fires in the larger landscape, resulting in more averaging of random variations in fire size – a consequence of the law of large numbers. For very large landscapes, Simard (1976) argued that the distribution would approach a single point discrete distribution at the expected proportion burned.

Here, we examine these conjectures in the context of a landscape composed of independent cells. This analysis is an extension of Boychuk (1993, ch. 4). Table 2 gives the expectation, variance, and coefficient of variation (CV) of the number of cells, area, and proportion burned in a landscape composed of independent cells. In part (a), the landscape size is fixed at F ha, and the cell size varies. In part (b), the cell size is fixed at C ha, and the landscape size varies. If the number of cells is n, and the probability that each burns in one year is p, then the expected number of cells burned, $E[N]$, is np, and the variance, $V[N]$, is $np(1 - p)$ (e.g., Ross 1972, p. 48). The CV is $[np(1 - p)]^{1/2}/(np)$. For the area burned, A, we use the transformation $A = N$ (Cell Size). For the proportion burned, P, we use the transformation $P = A/$(Landscape Size).

Of all the ways of characterising the amount burned, we believe that the expected proportion burned, $E[P]$, and the CV of proportion burned, $CV[P]$, are the most meaningful. CV is also called *relative variance* and this is effectively demonstrated because the CV is the same whether the statistic is the number of cells, the area, or the proportion burned; i.e., $CV[N] = CV[A] = CV[P]$. Also, the CV is the same for a given number of cells regardless of the size of the cell or landscape.

The CV can be interpreted as follows. The exponential distribution has a CV of one. Fire disturbance distributions with CV greater than one have a high probability of little fire disturbance, and low probability of very high fire disturbance. Distributions with a CV much less than one are more symmetrical, like a normal curve. Figure 2 shows a family of gamma distributions with the expected proportion burned and the all but the highest and lowest CV's from Table 2. For landscapes with few cells, there is a high probability of having a low proportion burned, and vice versa. As the number of cells increases, the probability distribution function of the proportion burned converges to the mean. (We arbitrarily chose the gamma distribution for convenience as it can represent continuous distributions with the required CVs. We could have used the binomial distribution directly in a series of analogous discrete graphs.) Figure 2 is a quantified version of Simard's (1976) conjecture.

The importance of the result in Table 2 is that – in models using cells – the number of cells greatly affects the model's results. The number of cells in the landscape determines the annual variability of the proportion burned, and – as demonstrated with FLAP-X below – this in turn affects the shape and stability of the forest age distribution over time. This could either be an unintended problem, or can be used to advantage to better represent the fire disturbance distribution of the landscape being studied. The number of cells used in landscape models should be chosen carefully to avoid unintended effects. We emphasise that the results of this section only pertain to models where the landscape is composed of independent cells. We relax this independence assumption in following sections.

Table 2: Annual fire disturbance statistics for a hypothetical landscape composed of independent cells. In both parts, each cell has an annual probability of burning of 0.01. In part (a), the landscape is of size F hectares, and the cell size varies. In part (b), each cell is of size C hectares, and the landscape size varies. In both cases, it is easiest to examine the expected proportion burned, $E[P]$, and the coefficient of variation, $CV[\ ]$. $CV[\ ]$ varies with the number of cells in the landscape, not the size of the cell or landscape.

(a) Landscape Size F (ha) (Cell Size Varies)

No. of Cells	Cell Size (ha)	No. of Cells Burned, N: $E[N]$	$V[N]$	Area Burned, A: $E[A]$	$V[A]$	Proportion Burned, P: $E[P]$	$V[P]$	$CV[N]$ $CV[A]$ $CV[P]$
1	F	10^{-2}	$9.9\text{x}10^{-3}$	$10^{-2}\ F$	$9.9\text{x}10^{-3}\ F^2$	10^{-2}	$9.9\text{x}10^{-3}$	9.95
10	$10^{-1}\ F$	10^{-1}	$9.9\text{x}10^{-2}$	$10^{-2}\ F$	$9.9\text{x}10^{-4}\ F^2$	10^{-2}	$9.9\text{x}10^{-4}$	3.15
10^2	$10^{-2}\ F$	1	$9.9\text{x}10^{-1}$	$10^{-2}\ F$	$9.9\text{x}10^{-5}\ F^2$	10^{-2}	$9.9\text{x}10^{-5}$	9.95×10^{-1}
10^3	$10^{-3}\ F$	10	9.9	$10^{-2}\ F$	$9.9\text{x}10^{-6}\ F^2$	10^{-2}	$9.9\text{x}10^{-6}$	3.15×10^{-1}
10^4	$10^{-4}\ F$	10^2	9.9x10	$10^{-2}\ F$	$9.9\text{x}10^{-7}\ F^2$	10^{-2}	$9.9\text{x}10^{-7}$	9.95×10^{-2}
10^5	$10^{-5}\ F$	10^3	$9.9\text{x}10^{2}$	$10^{-2}\ F$	$9.9\text{x}10^{-8}\ F^2$	10^{-2}	$9.9\text{x}10^{-8}$	3.15×10^{-2}
10^6	$10^{-6}\ F$	10^4	$9.9\text{x}10^{3}$	$10^{-2}\ F$	$9.9\text{x}10^{-9}\ F^2$	10^{-2}	$9.9\text{x}10^{-9}$	9.95×10^{-3}
10^7	$10^{-7}\ F$	10^5	$9.9\text{x}10^{4}$	$10^{-2}\ F$	$9.9\text{x}10^{-10}F^2$	10^{-2}	$9.9\text{x}10^{-10}$	3.15×10^{-3}

(b) Cell Size C (ha) (Landscape Size Varies)

No. of Cells	Land-scape Size (ha)	No. of Cells Burned, N: $E[N]$	$V[N]$	Area Burned, A: $E[A]$	$V[A]$	Proportion Burned, P: $E[P]$	$V[P]$	$CV[N]$ $CV[A]$ $CV[P]$
1	C	10^{-2}	$9.9\text{x}10^{-3}$	$10^{-2}\ C$	$9.9\text{x}10^{-3}\ C^2$	10^{-2}	$9.9\text{x}10^{-3}$	9.95
10	$10\ C$	10^{-1}	$9.9\text{x}10^{-2}$	$10^{-1}\ C$	$9.9\text{x}10^{-2}\ C^2$	10^{-2}	$9.9\text{x}10^{-4}$	3.15
10^2	$10^2\ C$	1	$9.9\text{x}10^{-1}$	C	$9.9\text{x}10^{-1}\ C^2$	10^{-2}	$9.9\text{x}10^{-5}$	9.95×10^{-1}
10^3	$10^3\ C$	10	9.9	$10\ C$	$9.9\ C^2$	10^{-2}	$9.9\text{x}10^{-6}$	3.15×10^{-1}
10^4	$10^4\ C$	10^2	9.9x10	$10^2\ C$	$9.9\text{x}10\ C^2$	10^{-2}	$9.9\text{x}10^{-7}$	9.95×10^{-2}
10^5	$10^5\ C$	10^3	$9.9\text{x}10^{2}$	$10^3\ C$	$9.9\text{x}10^{2}\ C^2$	10^{-2}	$9.9\text{x}10^{-8}$	3.15×10^{-2}
10^6	$10^6\ C$	10^4	$9.9\text{x}10^{3}$	$10^4\ C$	$9.9\text{x}10^{3}\ C^2$	10^{-2}	$9.9\text{x}10^{-9}$	9.95×10^{-3}
10^7	$10^7\ C$	10^5	$9.9\text{x}10^{4}$	$10^5\ C$	$9.9\text{x}10^{4}\ C^2$	10^{-2}	$9.9\text{x}10^{-10}$	3.15×10^{-3}

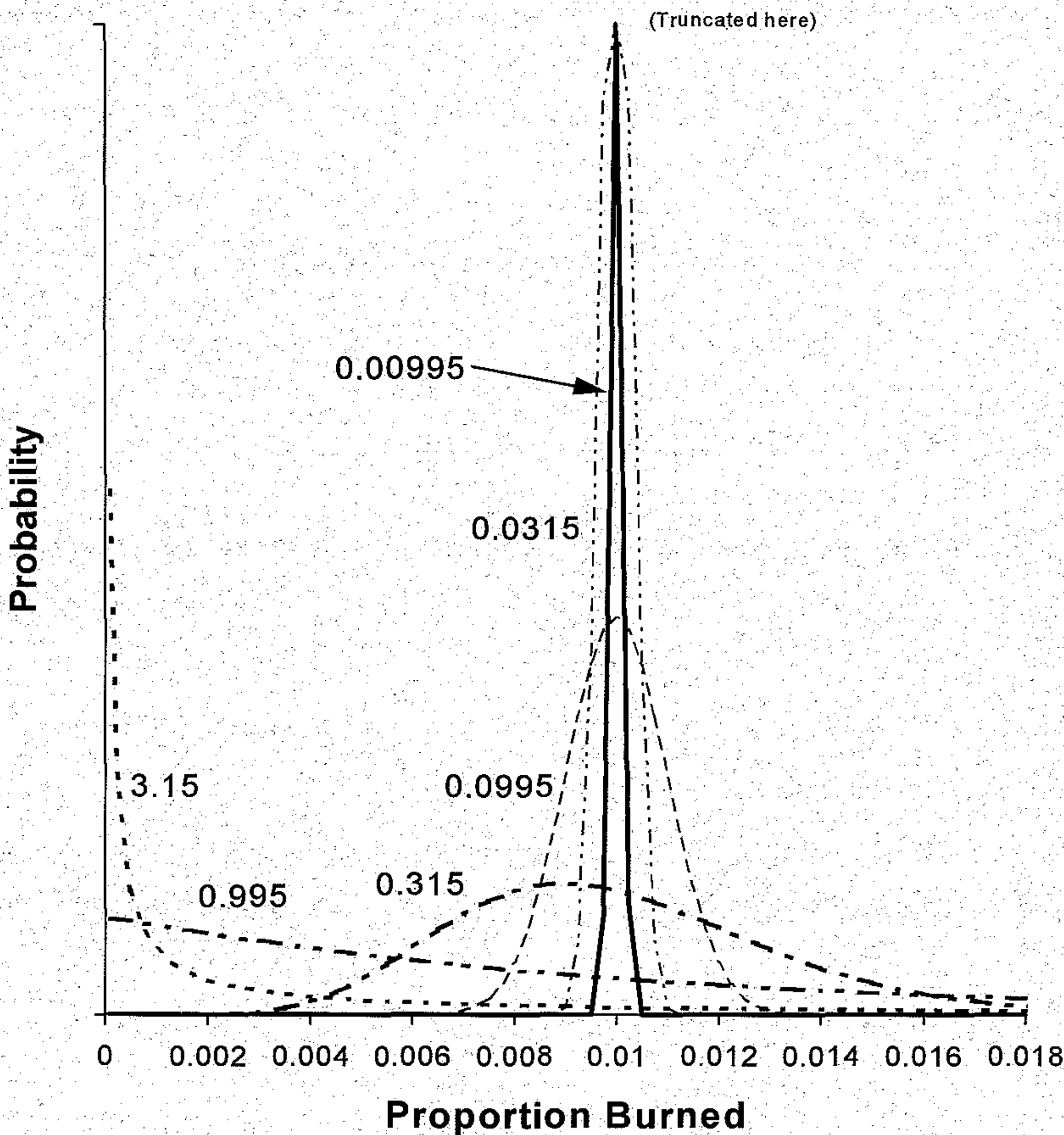

Figure 2: Illustration of a family gamma probability density functions, each with an expectation of 0.01, and a different coefficient of variation from Table 2.

3.5. Proof of Exponential Stand Age without Independent Cells: FIRECO

The occurrence of large fires leads us to explore landscape-level models in which fires do not burn independently in individual cells. Due to fire growth, the burning of adjacent cells is not independent. In this section we show that, under certain conditions, fire growth does not refute the (temporal) exponential age model for a single point. We describe a simulation model called FIRECO, outline the computational result, and give an analytical proof that the age of any *point* in a flammable landscape is exponentially distributed (over time) – under the given assumptions.

FIRECO has a square landscape subject to random fires. A series of circular fires is generated by a stationary spatial Poisson process, each randomly located in the landscape using a uniform probability distribution, and each with a size randomly generated from an exponential distribution. The simulation model records the times that an arbitrary point in the landscape burns. Statistical analysis of the computational results showed that the time between fires is exponentially distributed. This result is not intuitive, because the burning of the point is accomplished by a complex spatial process, unlike in the cell-based models that have independent cells.

Intuitive insight into the result can be obtained from an analytical proof. The system can be modelled as a filtered Poisson process as follows. Our assumptions are:

1. The landscape is very large relative to the size of fires so that we can ignore edge effects, and assume that the entire burned area of every fire that starts in the landscape falls entirely within the landscape.

2. Fires occur according to a Poisson fire arrival process at a constant rate of λ fires per year.

3. The probability distribution function of fire size is $f_A(a)$ with a mean of μ hectares. The distribution and its mean are independent of the vegetation at and near the point where the fire occurs. (While discrete years are not a feature in this proof, we note that fires can burn over recently burned area even within one year. Thus, the sum of the area burned by all the fires

that occur during a year may be greater than the total area "burned" during the year – or more correctly, reset to age zero during that year.)

When a fire occurs in the landscape, we observe whether or not it burns an arbitrary point in the landscape. Let the event that the fire burns the arbitrary point be B, and the event that it does not burn the arbitrary point be N. The sample space for the observation is therefore $\{B, N\}$.

Further, let

$\Pr\{B\}$ be the probability that the fire burns the arbitrary point
A be a random variable representing the burned area of the fire, and
s be the landscape size.

To find $\Pr\{B\}$, we first condition upon the fire size, A, i.e., $\Pr\{B \mid A\}$. Because fires are randomly located in the landscape, the probability that the fire burns the arbitrary point is $\Pr\{B \mid A = a\} = a / s$.

Removing the conditioning upon A, we have

$$\begin{aligned}
\Pr\{B\} &= \int_{a=-\infty}^{\infty} \Pr\{B \mid A = a\} f_A(a)\, da \\
&= \int_{a=-\infty}^{\infty} \frac{a}{s} f_A(a)\, da \\
&= \frac{1}{s} \int_{a=-\infty}^{\infty} a\, f_A(a)\, da \\
&= \frac{E[A]}{s} \\
\therefore \quad \Pr\{B\} &= \frac{\mu}{s}
\end{aligned}$$

Because fires arrive in a Poisson process, and the probability that each fire burns the arbitrary point is constant, the burning of that point is a Poisson process. In particular, the burning of that point is a filtered Poisson process with random selection, where the probability that a fire burns the point is

μ/s. The Poisson fire arrival rate for the arbitrary point is therefore $\lambda\,\mu/s$. Finally, the probability distribution of the fire interarrival time is therefore exponential with a mean of $s/(\lambda\,\mu)$.

The significance of this result is that fire interarrival times, and hence, the probability distribution of the vegetation age of an arbitrary point in the landscape is therefore exponential (over time), even with a complex spatial fire growth process. This exponential result has only been previously developed for points or cell-based models with no fire growth. We relaxed one assumption of the classical landscape-level exponential model: the assumption of burning cells independently, and showed that the (temporal) exponential model is still applicable under the other given assumptions.

We emphasise that this result is for a single point in the landscape. This model does not address the forest age distribution: it examines a point in space over time rather than the landscape at a point in time. The result here should not be used to infer that the forest age distribution has an exponential shape. We extend the scope of our analysis to the landscape level in our theoretical models implemented in FLAP-X, and show that the forest age distribution indeed does not, in general, have the exponential shape at a point in time.

3.6. A Note on the Distinction between Stand-Level and Landscape-Level Algebra

In this section we compare and contrast the algebra that describes stand-level and landscape-level probabilities of disturbance. In a stand-level model, if the probability that the stand is burned in any one year is p, independently of whether it burns in any other year, then the probability that the stand is burned at least once in an L year period is

$$f(L) = 1 - (1 - p)^L$$

derived as follows. The quantity $1 - p$ is the probability that the stand does not burn in the first year, and $(1 - p)(1 - p)$ is the probability that it does not burn in either the first or second years, and so forth for L years. Note that $f(2) = 2p - p^2$. This is the sum of the probabilities that the stand burns in the first or second years; minus the probability that it burns in both years, to avoid double counting. Similarly, $f(3) = 3p - 3p^2 + p^3$. This is the sum of the probabilities that the stand burns in the first, second, or third years; minus the probabilities that it burns in (a) the first and second years, (b) the second and third years, and (c) the first and third years; plus the probability that it burns in all three years (see, e.g., Ross 1972, p. 6). Regardless of L, the entire stand is either burned or spared in any given year.

In the landscape-level case, only part of the landscape burns in any given year. The algebra shown for this case is based on Reed and Errico (1986) and Gassmann (1989); see Boychuk (1993) for further discussion. Suppose in a landscape, the *expected* proportion burned in a year is P. Then the expected proportion burned in L years is

$$g(L) = 1 - (1 - P)^L$$

derived as follows. The quantity $1 - P$ is the expected proportion not burned in the first year, and $(1 - P)(1 - P)$ is the expected proportion not burned in either the first or second years, and so forth for L years. Note that $g(2) = 2P - P^2$. This is the sum of the expected proportion burned in the first and second years; minus the expected proportion burned in both years due to overlap.

Similarly, $g(3) = 3P - 3P^2 + P^3$. This is the sum of the expected proportions burned in the first, second, and third years; minus the expected proportion burned, due to overlap, in both (a) the first and second years, (b) the second and third years, and (c) the first and third years; plus the expected proportion burned in all three years.

While the two cases are superficially similar in terms of the algebra, they are distinctly different models. They are based in different spatial settings, and their parameters, p and P, have very different meanings, i.e., probability that a point burns in a year vs. the proportion of a landscape that burns in a year. Care must be taken to avoid the inappropriate application of stand-level results at the landscape level, and vice versa.

4. Theoretical Models of Disturbance and Age Distribution

4.1. Introduction and Description of Disturbance Patterns

The motivation for our theoretical models is the following characteristics of fire disturbance patterns. First, individual fire sizes are highly variable, and the largest fires can be significant in relation to the size of landscape areas of concern. Second, there is a very large year-to-year variability of the proportion burned. It can be observed that stands in the landscape do not burn independently of each other for the following reasons:

- the burning of adjacent stands is statistically dependant due to significant weather-driven growth of individual fires, and
- aggregate fire disturbances vary significantly from year-to-year, with considerable correlation of high and low disturbances over large areas, again due to weather.

FLAP-X is based on the cell models by Van Wagner (1978, 1983, 1986), Wilson (1983), and Antonovski et al. (1992). We can specify wide ranges of various parameters, namely the:

- number of cells in the landscape
- expected fire size
- variability of fire size
- expected annual proportion of the landscape burned, and
- variability of expected annual proportion of the landscape burned (or fire ignitions) over time.

We have particularly tried to evaluate the effect of two kinds of dependencies of burning among cells or disturbance patterns: burning adjacent cells due to fire growth, and the correlation of high and low disturbances over large areas due to weather. We organised the many possible alternatives into four spatial and temporal disturbance patterns which are listed in Table 3 and illustrated in Figure 3.

Table 3: The four spatial and temporal disturbance patterns used in FLAP-X.

		Temporal Variability	
		Years Same	Years Different
Burning Adjacent Cells	Fires All One Cell	Disturbance Pattern 1: YS-F1	Disturbance Pattern 3: YD-F1
	Fires Grow	Disturbance Pattern 2: YS-FG	Disturbance Pattern 4: YD-FG

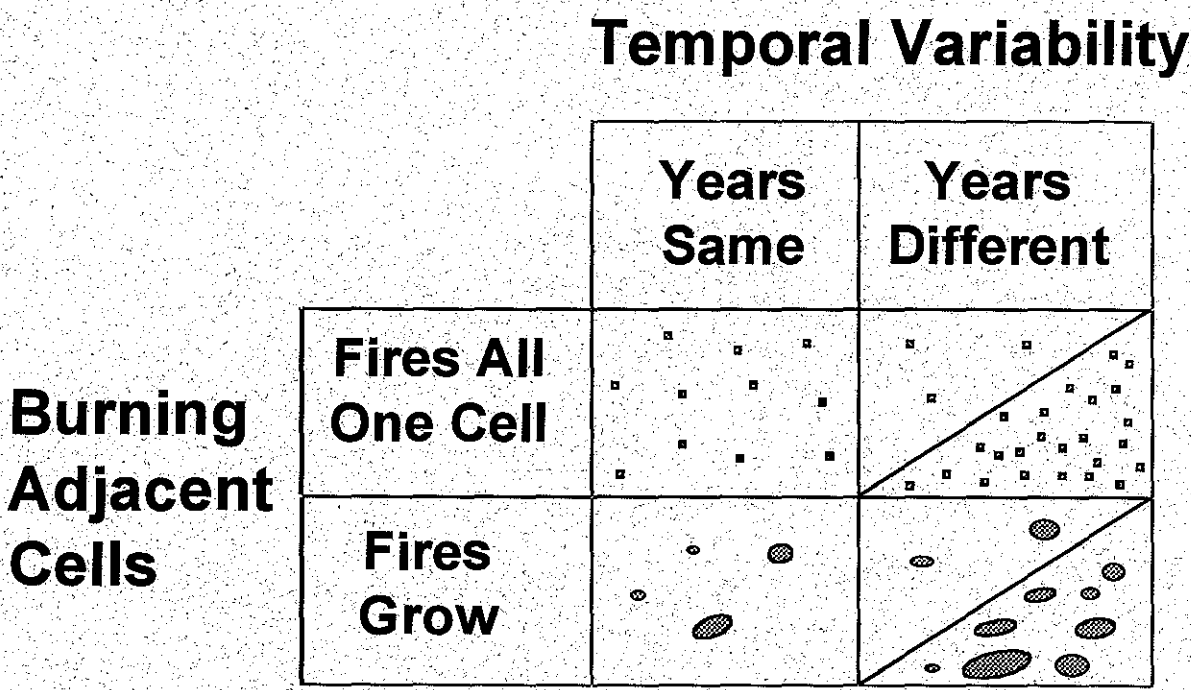

Figure 3: Illustration of fires for the four spatial and temporal disturbance patterns used in FLAP-X.

Disturbance Pattern 1: Years Same – Fires One Cell: Fires burn only one cell each, and each year has the same expected proportion burned.

Disturbance Pattern 2: Years Same – Fires Grow: Fires grow to adjacent cells, and each year has the same expected proportion burned.

Disturbance Pattern 3: Years Different – Fires One Cell: Fires burn only one cell each, and some years have a higher expected proportion burned than other years. In the years with the higher expected proportion burned, all cells have a higher probability of burning due to a larger expected number of fires in the landscape.

Disturbance Pattern 4: Years Different – Fires Grow: Fires grow to adjacent cells, and some years have a higher expected proportion burned than other years.

We can specify the expected annual proportion burned for all four disturbance patterns in advance. This way, we can compare the effect of the disturbance pattern on an equal basis.

Disturbance Pattern 1 represents the classical stand-level (for one cell) or landscape-level (for more than one cell) exponential models, while Disturbance Pattern 4 is the most realistic case that FLAP-X can represent. Disturbance Patterns 2 and 3 are intermediate cases that are intended only to illustrate separately the effects of fire growth and different years.

4.2. Description of FLAP-X

All four disturbance patterns for our theoretical models are implemented together in FLAP-X. The specific disturbance pattern invoked depends upon the input parameters for that run. The parameters and components of FLAP-X are as follows:

Cell and Landscape Size: The landscape is square and is composed of N square cells, where $N = I^2$, $I = 1, 2, 3, \ldots, 999$. The upper limit of I is 440 in the graphical version and 999 in the non-graphical version of FLAP-X, giving maximum landscape sizes of 193 600 and 998 001 cells, respectively. The size unit of cells is not specified, as it can represent any desired size. The cell size is intentionally not specified because the actual size does not affect the results, only the relative size of the cells (or fires) and the landscape. Cells may be thought of as being about one to 10 ha or more.

Cell Display Size and Colour: Cells can be displayed in a range of sizes from 1 x 1 pixel to the largest size that allows the landscape display to fit the screen. The displayed cell size does not affect the results. The cell colour indicates the cell age class.

Initial Condition: For the initial condition, cell ages are randomly generated from the geometric distribution corresponding to the overall expected proportion burned (see below). For example, for an overall expected percentage burned value of 1%, the mean stand age is 100 years.

Simulation Run Lengths: The simulation duration is the total of the transient and main simulation run lengths. To reduce the effect of the initial conditions, the simulation first runs over several years, s, $s = 1, 2, 3, \ldots, S$ in the transient phase. Those results are discarded except for illustrative purposes, i.e., the expected age distribution graph is re-initialized at the beginning of the main simulation run. The main simulation runs over several years, t, $t = 1, 2, 3, \ldots, T$.

Number of Replications: For some statistical analyses, the simulations must be replicated. Each simulation over T years is one replication, r, $r = 1, 2, 3, \ldots, R$.

Overall Expected Annual Percentage Burned: This can range from 0.1% to 29.99%. Typical values are 1.5% without fire suppression, and 0.02% with fire suppression.

Expected Annual Percentage Burned in the Three Highest Disturbance Years out of 10: Due to weather, years can vary in the expected percentage burned. This is represented in FLAP-X by having two types of years: low and high disturbance years. This approximates the random severity by a two-point discrete distribution. Low disturbance years are arbitrarily defined as the seven out of 10 years with the least disturbance. High disturbance years are the three out of 10 years with the greatest disturbance. If all years have the same expected percentage burned, then this value is the same as the overall expected percentage burned above.

FLAP-X represents this internally as follows. The overall expected annual proportion burned in the landscape is p_O. The two types of disturbance years, low and high, occur with probability 0.7 and 0.3, respectively. The type of each year is independent of the types of previous years. The expected annual proportion burned in low and disturbance high years is p_L and p_H, respectively.

$$p_O = 0.7\, p_L + 0.3\, p_H$$

The value of p_O is specified first from the range $0.001 \leq p_O \leq 0.2999$, then p_H is selected from the range $p_O \leq p_H \leq p_O / 0.3$. At the lower bound ($p_H = p_O$), all years are identical in terms of the expected proportion burned. At the upper bound ($p_H = p_O / 0.3$), $p_L = 0$, i.e., there are no fires in low disturbance years.

Fire Size Distribution, and Expected Fire Size: Fire sizes are either a constant or random number of cells. If constant, fire size, F, is in the range $1 \leq F \leq N$ cells. If a random variable, fire size is geometrically distributed with mean $E[F]$ in the range $1 \leq E[F] \leq N$. Due to the properties of the geometric distribution, if $E[F] = 1$, then all fires are 1 cell in size.

Fire Shape, Rate of Spread, and Fire Direction: The fire growth model is based on Xu and Lanthrop (1994). Fire shape is primarily elliptical with a small random component for variety. To reduce edge effects, fires that burn off the end of the landscape continue burning at the opposite edge. For the fire growth model, we use arbitrary units of length, time and speed. The length unit is "cell length," and the time unit is unspecified, so the speed unit is cell lengths per unit time. The actual speed and duration of each fire are not relevant in this model.

The Rate of Spread (*ROS*) has two components: *Base ROS* and *Maximum ROS*.

$$\textit{Base ROS} = 5 \text{ (cell lengths/unit time)}$$
$$\textit{Maximum ROS} = 3\,(\textit{Base ROS}) + \textit{Extra ROS}$$

where *Extra ROS* is a geometrically distributed random variable with a mean of 6 (cell lengths/unit time).

The *Maximum ROS* occurs in the *Fire Direction* which is a normally distributed random variable, $\varnothing_{MAX}$, with a mean bearing of 100° and a standard deviation of 30°. The *ROS* in any direction $\varnothing$ is given by:

$$ROS(\varnothing) = \textit{Base ROS} \,/\, (1 - (1 - \textit{Base ROS} \,/\, \textit{Maximum ROS}) \cos(\varnothing - \varnothing_{MAX})\,)$$

from Xu and Lanthrop (1994).

If a fire spreads past the cell of ignition, surrounding cells burn in order of earliest to latest burning time. The burning time of cell (i,j) is:

$$\textit{Burning Time}\ (i,j) = \textit{Distance}\ (i,j) \,/\, ROS(\varnothing) + \textit{Random Factor}$$

where *Distance* (i,j) is the straight line distance from the center of the ignition cell to the centre of cell (i,j), and *Random Factor* is a uniformly distributed random variable on the interval (0, 0.25] time units.

Fire Occurrence: The number of fires in a year is a Poisson distributed random variable whose mean depends on the expected proportion burned that year, the expected fire size, and the landscape size. It is derived as follows:

Expected Proportion Burned

= *Expected Burned Area* (cells) / *Landscape Area* (cells)

= (*Expected Number of Fires* (fires)) (*Expected Fire Size* (cells / fire))

/ *Landscape Size* (cells)

Then,

Expected Number of Fires (fires)

= (*Expected Proportion Burned*) (*Landscape Size* (cells))

/ *Expected Fire Size* (cells / fire)

In symbolic notation,

$$p_L = \mu_L \, E[F] \, / \, N$$

$$\mu_L = p_L \, N \, / \, E[F]$$

where μ_L is the expected number of fires in low disturbance years. Similar equations are used for the expected number of fires in high disturbance years.

Cell Ages: Each cell is characterized by the time since the last fire. That is, each cell has an age, a, $a = 1, 2, 3, \ldots, 32\,760$ which is the number of years since the last fire. (The upper limit is due to computer storage limitations.) Cell ages are defined as follows. Cell age is recorded in "spring" before any fires occur that year. Fires occur during the "summer." Just after a fire, the cell age is zero. All burned and unburned cells age one year "over the winter" so that the minimum recorded cell age is one.

Age Distribution: During the "spring" of each simulated year, t, at any replication, r, the landscape has an age distribution vector,

$$D(t,r) = [d_1(t,r), d_2(t,r), d_3(t,r), \ldots, d_A(t,r)]$$

where $d_a(t,r)$, $a = 1, 2, 3, \ldots, A-1$ is the proportion of the landscape that is age a. That is, $d_a(t,r)$ = (Number of Cells of age a) / N.

The value of $d_A(t,r)$ is different in that it is a "collecting" age:

$$d_A(t,r) = 1 - \sum_{a=1}^{A-1} d_a(t,r)$$

The value of A is chosen so that less than 0.5% of the theoretical expected age distribution (see below) lies in age A or above.

Theoretical Age Distribution: The theoretical age distribution used is the geometric analogue of the exponential distribution. $H(p_O) = [h_1(p_O), h_2(p_O), h_3(p_O), \ldots, h_A(p_O)]$ is the expected proportion of the landscape in each age a, $a = 1, 2, 3, \ldots, A-1$ when the overall expected proportion burned is p_O. As for $d_A(t,r)$, $h_A(p_O)$ is the collecting age.

Age Class Width: The age distribution can be grouped into age classes of width one to 40 years. When the age class width is greater than one:

- the value of A is increased to create a collecting age class that is equal in width to the other age classes while satisfying the 0.5% criterion above, and
- the proportions $d_a(t,r)$ are averaged within each age class for both the current and theoretical age distributions.

Note that grouping ages greatly affects the apparent variability of the forest age distribution. The following averages and similarity measures are computed identically regardless of the age class width.

Similarity Index: A similarity index from Baker (1989b) was used to indicate the degree to which any age distribution has an exponential shape.

$$SIM\{D(t,r)\} = 100\% \cdot \left(1 - 0.5 \sum_{a=1}^{A} |d_a(t,r) - h_A(p_O)| \right)$$

where $|x|$ denotes the absolute value of x. *SIM* ranges from 0 to 100%. The maximum error due to the collecting age class is a 0.5% overstatement of *SIM*.

Average Age Distributions: The average age distribution vector is calculated two ways:

- Over time within one replication (longitudinal)

$$AV_T[D(t,1)] = \frac{1}{T}\sum_{t=1}^{T} D(t,1)$$

- Across replications (cross sectional)

$$AV_R[D(T,r)] = \frac{1}{R}\sum_{r=1}^{R} D(T,r)$$

It is important to note that the longitudinal average consists of *dependent* observations while the cross sectional average does not. The longitudinal average represents what is experienced over time in one landscape.

Average Similarity Indices: The average similarity index is also calculated two ways:

- Over time within one replication (longitudinal)

$$AV_T[SIM\{D(t,1)\}] = \frac{1}{T}\sum_{t=1}^{T} SIM\{D(t,1)\}$$

- Across replications (cross sectional)

$$AV_R[SIM\{D(T,r)\}] = \frac{1}{R}\sum_{r=1}^{R} SIM\{D(T,r)\}$$

As for the average age distributions, the longitudinal average consists of *dependent* observations while the cross sectional average does not. The longitudinal average represents what is experienced over time in one landscape.

Relationship between Similarity of Average and Average Similarity: Figure 4 clarifies the relationship between the similarity of average age distribution, and the average similarity of age distribution.

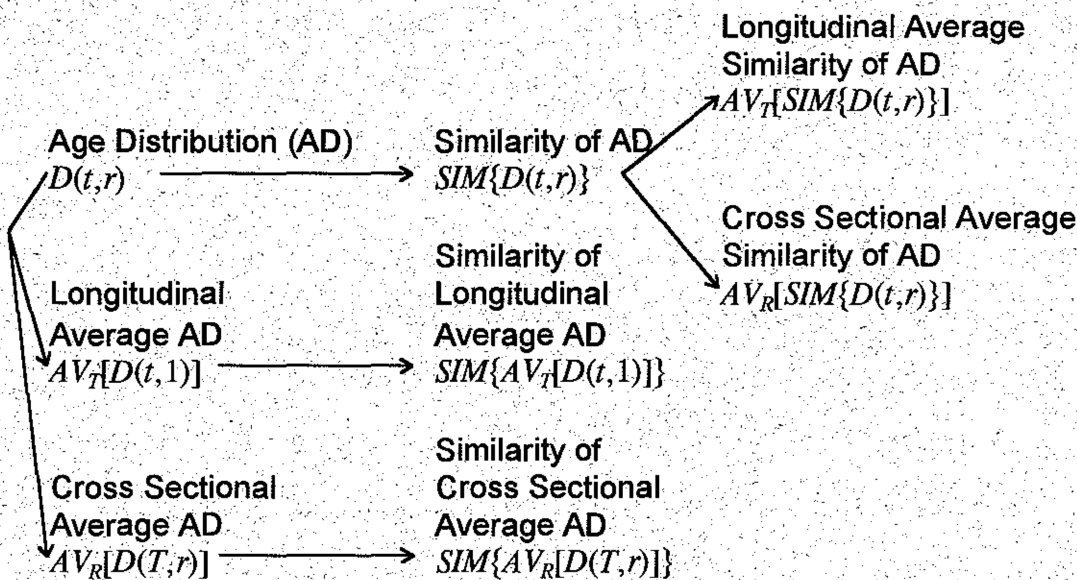

Figure 4: Illustration of the relationship between the similarity of average age distribution, and the average similarity of age distribution. (AD denotes age distribution.)

4.3. A Note on Burning the Entire Landscape

It is important to note that certain input parameter settings will lead to inaccurate results. In particular, if there is an attempt to burn more than the entire landscape area in any one year, then the burned area results will be understated, biasing the age distribution statistics. FLAP-X cannot burn more than the entire landscape area in any one year, so any attempt to burn more is ignored. Parameter settings where this can happen are those where:

- the expected fire size is large compared to the landscape size
- the expected annual proportion burned is large, and/or
- the landscape size is small.

The error in the average age class distribution induced by this limitation is very small and can be seen by running FLAP-X for long simulations with settings that attempt to burn more than the entire landscape during some years.

The small landscape size factor is due to the way that fires are generated in FLAP-X. The classical exponential model assumes that each cell has the same probability of burning each year. The number of cells burned per year is therefore binomially distributed. For a moderate to large number of cells, and a small probability of burning, the number of cells burned per year is approximately Poisson distributed (e.g., Gibra 1973, p. 172). FLAP-X generates the Poisson distributed total number of fires that will occur each year, and then randomly locates the fires. It does not determine independently for each cell whether or not that cell burns. This is both computationally more efficient, and necessary to correctly model the cases in which fires grow.

4.4. A Note on Changing the Fire Size vs. Number of Fires

There is a subtle but important assumption in FLAP-X with respect to the difference between low and high fire disturbance years. In reality, low disturbance years have both fewer and smaller fires than high disturbance years. In FLAP-X however, the number of fires is reduced while the expected fire size remains the same. The need for this simplification derives from the need to have FLAP-X represent the four different Disturbance Patterns consistently for the full range of values for all input parameters.

Recall that we specify the overall expected annual proportion burned, and the fixed or expected random fire size. FLAP-X then calculates the expected number of fires that will result in the specified expected annual proportion burned. For any case where years are different and all fires are one cell (Disturbance Pattern 3), we cannot reduce fire size except to zero which is equivalent to reducing the number of fires. For any case where years are different and all fires are fixed at, say, two cells (Disturbance Pattern 4), we could only reduce fires to zero or one cell. This still provides too little flexibility in reducing fire disturbance by reducing fire size, so we would still need to adjust the number of fires in most cases to generate precisely the required expected annual proportion burned.

FLAP-X could be changed so that where years are different and fires grow (Disturbance Pattern 4), users would specify the fixed or expected fire size and the expected proportion burned in the worst three of 10 years and the mildest seven of 10 years separately. We believe, however, that this would unnecessarily complicate our analyses by presenting another degree of freedom in the choice of input parameters for one Disturbance Pattern.[1] We prefer to leave this enhancement to more realistic landscape models in which we will not need to represent the cases where all fires are one cell (Disturbance Patterns 1 and 3).

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[1] It would also unnecessarily complicate the use of FLAP-X to represent the impact of fire suppression, described in Boychuk and Perera (1995).

4.5. Outline of the Simulation Algorithm

The simulation model operates as follows:

- Generate the initial landscape
  (i.e., initialize the age of each cell)
- For each simulated year:
  - Generate the type of year
    (i.e., high or low disturbance)
  - Generate the Poisson distributed random number of fires that year
  - Generate a random fire direction for fires that year
  - For each fire that year:
    - Generate a random fire location
    - Generate a random fire size (unless fire size is constant)
    - Generate the fire shape parameter
      (i.e., the Extra ROS*)*
    - Grow the fire to the required size
  - At the end of each simulated year, age the landscape one year, and update the displayed cell colours and age distribution graphs.

5. Computational Results

It is instructive to run FLAP-X with a large variety of settings to see the spatial pattern in the landscape and the age distribution graphs changing over time. Here, we present our principal results.

The length of the transient phase for our computational results was calculated as follows. Recall that the collecting age, A, is chosen so that it contains less than 0.5% of the theoretical landscape area. By default, the length of the transient phase is set to A years. At the end of the transient phase, each cell in the entire landscape will have either aged into the collecting age, or been burned at least once. Thus, the state of the landscape at the start of the main simulation is independent of the initial condition or state. This default transient phase run length was used for all of the computational results. Note also that in all of the results in this section, the age class width is one year.

Figure 5 shows the effect of the expected proportion burned on the forest age distribution. Two sample age distributions ($D(T,1)$) are shown. In both cases, the landscape size is 160 000 cells, all years are the same, and all fires are fixed at one cell (i.e., Disturbance Pattern 1: YS-F1). In one case the expected percentage burned is 2.5% and in the other case it is 1.5%. The age distribution graphs are not smooth due to randomness, but they both closely match the theoretical exponential distributions with their corresponding means.

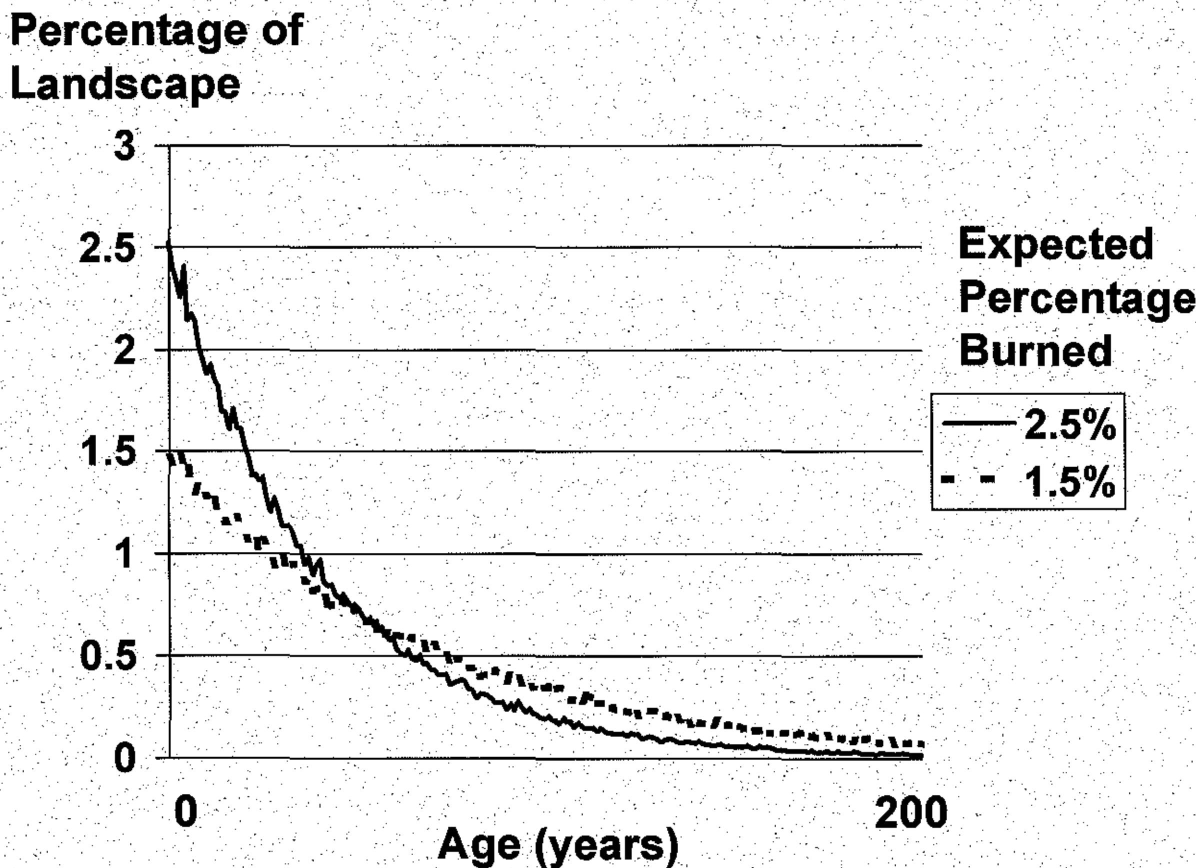

Figure 5: Sample age distributions showing the effect of the expected annual percentage burned. A higher value leads to more area in the younger ages and less area in the older ages.

Figure 6 shows the effect of the number of cells in the landscape on the forest age distribution. Three sample age distributions ($D(T,1)$) are shown. In all three cases Disturbance Pattern 1 is used (YS-F1) with an expected proportion burned of 1.5%. The results are given for landscape sizes of 400, 10 000, and 160 000 cells, along with the corresponding smooth theoretical age distribution ($H(p_O)$) curve. For relatively small landscapes, the age distribution varies widely around the exponential shape due to the small sample size.

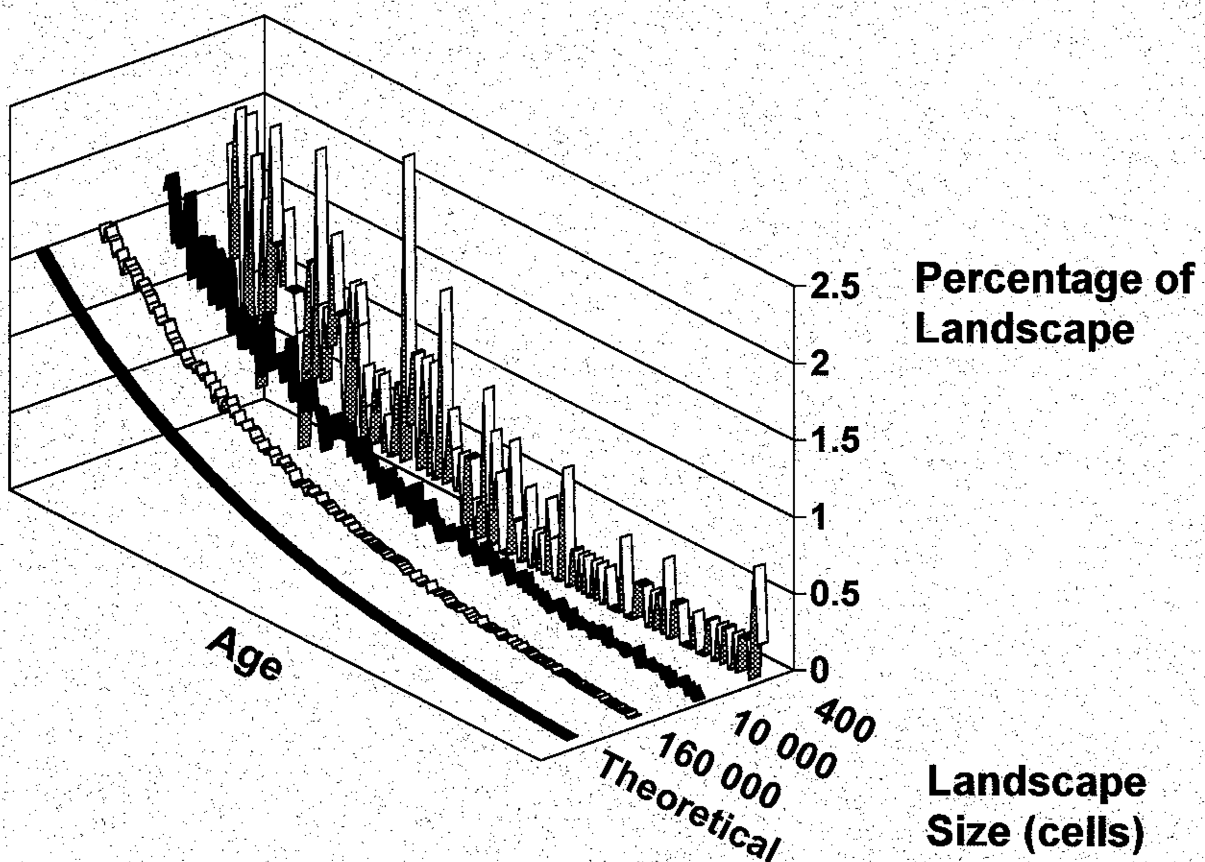

Figure 6: Sample age distributions showing the effect of the landscape size.

Figure 7 shows the effect of the four disturbance patterns on the forest age distribution ($D(T,1)$). In all four cases, the landscape size is 40 000 cells and the overall expected proportion burned is 1.5%. For the cases in which fires grow (FG), the expected fire size is 200 cells, or 0.5% of the landscape area. For the cases in which years are different (YD), the expected proportion burned in the 3 of 10 worst years is 4%. Even for this relatively large landscape, where years are different (YD), the age distribution is highly variable. Figure 7 is a key illustration of the effects of relaxing the classical exponential model's assumptions of independent cell burning, and constant annual probability of burning. For all but the classical model (YS-FG), fire growth, annual variability, or both cause the annual proportion burned to vary widely. The forest age distribution shows the history of previous variations.

Due to the probability of re-burning, older ages are generally less prevalent than younger ages. This is entirely consistent with the fundamental insight gained from Van Wagner's (1978) original exponential model. Indeed, it is significant that the age distribution *averaged over time* ($AV_T[D(t,1)]$) is precisely exponential for all for disturbance patterns. This implies that the replication-average age distribution ($AV_R[D(T,r)]$) is also exponential. For the time-average age distribution, consecutive observations are dependent, whereas for the replication-average age distribution, they are independent. In both cases, however, the observations come from the same distribution. For the replication-average age distribution, the observations can be thought of as being taken far enough apart in time to be independent.

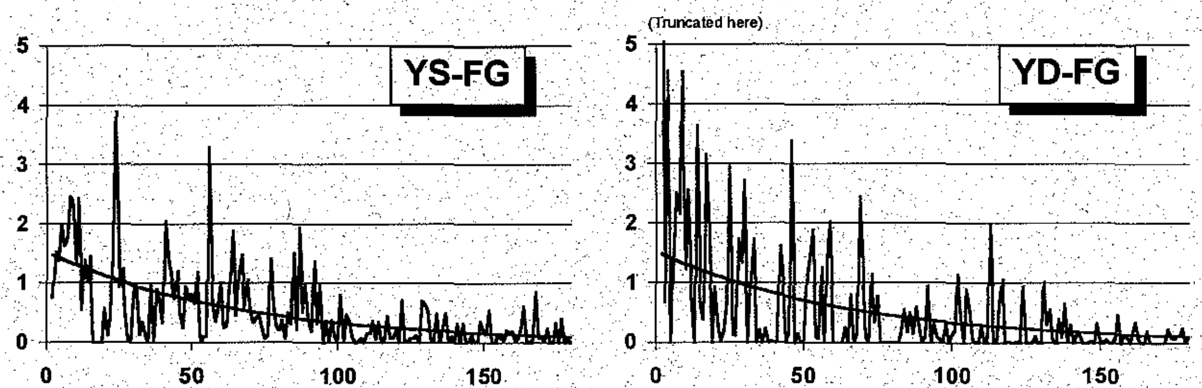

Figure 7: Four sample age distributions showing the effect of the disturbance pattern. Each plot shows the percentage of landscape area vs. age.

One of the principal questions in our analysis is whether the forest age distribution approaches the exponential shape for "sufficiently large" landscapes. For this we used *SIM*, the similarity index defined above, to measure the degree to which any given age distribution matches the corresponding theoretical age distribution. *SIM* ranges from 0% to 100%. The sample age distributions in Figure 7 have the following *SIM* (specifically, $SIM\{D(T,1)\}$) values:

YS-F1, *SIM* = 96%	YD-F1, *SIM* = 50%
YS-FG, *SIM* = 64%	YD-FG, *SIM* = 48%

Figure 8 shows the replication-average similarity index ($AV_R[SIM\{D(T,r)\}]$) vs. landscape size for the four disturbance patterns. $R = 6$ for landscapes up to and including 62 500 cells, and $R = 5$ for larger landscapes. For Disturbance Patterns 1 and 2 (YS-F1, YS-FG), *SIM* approaches 100% for sufficiently large landscapes. For Disturbance Pattern 2 (YS-FG), however, the landscape needs to be roughly 15 times larger than for Disturbance Pattern 1 (YS-F1) to get the same *SIM*, because its average fire size is 15 cells. For the cases in which years are different (YD), *SIM* never approaches 100% because the disturbances are correlated throughout the landscape. The age distribution always varies around the theoretical exponential shape. For these cases, *SIM* is asymptotic to a level of about 50%. Thus, under the assumptions of the classical exponential model (Disturbance Pattern 1), the forest age distribution approaches the exponential shape for sufficiently large landscapes. For the most realistic case represented in FLAP-X (Disturbance Pattern 4), the forest age distribution does not approach the exponential shape even for arbitrarily large landscapes.

Note that the specific results depend on the expected proportion burned, the expected fire size, how much years differ, and the age class width. The relative positions of the curves in Figure 8 can change significantly for different parameter settings.

Figure 9 shows a subset of the results from Figure 8 with 98% confidence intervals. For clarity only the results for landscapes up to 40 000 cells is shown. The confidence intervals are based on the t-statistic with five degrees of freedom.

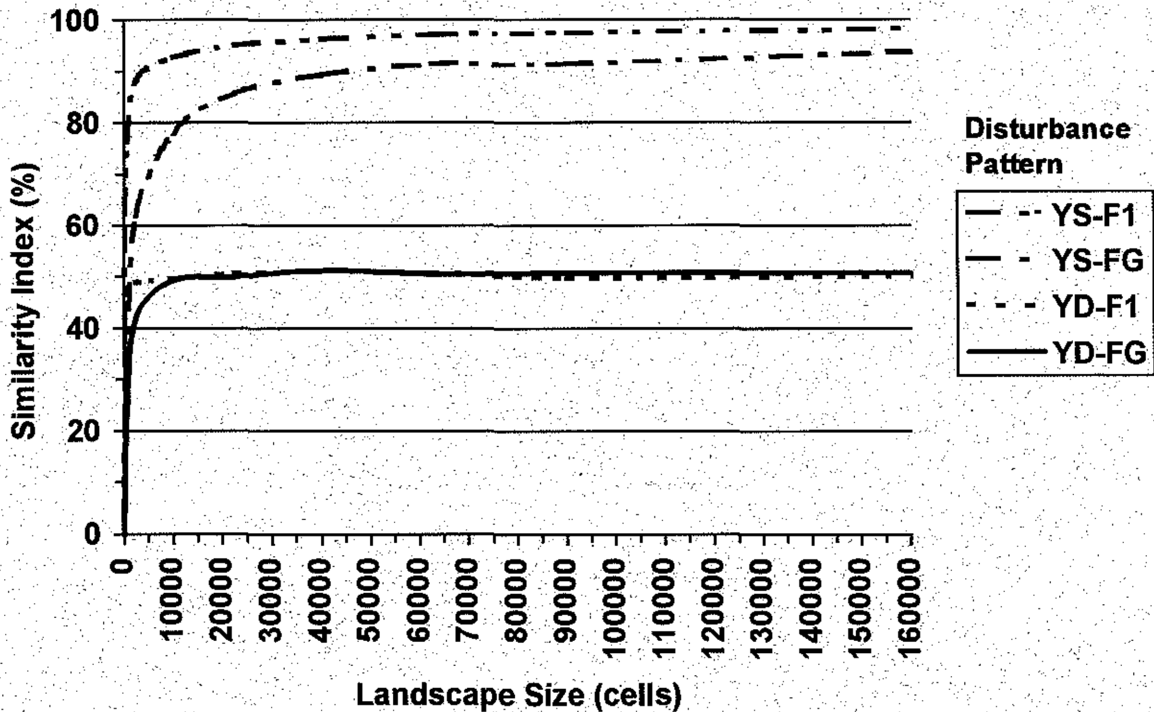

Figure 8: Replication-average similarity index vs. landscape size for four disturbance patterns.

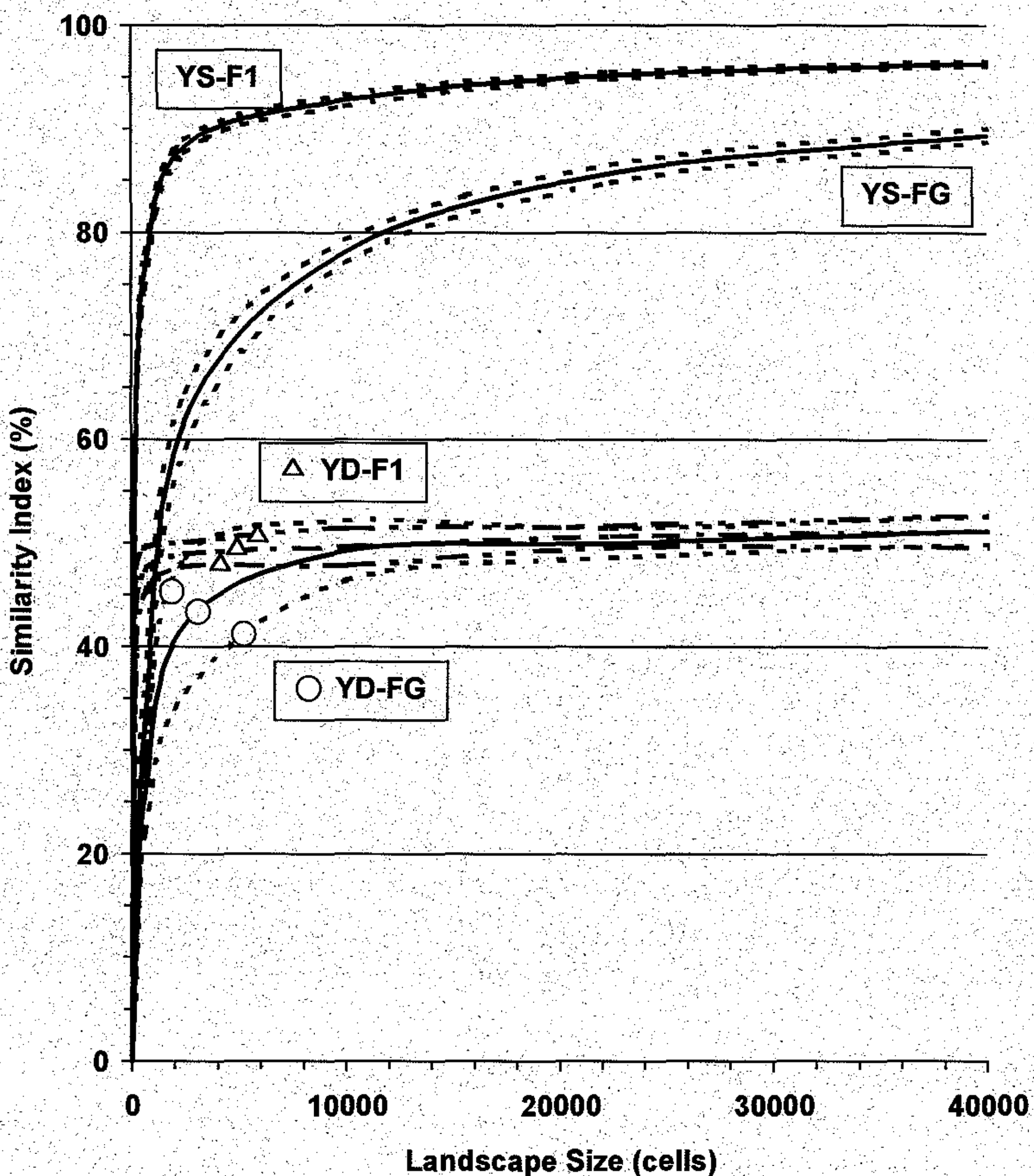

Figure 9: Replication-average similarity index vs. landscape size for the four disturbance patterns showing 98% confidence intervals based on the t-statistic with 5 degrees of freedom.

6. Case Study Example

We present a brief example of the application of FLAP-X to the analysis of a specific landscape unit. We emphasise that FLAP-X is theoretical and that its results should be used for insight only. Consider the extensive protection zone of Red Lake District in Ontario whose size and recent fire history are given in Table 4. In the extensive protection zone, fires are generally only suppressed if they threaten communities or specific isolated values. From Table 4, we use the following data to prepare input to FLAP-X:

- Extensive protection zone area: 3 266 186 ha
- Overall average annual percentage burned: 1.68%
- Average annual percentage burned in worst 5 of 18 years: 4.67%
- Average fire size: 1992 ha

We can assume any cell size in FLAP-X. We arbitrarily choose a cell size that will give us a landscape of 200 x 200 = 40 000 cells. The cell size is 3 266 186 ha / 40 000 cells = 81.65 ha. The average fire size is 1992 ha / 81.65 ha = 24.4 cells. Note that the worst five of 18 years is not the required worst three of 10 years (0.278 vs. 0.300), but it is sufficient for this purpose.

We used these parameters and a 20-year age class width in FLAP-X to generate sample age distributions, illustrated in Figure 10. Seven independent sample age class distributions are shown, along with the theoretical age class distribution. Note that the age class distribution extends further, but the higher age classes are absent from the graph.

If we used the assumptions in the classical exponential model, with all years being the same and fire sizes of one cell (i.e., Disturbance Pattern 1: YS-F1), then the sample age distributions would be extremely close to the theoretical age class distribution as can be seen in Figures 7 and 8.

FLAP-X could be used to explore the variability of forest age distribution for different sized landscapes with the same fire regime by changing only the number of cells in the landscape. For example, a landscape with 70 x 70 = 4 900 cells which are each 81.65 ha has an area of 400 085 ha. This is close to the size of the BWCA studied by e.g., Baker (1989b).

Table 4: Area and percentage burned in the extensive protection zone in Red Lake District of Ontario, 1976 to 1993. The years are sorted from highest to lowest area burned. Data were provided by the Aviation, Flood and Fire Management Branch, Ontario Ministry of Natural Resources.

Year	Area Burned (ha)	Percentage Burned[1] (%)	Percentage Burned by Group (%)
1988	243 037.9	7.44	
1989	218 221.8	6.68	
1977	142 220.1	4.35	(Five
1984	91 269.0	2.79	highest disturbance
1986	68 479.2	2.10	years) 4.67
1976	56 317.8	1.72	
1991	50 731.0	1.55	
1983	40 041.6	1.23	
1993	30 554.4	0.94	
1981	19 535.0	0.60	
1980	11 458.0	0.35	
1987	6 917.7	0.21	
1990	5 854.8	0.18	
1979	824.2	0.03	
1992	346.3	0.01	
1985	9.0	0.00	(Thirteen
1978	0.2	0.00	lowest disturbance
1982	0.1	0.00	years) 0.52
Total	985 818.1		
Average	54 767.7	1.68	1.68

[1] The extensive protection zone area is 3 266 186 ha.

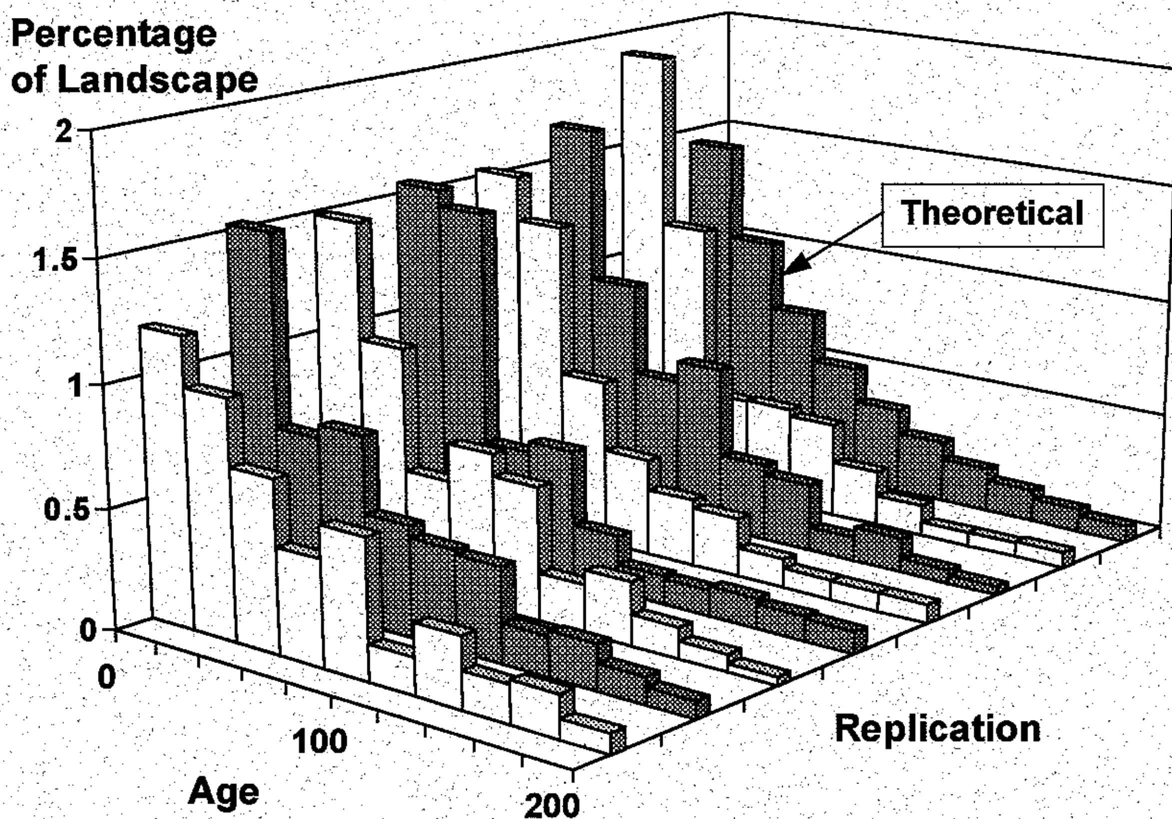

Figure 10: Illustration of the variability of the age class distribution for the case study example. Seven independent sample age class distributions are shown, along with the theoretical age class distribution. Note that the age class distribution continues into older classes which are not shown.

7. Discussion

Our purpose was to gain insight into the dynamics of a system subject to random disturbances. We believe that this research gives useful insights concerning our understanding the dynamics of landscapes that are subject to fire disturbances. Our results suggest that, under plausible natural boreal disturbance regimes, we should not expect to find forest age distribution stability even at very large scales due to spatial and temporal dependence of disturbances. We note, however, that the exponential model represented the *time-average* age distribution (and therefore the replication-average age distribution also) for all disturbance patterns.

We emphasise that this model and its results and conclusions are theoretical. We have analysed the consequences of the assumptions in our model. For example, the exponential shape of the time-average age distribution depends on age-independent fire growth, and the fire occurrence process we used. We can make no claims about the dynamics of actual forest age distributions from this work alone. We are, however, developing refined models which we plan to test with empirical data. A significant amount of work has been done previously on the analysis of empirical age distribution data, much of which used Van Wagner's exponential model in some way (or Johnson and Van Wagner's (1985) Weibull model). The insights gained from our theoretical work might aid in the further analysis of empirical data. In particular, we demonstrated that it is important to represent the spatial and temporal correlations of disturbances in landscape simulation models.

Based on previous theoretical and empirical studies and our theoretical work presented here, we believe that it is necessary to allow for variability – perhaps significant variability – when attempting to identify the natural state of the forest landscape. In the presence of variable disturbances and spatial and temporal dependence, applying the exponential age distribution model to landscape management could lead to problems. For example, if the landscape was managed to maintain an exponential age distribution at relatively small spatial scales, there would be an absence of large disturbances. Conversely, if the variability of disturbance sizes was recognised, but the effect of spatial scale was not considered, another type of problem could result. For example, decision-makers at every local landscape unit could, in principle, attempt to justify a large clearcut on the basis that such large disturbances are natural in their landscape. At the spatial scale of the larger landscape, however, the sum or cumulative effect of the individual clearcuts of that size might be very "unnatural."

We note that our representation of fire disturbance variability was deliberately conservative, i.e., understated. Each of our simulated years had either a "low" or "high" expected proportion burned, whereas an informal review of historical fire disturbance shows greater variability (e.g., Van Wagner 1988). Many studies have shown abrupt climate-driven changes in fire regimes in the past, e.g., Green (1982), Cwynar (1987), Clark (1988, 1989, 1990), Bergeron (1991), Johnson and Larsen (1991), and Campbell and McAndrews (1993). But even relatively short-term weather-driven variability of fire disturbance appears greater than that used in our analyses, e.g., Romme and Despain (1989) and Harrington and Flannigan (1993). Greater variability in fire disturbance regime leads to greater variability in the forest age distribution (e.g., Baker 1989b).

We have examined the effect of spatial scale and the relative size of the landscape disturbances on the forest age distribution. FLAP-X can also be used to examine the effect of model resolution or "grain size" on the forest age distribution. While it is an important question, we have not addressed it here as it is a modeling and data issue, rather than a landscape management issue. Our approach would be to simply set the grain size small enough so that it would not significantly affect the results. The issue of resolution was addressed by, e.g., Turner et al. (1989) and Wiens (1989).

The problem of relating fire disturbance to age distribution is especially complicated by apparent age dependency of fire ignition and fire growth. We deliberately assumed age independence so that the effects of spatial and temporal dependence would not be needlessly obscured. We plan to modify FLAP-X to test age-dependent ignition and growth, and we anticipate that we can demonstrate Johnson and Van Wagner's (1985) and Johnson and Gutsell's (1994) results. Specifically, the expected age distribution would be an "S-shaped" variation of the exponential shape with more area in younger ages and less in older ages. We also believe that the high degree of variability that we have demonstrated in the age-independent case would still occur in the age-dependent case.

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