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Stand density management diagrams: modelling approaches, variants, and exemplification of their potential utility in crop planning[1]

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Abstract: The evolving shift in forest management objectives towards the collective consideration of volumetric yield, end-product quality and value, and ecosystem service outcomes, while accounting for the impacts of anthropogenic climate change, has resulted in innovative advancements in decision-support models used in stand density management. This review provides a synopsis of these efforts with respect to static, dynamic, and structural stand density management diagrams (SDMDs). More precisely, the scope of this review includes an ecology-based perspective of stand density management, summarization of the foundational quantitative relationships along with their utilization within the analytical structure of the SDMD, examination of SDMD compliance with underlying ecological constructs and empirical prediction expectations, exemplification of a climate-sensitive structural SDMD variant in boreal crop planning, and identification of outstanding analytical challenges and plausible future research directions for advancing the SDMD modelling approach and its utility in stand-level management planning. Collectively, this account of the conceptual basis, historical analytical evolution, ecological integrity, predictive ability, application diversity, and demonstrated utility of the various SDMD variants solidifies the prerequisite evidentiary foundation for the continued development and deployment of SDMD-based crop planning decision-support models.

Key words: competition, static, dynamic and structural SDMD variants, climate change, conifers.

Résumé : Le changement évolutif des objectifs d'aménagement forestier vers des considérations collectives axées sur la production volumétrique, la qualité et la valeur des produits transformés et les services écosystémiques qui en résultent, tout en tenant compte des impacts des changements climatiques anthropiques, a permis des avancées innovatrices dans le domaine des modèles d'aide à la décision utilisés dans la gestion de la densité des peuplements. Cette synthèse résume les efforts consentis dans le domaine des diagrammes de gestion de la densité des peuplements (DGDP), qu'ils soient statiques, dynamiques ou structurels. Plus précisément, la portée de cette synthèse comprend une perspective écologique de la gestion de la densité des peuplements, un résumé des relations quantitatives fondamentales utilisées par la structure analytique des DGDP, un examen de la conformité des DGDP avec les concepts écologiques sous-jacents et les attentes de prévision empirique, un exemple de variante structurelle d'un DGDP sensible au climat pour planifier la récolte en forêt boréale, et l'identification des défis analytiques uniques et des pistes plausibles de recherche visant à faire progresser l'approche de modélisation par les DGDP et leur utilité dans la planification de l'aménagement à l'échelle du peuplement. Globalement, ce compte rendu des fondements conceptuels, de l'évolution analytique historique, de l'intégrité écologique, de la capacité prédictive, de la diversité des applications et de l'utilité démontrée des différentes variantes de DGDP solidifie les prémisses de base requises pour le développement et le déploiement continus de la plate-forme analytique fondée sur les DGDP pour la prise de décision en matière de planification des récoltes. [Traduit par la Rédaction]

Mots-clés : concurrence, variantes statique, dynamique et structurelle du diagramme de la gestion de la densité des peuplements, changements climatiques, conifères.

1. Introduction

Conceptually, stand density management is the process of regulating site occupancy via the informed manipulation of the occurrence and intensity of inter-tree competitive interactions to achieve a desired stand-level management objective (sensu Newton and Weetman 1993). Optimally, the localized stand-level management objective is one that is derived within an integrated forest-level management planning and policy context, and hence its attainment contributes to the realization of a stated global forest-level objective (sensu Erdle and Baskerville 1986). Stand-level density control strategies that have been used to address forest-level objectives include the following applications: (1) pre-commercial thinning within overstocked juvenile coniferous

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stands that accelerate the attainment of stand operability status and thus assist in mitigating forecasted future wood-supply deficits (e.g., Erdle 2000); (2) initial espacement treatments that maximize volumetric yields to increase available harvest volumes through the allowable cut effect (e.g., Erdle 2000); (3) initial espacement and subsequent thinning treatments that maximize end-product diversity thus enhancing secondary manufacturing opportunities (e.g., Middleton et al. 1995); (4) multiple thinning treatments throughout the entire rotation to maximize carbon sequestration potential (e.g., increasing net production (growth) and (or) carbon storage (dimensional lumber yields)) as part of regional or national climate change mitigation strategies (sensu Bourque et al. 2007); and (5) a combination of initial espacement and thinning treatments that create a temporal and spatial continuum of stand structures across the forest landscape that are conducive to wildlife survival (provision of habitat and migration routes) or fire risk reduction (e.g., Sturtevant et al. 1996 and Hirsch et al. 2001, respectively).

Since its acceptance as a valid silvicultural treatment in central Europe more than 200 years ago (sensu Puettmann et al. 2009), stand density management continues to be one of the most consequential and frequently applied treatments used by forest managers throughout the temperate and boreal forest regions. In Canada, three primary manipulation mechanisms are deployed to regulate competitive interactions within forest stands: initial espacement (IE), precommercial thinning (PCT), and commercial thinning (CT) treatments. More specifically, IE treatments are implemented at the time of plantation establishment usually on recently disturbed sites that have incurred a stand-replacing disturbance (e.g., wildfire or harvesting). Operationally, IE frequently involves the planting of genetically improved seedling stock arranged within a square, rectangular, or hexagonal spatial configuration at initial densities ranging from approximately 1000 to 2500 seedlings·ha^{-1} depending on the species, site quality, stand-level objective, and silvicultural intensity (sensu British Columbia Ministry of Forests 1999a; OMNRF 2015). PCT treatments are commonly implemented within natural-origin or artificially regenerated (e.g., seeded) density-stressed stands during the sapling stage or early post-crown-closure stem exclusion phase of development. This involves the removal of the smallest non-commercial-sized individuals in a manner that attempts to allocate the newly available growing space equitably among the residual crop trees. Structurally, selection thinning-from-below treatments commonly result in left-truncated horizontal and vertical size distributions and a pseudorandom spatial arrangement of residual crop trees. CT treatments are implemented during the semi-mature stage of stand development normally within plantations or natural-origin stands previously subjected to PCT. A hybrid selection-systematic thinning treatment is commonly deployed during which trees within equal-distance corridor-like rows are harvested to enable machine access followed by selection thinning between the corridors with a bias towards removing the smallest commercially sized individuals (i.e., thinning-from-below). Depending on the jurisdiction, stand type, and silvicultural system, CT is considered either a stand tending improvement practice or partial harvesting event, which are statutorily accepted as either a conventional treatment or silvicultural exception that requires additional scrutiny via posttreatment monitoring (e.g., British Columbia Ministry of Forests 1999b and Kayahara et al. 2006, respectively). Furthermore, in some provinces such as Ontario, candidate stands for CT must attain regulatory-specified structural and occupancy thresholds. For example, jack pine (*Pinus banksiana* Lamb.) and black spruce (*Picea mariana* (Mill.) B.S.P.) candidate stands must have a live crown ratio not less than 35%, height/diameter ratio less than 100, and basal area of at least 25 m^2·ha^{-1} (OMNRF 2015), and if selected, the degree of CT removal is similarly regulated (e.g., ≤35% of the stand's
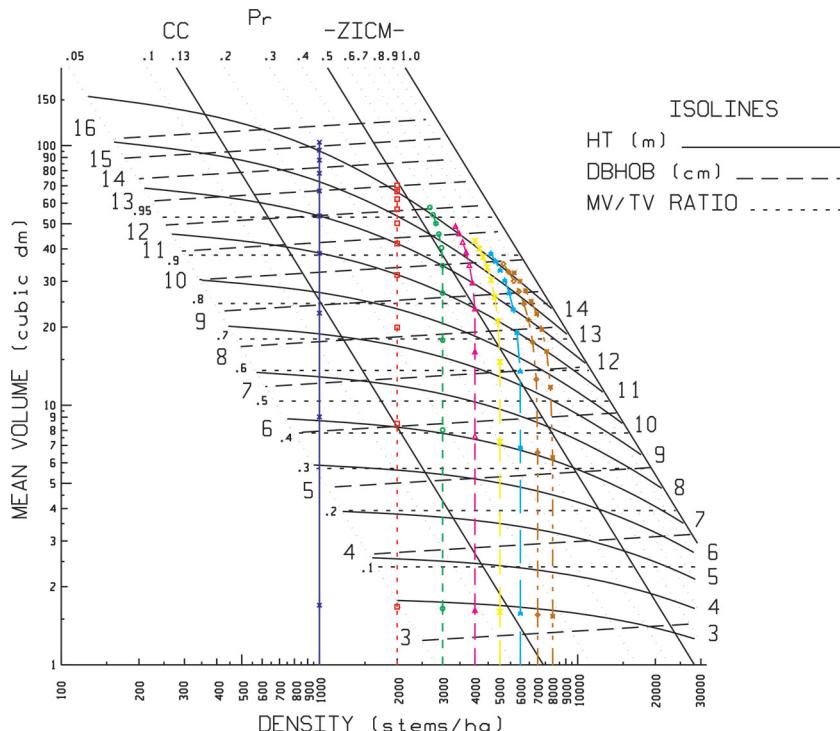
pretreatment basal area (OMNRF 2015)). Available annual rates of reforestation and thinning treatments in Canada over the 1990–2017 period indicate that approximately 500–800 million seedlings were planted on some 350 000–500 000 ha, and 59 000–219 000 ha were subjected to PCT treatments (CCFM 2020). Notably, this annual density management treatment area is comparable to the size of Canada's smallest province (Prince Edward Island; ≈5800 km^2) and is more than double the area of the next closest silvicultural treatment used in Canada, vegetation management (i.e., approximately 2100 km^2 of productive forests receive release treatments annually (CCFM 2020)). Furthermore, if aspirational afforestation and reforestation intentions to plant 2 billion additional trees by 2030 as part of a national-level climate change mitigation plan are realized (sensu FPAC 2020), tree planting will soon approach unprecedented levels in Canada.

Historically, density management treatment protocols in Canada have largely been informed from a combination of (1) theoretical forest production constructs arising from research studies in Europe (e.g., Langsaeter 1941; Möller 1954; Assmann 1970); (2) site-invariant empirically derived spacing indices and guides (e.g., Reineke's (1933) stand density index (maximum number of trees per unit area for a given quadratic mean diameter) and Wilson's (1946) mean dominant height/intertree spacing index); (3) variable-density yield tables (e.g., Johnstone 1976; Berry 1978; Beckwith et al. 1983); and (4) empirical observations, analytical results, and associated inferences derived from numerous individual experimental and operational field trials investigating the magnitude, pattern, and duration of volumetric yield responses to IE, PCT, and CT treatments at the individual-tree and (or) stand levels. This latter source included results extracted from a set of large-scale and long-term field experiments established during the 1950–1970 period as exemplified by IE, PCT, and CT studies in lodgepole pine (*Pinus contorta* Douglas ex Loudon) and jack pine throughout Alberta, Saskatchewan, and Manitoba (e.g., Cayford 1961, 1964; Bella and De Franceschi 1974a, 1974b); IE red pine (*Pinus resinosa* Aiton) experiments in Ontario (e.g., Stiell and Berry 1977); and the Green River PCT trial for balsam fir (*Abies balsamea* (L.) Mill.) in New Brunswick (Baskerville et al. 1960). Although the empirical results and associated inferences derived from field experiments were of consequential importance in operational stand density management decision-making when locale, species, site quality, stand age, and treatment characteristics were comparable, the requirement for an analytical-based decision-support platform for simultaneously forecasting and contrasting rotational outcomes to a much broader array of density management scenarios was clearly evident. This requirement became even more acute during the 1980s when IE and PCT were increasing rapidly under the auspices of numerous federal–provincial resource management agreements (sensu Honer 1986).

Coincidentally, the static stand density management diagram (SDMD) was being introduced to the North American forest science literature by Drew and Flewelling (1977, 1979) as part of an industrial-based research initiative in the Pacific Northwest region. Briefly, the original static SDMD, conceptualized, formulated, and advanced by Japanese forest scientists during the 1960s, falls within the stand-level distance-independent average-tree yield model class (sensu Porté and Bartelink 2002). The SDMD has a strong ecological underpinning given that a majority of its principal relationships are derived from theoretical axioms of plant population biology and forest stand dynamics (e.g., reciprocal size–density relationships, self-thinning theory, allometric scaling rules, and forest production concepts).

The SDMD graphic provides a species-specific illustration of the quantitative interrelationships between mean tree yield attributes and stand density within an ecologically bounded two-dimensional (2D) size–density space where density dependence dominates stand dynamical processes and associated structural development. As exemplified by the dynamic SDMD developed

Fig. 1. Exemplification of the dynamic stand density management diagram (SDMD) developed for the natural-origin black spruce stand type of central insular Newfoundland (Newton and Weetman 1993). Graphically illustrating (1) universal site-independent core yield–density relationships applicable to this specific stand type inclusive of the (*i*) approximate crown closure line (CC) at a relative density index (P_r) of 0.13 (innermost left-hand-side diagonal solid line), (*ii*) lower limit of the zone of imminent competition mortality (ZICM) at a P_r value of 0.50 (mid-graph (right-hand-side) diagonal solid line), (*iii*) biological asymptotic size–density relationship during self-thinning at a P_r value of 1.0 (self-thinning rule; outermost right-hand-side diagonal solid line), (*iv*) reciprocal equation of the C–D effect by dominant height class (HT) (horizontal-like curved solid lines for height values from 3 to 14 m by 1 m increments), (*v*) set of quadratic mean diameter (DBHOB) isolines (horizontal-like linear long-dash lines for DBHOB values from 3 to 16 cm by 1 cm increments), (*vi*) merchantable volume/total volume isolines (MV/TV) (horizontal-like linear short-dash lines for ratio values from 0.1 to 0.95 by 0.05 or 0.1 increments), and (*vii*) P_r isolines (diagonal dotted lines for P_r values from 0.05 to 1.0 by 0.05 or 0.1 increments) and (2) expected 100-year dynamic size–density trajectories for eight specified initial densities (1000, 2000, . . ., 8000 stems·ha⁻¹; vertical linear/non-linear symbol-specific and colour-coded lines with marked 10-year intervals denoted) superimposed for stands growing on medium-quality sites (site index of 12 m at 50 years (stand age); Newton 1992). Source: SDMDSAO.EXE algorithm (Newton 2006*b*).

for natural-origin black spruce stands of central insular Newfoundland (Fig. 1; Newton and Weetman 1993), the principal interrelationships commonly illustrated within a given species-specific SDMD include (1) categorized 2D diagonal mean volume – density relationships that demarcate the (*i*) beginning curve of competition (Ando 1962) or crown closure phase (Drew and Flewelling 1979), (*ii*) initiation of natural thinning (Ando 1962) or lower limit of the zone of imminent competition mortality (ZICM; Drew and Flewelling 1979), (*iii*) lower and upper limits of the zone of optimal site occupancy where net production is maximized (Drew and Flewelling 1979; Newton 2006*b*), and (*iv*) full density condition (Ando 1962) or asymptotic self-thinning phase (Drew and Flewelling 1977); (2) family of three-dimensional (3D) mean volume – mean dominant height – density relationships, which are presented in a 2D context using height-based isolines (i.e., reciprocal equation of the competition–density (C–D) effect (Ando 1962) or empirical regression-based analogues (Drew and Flewelling 1979)); (3) family of 3D yield index (Ando 1962) or relative density index (Drew and Flewelling 1979) relationships, which reflect the relative degree of density stress with respect to the asymptotic size–density condition (self-thinning rule) and are presented in a 2D format via the use of index-based isolines; (4) family of 3D quadratic mean diameter – mean volume – density relationships, which are presented in a 2D format deploying diameter-

based isolines; and (5) given 1–4, superimposed site-driven mean volume – density temporal stand development trajectory for a specified crop plan that either (*i*) negates accounting for mortality until the asymptotic self-thinning line is intersected (i.e., static SDMD; sensu Newton 2003*a*) or, conversely, (*ii*) explicitly accounts for density-dependent mortality throughout all stages of development via the incorporation of a mortality submodel (i.e., dynamic SDMD; sensu Newton 2003*a*). Once a given stand of known site quality is positioned within its species-specific SDMD using its mean dominant height and density coordinates, its developmental stage, relative degree of site occupancy, likelihood of incurring competition-induced mortality, mean tree size metrics, per unit area yields, and probable future size–density trajectory can be ascertained.

The ecological foundation of the SDMD along with its deployment of functional relationships that are universally applicable across species and sites, compatible calibration requirements with respect to the most commonly available sources of forest resource information that readily facilitate the parameterization of the principal yield–density and self-thinning relationships (e.g., permanent sample plot (PSP) systems consisting of remeasured tree-list or diameter-class mensurational metrics), and demonstrated utility in stand density management decision-making has provided the prerequisite foundation for its adoption and

acceptance as one of the preferred density management decision-support platforms in Canada (e.g., OMNRF 2015). More precisely, provincial-specific SDMDs have been developed and used in the management of black spruce, jack pine, and red pine in central and eastern Canada (e.g., Newton and Weetman 1993, 1994; Archibald and Bowling 1995; Smith and Woods 1997; Sharma and Zhang 2007) and of coastal Douglas-fir (*Pseudotsuga menziesii* (Mirb.) Franco), lodgepole pine (*Pinus contorta* var. *latifolia* Engelm. ex S. Watson), white spruce (*Picea glauca* (Moench) Voss), western hemlock (*Tsuga heterophylla* (Raf.) Sarg.), western red cedar (*Thuja plicata* Donn ex D. Don), interior Douglas-fir (*Pseudotsuga menziesii* var. *glauca* (Mayr) Franco), and Sitka spruce (*Picea sitchensis* (Bong.) Carrière) in western Canada (e.g., Farnden 1996, 2001). Apart from most other provincial jurisdictions, which generally have a single density management support model available for operational use at best, the SDMDs presented by Farnden (1996, 2001) for British Columbia conifers were calibrated employing output extracted from variable-density managed yield tables that were generated using an individual-tree growth model (i.e., species-specific natural and managed stand-type variants of the Tree and Stand Simulator (TASS) model (Mitchell 1975)). This approach ensured predictive compatibility, which is of paramount importance in terms avoiding decision-making ambiguity due to varying volumetric forecasts arising from different models for the same crop plan. Furthermore, the successful transformation of the simulator output into a conventional SDMD suggests shared concordance in terms of their underlying ecological foundations (e.g., density dependency of yield relationships and self-thinning-driven mortality patterns).

Globally, SDMDs have also been developed for numerous commercially important coniferous (evergreen softwoods) and deciduous (broadleaf hardwoods) tree species throughout the boreal and temperate forest biomes. Although varying in their analytics and presentation, representative examples of these efforts include the SDMDs developed for Japanese red pine (*Pinus densiflora* Siebold & Zucc.), Sugi (*Cryptomeria japonica* (L. F.) D. Don.), Hinoki cypress (*Chamaecyparis obtusa* (Siebold & Zucc.) Siebold & Zucc. ex Endl.), and Japanese larch (*Larix leptolepis* (Siebold & Zucc.) Gordon) in Japan (Ando 1968); radiata pine (*Pinus radiata* D. Don.) in New Zealand (Drew and Flewelling 1977); pitch pine (*Pinus rigida* Mill.) in South Korea (Shim et al. 1987); Scots pine (*Pinus sylvestris* L.), Austrian black pine (*Pinus nigra* Arnold), downy birch (*Betula pubescens* Ehrh.), and English oak (*Quercus robur* L.) in Europe (Stankova and Shibuya 2007; Stankova and Diéguez-Aranda 2020); and Chinese fir (*Cunninghamia lanceolata* (Lamb.) Hook.) in China (Tang et al. 2016). In addition to their prime importance as a crop planning decision-support tool for use in addressing largely volumetric and end-product yield objectives, SDMDs have also been utilized to address a diversity of non-timber-based objectives. For example, these have included (1) designing crop plans that maximize wildlife habitat potential for the American pine marten (*Martes americana* (Turton, 1806)) in the coniferous forests of eastern Canada via the thinning-induced generation of coarse woody debris that increases small mammal forging potential (Sturtevant et al. 1996); (2) extracting density control schedules that mitigate the damaging effects of terrestrial disturbances through the creation of structurally stable buffer stands that maximize community protection within the mountainous regions of western Italy (Vacchiano et al. 2008); (3) minimizing crown fire risk through the application of density control treatments that create vertical stand structures and crown biomass distributions that are conducive to minimizing the potential of surface-to-crown fire progression within coniferous stands in western Europe (Gomez-Vasquez et al. 2014); and (4) minimizing windthrow and snow load mortality risk by designing crop plans that increase stand stability via managing height/diameter ratios through density management within radiata pine plantations in northwestern Spain (Castedo-Dorado et al. 2009). Operationally, the development and subsequent deployment of

SDMDs within statutory-based regulated forest management planning systems have been best exemplified by efforts in Japan and Canada. In Japan, prefecture-specific SDMDs have been developed and utilized in the management of Japanese red pine, Sugi, Hinoki cypress, and Japanese larch (Forestry Agency of Japan 1978–1983). Likewise, in the Canadian provinces of British Columbia and Ontario, provincial planning guidelines have identified SDMDs as an applicable tool to be used in crop planning (e.g., British Columbia Ministry of Forests 1999*a* and OMNRF 2015, respectively).

To maintain the utility of the SDMD approach in light of the evolving paradigm shift in forest management objectives from essentially an overarching univariate volumetric yield maximization objective towards a trivariate collective objective involving the attainment of specified volumetric yield, end-product potential and ecosystem service outcomes (sensu Emmett 2006), coupled with consideration of anthropogenic climate change effects, consequential innovations in the SDMD modelling platform have been advanced since its first introduction some 60 years ago by Ando (1962). Thus, the goal of this review is to provide a synopsis of these innovations within a historical context and exemplify the potential utility of the resultant novel SDMD variants in addressing these new forest management realities. More precisely, the scope of this review includes a (1) prerequisite ecological perspective of stand density management; (2) analytical-based synopsis of the foundational functional and empirical relationships utilized in the development of the static, dynamic, and structural SDMD variants; (3) demonstration of SDMD compliance with underlying ecological constructs and empirical prediction abilities; (4) in-depth exemplification of the potential utility of a structural SDMD in boreal crop planning within the context of anthropogenic climate change; and (5) identification and discussion of the analytical challenges and plausible research directions for advancing the SDMD modelling approach and its utility in crop planning. Collectively, this updated historical account of the conceptual basis, analytical structures, innovative developmental pathways, ecological integrity, predictive ability, application diversity, utility, and outstanding analytical challenges aspires to solidify the evidentiary foundation for the continued development and deployment of the SDMD modelling platform for crop planning decision-making.

2. Prerequisites to stand density management and associated SDMD-based decision-support tools

2.1. Analytical foundation of the SDMD modelling approach: ecological basis and quantitative descriptions of competition effects on cumulative yield attributes, survivorship, and net production

Conceptually, competitive interactions among adjacent individual trees within even-aged monospecific stands are among the principal determinates underlying stand development and associated structural dynamics. In essence, the objective of stand density management is to achieve a desired stand-level objective via the manipulation of these competitive interactions and their effects via informed silvicultural interventions at the time of establishment (e.g., IE treatments) or during the later stages of stand development (e.g., PCT and CT treatments). Acknowledgement and appreciation of the type, intensity, and resultant consequences of these competitive interactions in terms of relative growth rate distributions, stand structural dynamics, and self-thinning patterns are essential crop planning prerequisites, particularly when using SDMDs.

Ecologically, resource competition relationships among individual trees have been described as a continuum ranging from a resource depletion to a resource pre-emption process (e.g., Newton and Jolliffe 2003). The resource depletion process is a symmetrical relationship in which individuals share the finite environmental

resources in direct proportion to their relative size. Conversely, the resource pre-emption process is an asymmetrical relationship in which larger-sized individuals obtain a disproportional relative share of resources at the expense of smaller-sized individuals. Experimental inferences suggest that the resource depletion process involves competition for belowground resources (soil nutrients and (or) moisture), whereas the resource pre-emption process involves competition for aboveground resources (solar radiation) (sensu Schwinning and Weiner 1998). These processes have differential effects on the distribution of individual-tree relative growth rates and survivorship patterns, which yields diverse stand structural changes as site occupancy increases. The resource depletion process may dominate the early stand development stages preceding crown closure during which all individuals passively acquire an equal relative-size-based share of the available belowground resources, resulting in greater uniformity in relative growth rates and hence lower size inequality at increasing levels of site occupancy. Conversely, the resource pre-emption process may dominate during the post-crown-closure stages during which larger competitors passively acquire a greater relative-size-based share of the solar resources at the expense of the smaller individuals, resulting in increased size-dependent relative growth rate variation. This eventually leads to increased rates of self-thinning and size structure stratification as site occupancy increases. Additionally, interactions may also involve the physical competition for space, where wind-induced crown abrasion among neighbouring trees leads to branch pruning and associated loss of photosynthetic biomass, with the greatest impacts rendered on the smallest trees (sensu Rudnicki et al. 2001). This type of interaction may produce mortality patterns analogous to those observed for an asymmetrical dominance–suppression competition relationship (sensu Newton 2006a). These competition processes are not mutually exclusive and may operate in unison during different stages of stand development, leading to a complex and temporal varying pattern of interactions. Essentially, stand density management attempts to regulate the temporal occurrence and intensity of these symmetrical and asymmetrical competitive interactions principally through the simultaneous control of individual-tree growing space (e.g., increasing the temporal period of the free-to-growth status of potential crop trees by reducing the likelihood of competitive interactions through reductions in population density stress levels and structuring spatial configurations of local neighbourhoods accordingly, via IE and PCT treatments).

Quantifying the consequences of these competitive interactions by measuring the species-specific responses of aboveground biomass yield to increasing population density stress levels across a wide spectrum of site conditions within even-aged monospecific plant populations at various stages of development was the central focus of a group of theoretical and experimental plant ecologists and forest scientists throughout Japan, Australia, and Great Britain during the 1950s and 1960s (e.g., Donald 1951; Kira et al. 1953; Hozumi et al. 1956; Shinozaki and Kira 1956, 1961; Koyama and Kira 1956; Bleasdale and Nelder 1960; Holliday 1960; Yoda et al. 1963). Collectively, the theoretical constructs, empirical results, and derived generalized relationships, produced from these pioneering studies along with consequential inferences attained from a series of site occupancy – forest productivity studies (e.g., Reineke 1933; Langsaeter 1941; Wilson 1946; Möller 1954), provided the ecological and analytical foundation for the development of the SDMD.

More specifically, the collective effect of increased symmetric and asymmetric sharing of the finite resources among individual trees arising from increased site occupancy for a given species, site quality, and stage of development is the depression of relative growth rates among individuals of all sizes. This results in a reduction in overall population yields and associated mean size with increasing levels of density stress. Kira et al. (1953) consistently observed such a size–density response through extensive plant experimentation involving a broad range of species and plant types (e.g., vegetable crop to forest tree species). They initially quantified the relationship between mean biomass per plant ($\bar{b}$) and total biomass per unit area (B), and population density (number of plants per unit area; N), at a given temporal stage of development (time since establishment), as a set of power functions: $\bar{b} = \alpha_1 N^{-\alpha_2}$ and $B = \alpha_1 N^{-\alpha_2 + 1}$ where α_1 is a constant specific to developmental stage that empirically varies by species, and α_2 is a varying power exponent ranging in value from zero to unity. Hozumi et al. (1956) later extracted from the latter relationship, the law of final constant yield, which states that as time approaches infinity, the power exponent approaches unity and hence total biomass per unit area approaches a constant asymptotic value (B_a) (i.e., $B \to \alpha_1$ as $\alpha_2 \to 1$, yielding B_a). Deploying logistic growth theory and the law of final constant yield, Shinozaki and Kira (1956) provided the conceptual foundation and associated functional specification for these reciprocal relationships. Specifically, the reciprocal equations of the competition–density (C–D) and yield–density (Y–D) effects: $\bar{b} = \alpha_3 + \alpha_4 N^{-1}$ and $B = N(\alpha_3 + \alpha_4 N)^{-1}$, respectively, where α_3 and α_4 are parameters specific to development stage, which empirically varies by species.

The reciprocal equations can be used to quantify the size–density relationship for a given species-specific population at any developmental stage. Thus, by utilizing mean dominant height as a developmental index given that it accounts for both age and site quality effects simultaneously, the reciprocal equations of the C–D and Y–D effects have functional applicability throughout a species size–density space. Analytically, this space is delineated by a lower size–density threshold relationship, which defines the beginning curve of competition (Ando 1962) or point of crown closure (Drew and Flewelling 1979), and an upper size–density threshold, which defines the asymptotic size–density condition, known as the full density curve (Ando 1962) or −3/2 power law of self-thinning (Yoda et al. 1963). These relationships were key to the development of the SDMD in terms of enabling the site-specific prediction of the size–density trajectory as it transitions throughout size–density space (e.g., the graphical progression of a given site-specific trajectory as it proceeds upwards within the SDMD is governed by the height-specific reciprocal equation of the C–D effect (dominant height isolines) as shown in Fig. 1).

The consequence of initial size differences among individuals growing within even-aged plant monospecific populations due to genetic, microsite, and temporal establishment variation combined with asymmetric sharing of finite aboveground resources among individual trees as site occupancy increases is the size-dependent differentiation of relative growth rates among individuals. As the intensity of both the resource depletion and pre-emption competitive interactions increase as populations attain greater degrees of site occupancy, the process of density-dependent mortality or self-thinning initiates within the smaller size classes. This leads to increased size inequality at increasing levels of density stress. As site occupancy approaches a species-specific maximum, a consistent inverse and non-linear size–density relationship emerges between mean biomass per surviving plant and survivor density. Experimentally, Kira et al. (1953) and Shinozaki and Kira (1956) were among the first to observe and quantify such a mean size–density relationship, which was later formalized as the −3/2 power law of self-thinning (self-thinning rule) by Yoda et al. (1963): $\bar{b} = \alpha_5 N^{-3/2}$ where α_5 is a species-specific constant of proportionality.

Yoda et al. (1963) demonstrated the general applicability of the relationship using experimental observations from a large number of diverse plant populations, ranging from cereal crops to forest tree species. To provide a conceptual foundation for this relationship, Yoda et al. (1963) also proposed a geometric-based derivation for explaining the asymptotic size–density relationship within plant populations incurring density-dependent mortality. Specifically, assuming that "self-thinning occurs only when the total coverage of a stand exceeds 100% and so operates

as to maintain 100% cover" and "plants of the same species are always geometrically similar in shape irrespective of growth stage or habitat conditions", the following formulation was proposed: (1) mean biomass per surviving plant is directly proportional to the cube of some unspecified linear plant size dimension (l), $\bar{b} \propto l^3$; (2) mean 2D spatial area occupied by a plant ($\bar{s}$) is directly proportional to the square of this linear dimension, $\bar{s} \propto l^2$, and inversely proportional to density, $\bar{s} \propto N^{-1}$; and (3) integrating the relationships in 1 and 2, $\bar{s} \propto l^2 \propto (l^3)^{2/3} \propto \bar{b}^{2/3} \propto N^{-1}$, yields an inverse non-linear mean biomass – density relationship, $\bar{b} \propto N^{-3/2}$, from which the asymptotic size–density relationship for self-thinning plant populations is obtained via the inclusion of a species-specific constant of proportionality. Although considerable scientific debate regarding the site independence of the intercept term (proportionality constant), constancy and species specificity of the self-thinning exponent, applicability of the underlying geometric model, and parameterization analytics in terms of data selection protocols and statistical approaches utilized has occurred since its formularization by Yoda et al. (1963) (e.g., Weller 1987a, 1987b; Zeide 1987; Newton and Smith 1990; Bi 2004; Pretzsch 2006; Newton 2006b), the relationship is generally regarded as an empirically validated universal theoretical construct in plant ecology (sensu Harper 1977).

Empirical results from a series of density control forestry experiments also yielded a valuable set of theoretical constructs regarding the relationship between site productivity (net production) and stand density. These constructs have informed stand density management decision-making in forestry since its inception (sensu Zeide 2001). To briefly summarize, for a given species, site quality and stage of development, net production as reflected by a surrogate measure, that is, stem volume increment per unit area per year, will increase with increasing site occupancy until a horizontal asymptotic tangential plateau is attained. This plateau, which has generalized as either narrow (sensu Assmann 1970) or wide (sensu Möller 1954) in terms of its tangential range, would approximately correspond to the size–density condition at which the law of final constant yield applies and hence before consequential density-dependent mortality occurs (i.e., self-thinning). Thus, net production remains constant, and only individual tree size varies (inversely with density stress) across this asymptotic site occupancy range. Increases in occupancy past this point result in declining net production due to reduced individual tree growth rates and increased rates of density-dependent mortality. Thus, regulating site occupancy via density manipulation so that a stand remains within this optimal asymptotic plateau allows for the (1) maximization of the duration of full occupancy status over a given rotation and thus maximizing stand-level volumetric yields and (2) maximum control of individual mean tree size and associated size-dependent end-product potentials (e.g., piece size).

Integrating these theoretical and empirical constructs within a unified size–density analytical framework for forest tree species yielded a suite of innovative crop planning tools, setting the stage for informed, objective, and ecologically sound stand density management decision-making. Specifically, initially denoted as density control diagrams when first introduced by Japanese scientists in the early 1960s (refer to fig. 1 in Newton (1997a) for a graphical illustration of the first static SDMD developed by Ando (1962)), SDMDs rapidly achieved considerable operational currency in stand-level management planning and silvicultural decision-making (e.g., Ando 1968). Later, during the late 1970s, Drew and Flewelling (1979) provided the conceptual linkage to the forest production constructs espoused by Langsaeter (1941) and Möller (1954) within the context of SDMDs. Similar in concept to Ando's (1962) relative yield index and Tadaki's (1964) management base line index, Drew and Flewelling (1979) also introduced a relative density index (P_r) for quantifying site occupancy (i.e., the ratio of the observed stand density to the maximum stand density attainable in a stand with the same mean tree volume). They deployed this index and an empirical-based regression relationship in place of the reciprocal equation of the C–D effect along with other Y–D relationships, including those defining the ZICM, to develop the first static SDMD for coastal Douglas-fir plantations. Furthermore, they delineated relative density levels that corresponded to key phases of stand development from which four key density management tenets were derived (refer to fig. 2 in Newton (1997a) for a graphical illustration): (1) stands with a P_r of <0.15 (crown closure) were not fully utilizing the site, and hence densities could be increased without decreasing mean tree growth; (2) stands managed at a P_r of 0.15 would yield trees of maximum size; (3) stands managed at a P_r of 0.40–0.55, corresponding to the optimal asymptotic net production plateau, would yield maximum total volumes per unit area and enable control of individual tree size without affecting overall volumetric yields; and (4) a P_r should not exceed 0.55 except immediately prior to harvest to maximize total net production and avoid extensive self-thinning mortality. Conceptually, the incorporation of the functional relationships (i.e., reciprocal equations of the C–D and Y–D effects (Shinozaki and Kira 1956) and the self-thinning rule (Yoda et al. 1963)) along with the forest production theoretical linkage espoused by Drew and Flewelling (1977, 1979) collectively affords the SDMD its ecological foundation.

2.2. Recognizing the historical evolution of the SDMD modelling approach: from static (1962) to dynamic (1993) to structural (2009) SDMD variants

The sequential evolution of the SDMD modelling approach, which has been characterized by the continuous introduction of increased analytical complexity, has led to three generalized SDMD model types or variants: 2D (size–density) static SDMDs, 3D (size–density–time) dynamic SDMDs, and n-dimensional (size–density–time–distributional) structural SDMDs. Categorizing the model-types within the hierarchical system used to classify growth and yields models (sensu Porté and Bartelink 2002), the static and dynamic variants would fall within the stand-level distance-independent average tree yield model class, whereas the structural variant could be considered a hybrid stand-level distance-independent average tree and size-distribution yield model. More precisely, the static SDMDs, which are analogous to traditional Reineke-based stocking charts, graphically illustrate the interrelationships between average tree size (mean stem volume, quadratic mean diameter, and mean dominant height) and stand density (stems per unit area) at various stages of stand development. For any specified size–density coordinate within a species' biologically defined 2D size–density space, mean tree metrics and stand-level volumetric yield estimates can be extracted. This graphical tool can be quite useful in forecasting size–density developmental pathways before the commencement of density-dependent mortality (e.g., predicting the time of crown closure for a given site and establishment density). Likewise, they can be used to quantify density stress levels within surveyed stands by positioning them within their size–density space. Such information can be used to determine if thinning treatments are required to avoid consequential mortality (e.g., CT within density-stressed stands about to incur mortality (below the ZICM) to capture expected merchantable volume losses). Although the dynamic SDMD is similar in structure to the static SDMD, it includes a site-driven mortality submodel that enables the prediction of the temporal size–density trajectory of a given stand throughout the entire size–density space, inclusive of stages of development in which density-dependent mortality is expected to occur (e.g., ZICM). The ability of the dynamic SDMD to predict the site-specific size–density trajectory and associated yield metrics for a given stand from establishment to rotation provides the decision-support foundation for evaluating the likelihood of a given crop plan of attaining a specified volumetric-based objective.

The structural SDMD retains all the analytical components of the dynamic SDMD but also includes size-distribution recovery models that enable the extraction of the underlying diameter and height distributions at any point in a stand's developmental trajectory. Given these distributions along with other stand-level variables, a multitude of other volumetric, end-product, and ecological-relevant metrics can be derived via the employment of composite allometric-based relationships that simultaneously account for both individual (tree size) and population (density stress and developmental stage) effects. For example, the hierarchical-based structural SDMD introduced by Newton (2009) consisted of six sequentially linked estimation modules: module A, dynamic SDMD; module B, diameter and height recovery; module C, taper analysis and log estimation; module D, biomass and carbon estimation; module E, product and value estimation; and module F, fibre attribute estimation. A schematic illustration of this structural variant inclusive of the interrelationships and generalized sequential flow of computations among the individual modules is summarized in Fig. 2. Currently, such models have been developed for a number of even-aged stand types, including monospecific natural-origin and managed (plantations) jack pine stands (Newton 2009, 2019), mixed natural-origin upland black spruce and jack pine stands (Newton 2012a), monospecific natural-origin lowland black spruce stands (Newton 2012b), and monospecific natural-origin and managed (plantations) upland black spruce stands (Newton 2012c, 2019). Additionally, a subset of climate-sensitive model variants have been developed for the black spruce and jack pine plantation types (Newton 2016a). All of these structural models were calibrated using Ontario-centric data sets derived from a multitude of boreal-based PSP systems and density control experiments, originally established by various forest sector corporations and governmental agencies during the last century (see Newton (2009, 2012a, 2012b, 2012c) for details or Table A1 (dependent data sets) for a generalized listing).

Analytically, module A is formulated deploying the traditional dynamic SDMD modelling framework (sensu Newton 1997a) and hence includes a broad array of stand-type-specific static and dynamic yield–density relationships, including the self-thinning rule (Newton 2006a). A biophysical site-specific height–age function for accounting for climate change effects (Sharma et al. 2015) and a set of submodels to address genetic worth and thinning growth responses (Newton 2015a and 2015b, respectively) are also incorporated. Module B consists of a stand-type-specific parameter prediction equation system for diameter distribution recovery (sensu Hyink and Moser 1983). This system was parameterized using the cumulative density function regression approach in which the location, shape, and scale parameters of the Weibull (1951) probability density function that is used to characterize the diameter frequency distribution are expressed as a function of stand-level variables (Newton and Amponsah 2005). Additionally, stand-type-specific composite allometric-based prediction equations for estimating diameter-class heights (Newton and Amponsah 2007) are also embedded in module B. Module C deploys species-specific non-linear mixed-effects dimensional compatible taper equations (Sharma and Zhang 2004; Sharma and Parton 2009) to predict stem product yields (number of pulplogs and sawlogs) and total and merchantable stem volumes at the individual tree, diameter class, and stand levels. Module D employs species-specific composite allometric-based biomass equations (Newton 2006b, 2009) to generate estimates of total and component-specific (bark, stem, branch, and foliage) aboveground biomasses and associated carbon-based mass equivalents at the individual tree, diameter class, and stand levels. Module E utilizes species- and sawmill-specific (stud and random-length mill) product and value equations, parameterized using Optitek sawing simulator (Forintek Canada Corp 1994) output (Liu and Zhang 2005; Zhang et al. 2006; Newton 2009), to generate estimates of the recoverable chip and lumber volumes and associated monetary values at the individual

tree, diameter class, and stand levels. Module F deploys species-specific (1) composite functions to generate estimates of mean whole-stem wood density and mean maximum branch diameter within the first-order sawlog (Newton 2009, 2012a, 2012b, 2012c) and (2) hierarchical mixed-effect attribute prediction models (Newton 2016b, 2019) to generate rotational end-product-related fibre quality attribute estimates (i.e., mean ring-area-weighted breast-height values of wood density, microfibril angle, modulus of elasticity, fibre coarseness, tracheid wall thickness, tracheid radial diameter, tracheid tangential diameter, and specific surface area) at the individual tree, diameter class, and stand levels. This analytical structure enabled the prediction of a multitude of annual and rotational metrics related to volumetric productivity, biomass and carbon outcomes, log product distributions, sawmill volumetric yields, and end-product potential, thus providing the prerequisite information for evaluating and comparing crop plans in terms of their ability to realize a specified volumetric, end-product, and (or) ecological-based stand-level management objective. Furthermore, when augmented by the inclusion of biophysical site index functions, the structural SDMD provides the functionality to crop plan under various climate change scenarios.
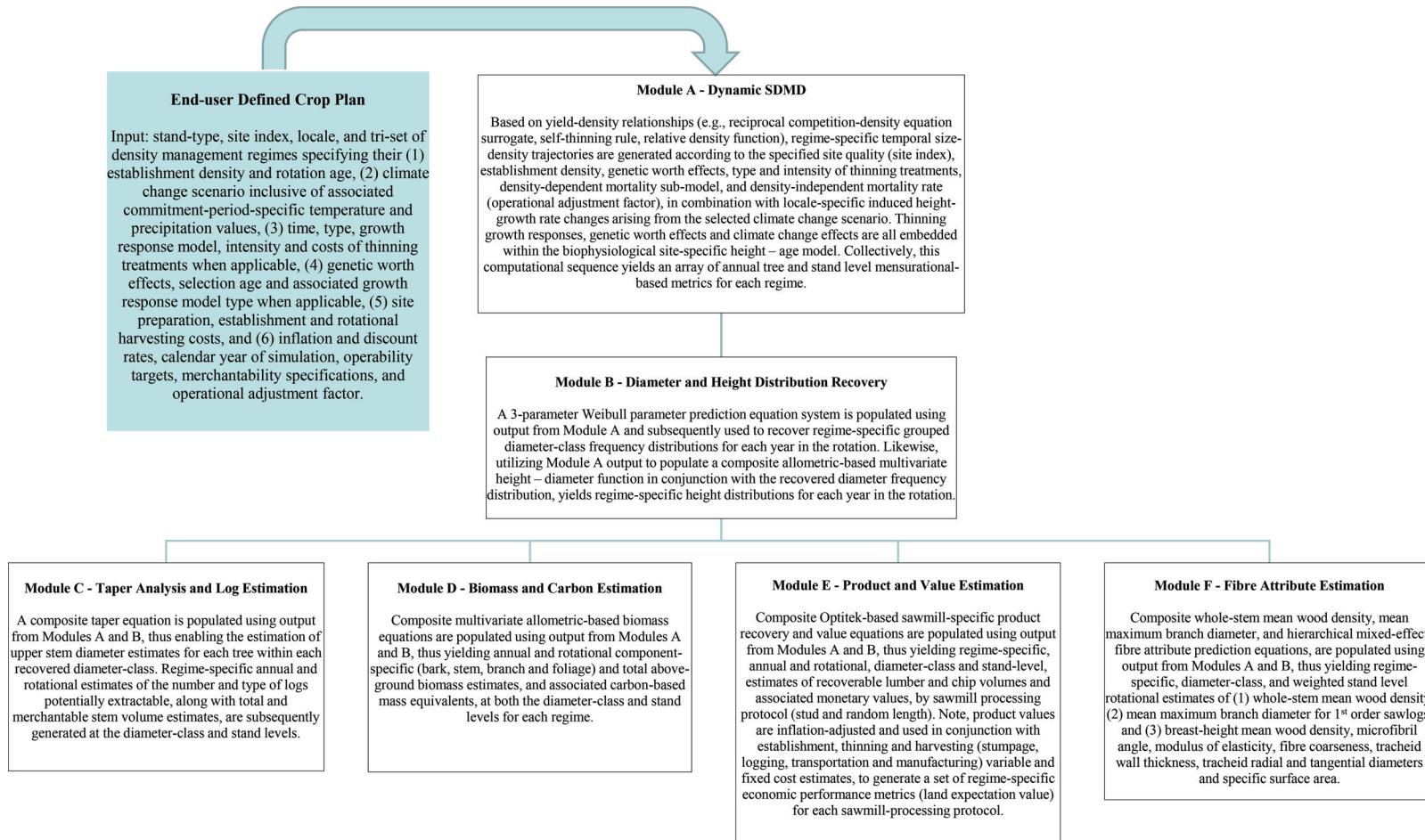
Although structural SDMD modelling activity has been largely concentrated in Canada, it is noteworthy to appreciate the recent innovative efforts in Europe. Specifically, deploying an alternative analytical approach with regard to the core dynamic elements of the model in which Hagihara's (1999, 2000) generalized C–D concept is deployed within a state-space whole-stand unified modelling framework, along with recoverable two-parameter Weibull-based diameter distribution submodels, as well as height-diameter functions, Stankova and Diéguez-Aranda (2020) presented dynamic structural SDMD variants for plantations of radiata pine and Scots pine and natural-origin stands of downy birch and English oak. The utility of these resultant dynamic structural SDMDs within the context of managing abiotic mass accumulations and evaluating established regulatory-based IE and thinning practices in reference to structural yield outcomes was demonstrated. Although not currently as comprehensive nor reflective of anticipated climate change impacts as the boreal variants, these analytical efforts afford modellers with an alternative and somewhat more unified approach with respect to the primary Y–D relationships (reciprocal Y–D and self-thinning relationships) for developing structural SDMDs.

In summary, the SDMD modelling approach has demonstrated considerable temporal resilience and analytical robustness as it has evolved over the last 60 years, particularly in terms of its diversity, complexity, and utility. As exemplified by the static, dynamic, and structural variants, the ecological, quantitative universality and allometric underpinnings of the SDMD modelling approach have enabled innovative analytical advancements through model modification and expansion. These efforts have yielded a wide array of comprehensive decision-support systems for use in addressing a wide spectrum of resource management objectives across the globe.

2.3. Appreciating ecological compliance and predictive performance expectations of SDMDs

The ecological basis, analytical foundation, and operational utility of the static, dynamic, and structural SDMD variants have been presented in a series of foundational articles (e.g., Ando 1962, 1968; Drew and Flewelling 1977, 1979; Newton 2009; Stankova and Diéguez-Aranda 2020) and comprehensive reviews (e.g., Drew and Flewelling 1977; Jack and Long 1996; Newton 1997a). These contributions have highlighted their ecological lineage, geographical adoption pattern, diversity of model forms and associated parameterization approaches, computational analytics, and operational utility. For example, these accounts have stressed the importance of the reciprocal equations of the C–D and Y–D effects as formally specified by Shinozaki and Kira (1956) and later generalized by

Fig. 2. Schematic illustration of the modular-based structural SDMD for boreal conifers (Newton 2009, 2012*a*, 2012*b*, 2012*c*, 2016*a*, 2019).

End-user Defined Crop Plan

Input: stand-type, site index, locale, and tri-set of density management regimes specifying their (1) establishment density and rotation age, (2) climate change scenario inclusive of associated commitment-period-specific temperature and precipitation values, (3) time, type, growth response model, intensity and costs of thinning treatments when applicable, (4) genetic worth effects, selection age and associated growth response model type when applicable, (5) site preparation, establishment and rotational harvesting costs, and (6) inflation and discount rates, calendar year of simulation, operability targets, merchantability specifications, and operational adjustment factor.

Module A - Dynamic SDMD

Based on yield-density relationships (e.g., reciprocal competition-density equation surrogate, self-thinning rule, relative density function), regime-specific temporal size-density trajectories are generated according to the specified site quality (site index), establishment density, genetic worth effects, type and intensity of thinning treatments, density-dependent mortality sub-model, and density-independent mortality rate (operational adjustment factor), in combination with locale-specific induced height-growth rate changes arising from the selected climate change scenario. Thinning growth responses, genetic worth effects and climate change effects are all embedded within the biophysiological site-specific height – age model. Collectively, this computational sequence yields an array of annual tree and stand level mensurational-based metrics for each regime.

Module B - Diameter and Height Distribution Recovery

A 3-parameter Weibull parameter prediction equation system is populated using output from Module A and subsequently used to recover regime-specific grouped diameter-class frequency distributions for each year in the rotation. Likewise, utilizing Module A output to populate a composite allometric-based multivariate height – diameter function in conjunction with the recovered diameter frequency distribution, yields regime-specific height distributions for each year in the rotation.

Module C - Taper Analysis and Log Estimation

A composite taper equation is populated using output from Modules A and B, thus enabling the estimation of upper stem diameter estimates for each tree within each recovered diameter-class. Regime-specific annual and rotational estimates of the number and type of logs potentially extractable, along with total and merchantable stem volume estimates, are subsequently generated at the diameter-class and stand levels.

Module D - Biomass and Carbon Estimation

Composite multivariate allometric-based biomass equations are populated using output from Modules A and B, thus yielding annual and rotational component-specific (bark, stem, branch and foliage) and total above-ground biomass estimates, and associated carbon-based mass equivalents, at both the diameter-class and stand levels for each regime.

Module E - Product and Value Estimation

Composite Optitek-based sawmill-specific product recovery and value equations are populated using output from Modules A and B, thus yielding regime-specific, annual and rotational, diameter-class and stand-level, estimates of recoverable lumber and chip volumes and associated monetary values, by sawmill processing protocol (stud and random length). Note, product values are inflation-adjusted and used in conjunction with establishment, thinning and harvesting (stumpage, logging, transportation and manufacturing) variable and fixed cost estimates, to generate a set of regime-specific economic performance metrics (land expectation value) for each sawmill-processing protocol.

Module F - Fibre Attribute Estimation

Composite whole-stem mean wood density, mean maximum branch diameter, and hierarchical mixed-effects fibre attribute prediction equations, are populated using output from Modules A and B, thus yielding regime-specific, diameter-class, and weighted stand level rotational estimates of (1) whole-stem mean wood density, (2) mean maximum branch diameter for 1st order sawlogs, and (3) breast-height mean wood density, microfibril angle, modulus of elasticity, fibre coarseness, tracheid wall thickness, tracheid radial and tangential diameters and specific surface area.

Hagihara (1999, 2000), the −3/2 power law for self-thinning as established by Yoda et al. (1963), and the linkages to forest production theory (Drew and Flewelling 1977, 1979). The historical evolution of the static and dynamic SDMDs and their adoption by forest managers throughout temperate and boreal forest regions have also been extensively summarized (e.g., from Japan and chronologically onwards to South Korea, New Zealand, United States of America (USA), Canada, Europe, and China). The diversity of the analytical approaches and resultant graphical variants, ranging from the widely adopted (Asia, Canada, and Europe) mean volume – density model structure utilizing the volume-based relative density indices proposed by Ando (1962; relative yield index) or Drew and Flewelling (1979; relative density index) to the somewhat more narrowly adopted (USA) quadratic mean diameter – density structure deploying Reineke's (1933) stand density index as proposed by McCarter and Long (1986) (refer to fig. 3 in Newton (1997a) for a graphical illustration of this variant), have been presented. A wide range of applications of these SDMD types in terms of addressing diverse resource management objectives have also been exemplified. Combined with species-specific monograph-like contributions documenting analytical developments within each of the SDMD modelling platforms, these efforts have consequentially advanced SDMD analytics and provided the prerequisite methodology for model replication. Representative contributions for the traditional volume–density-structured SDMDs initially introduced by Ando (1962) would include those of Drew and Flewelling (1979) and Smith and Woods (1997) for static SDMDs, Newton and Weetman (1993) and Stankova and Shibuya (2007) for dynamic SDMDs, and Newton (2009) and Stankova and Diéguez-Aranda (2020) for structural SDMDs.

The deployment of known forest dynamic axioms within the generic SDMD structure provides a pathway for their validation upon parameterization. Collectively assessing temporal stand development patterns and associated yield attributes derived from multiple SDMD crop plan simulations across a wide range of initial establishment densities, rotational ages, and site qualities, expectations for acceptance of a given SDMD have included predictive compliance with the Sukatsckew effect (Harper 1977), Eichhorn's rule (1902, as cited by Pretzsch (2009)), and site-invariant yield–density allometric relationships and self-thinning patterns. Employment of stand property graphics such as modified versions of the Bakuzis matrix (sensu Leary 1997) can provide an inferential framework for evaluating such expectations. Newton (2015c) demonstrated the utility of this approach in an ecological-orientated evaluation of the core dynamic elements of the structural SDMD models developed for coniferous stand types. The results of this evaluation suggested concurrence with the expected Sukatsckew effect (i.e., temporal rate of density-dependent mortality or self-thinning increases with increasing site quality) and Eichhorn's rule (i.e., total stand volume for a given mean stand height is constant across a range of site classes). The only departure from expectation among the 10 relationships examined was the presence of a possible site-quality effect on the asymptotic size–density relationships. Essentially, the simulations revealed that stands grown on the higher quality sites could tolerate a greater degree of density stress before self-thinning than stands grown on the lower quality sites, suggesting that asymptotic site occupancy may vary directly with site quality. Consequently, the position of the self-thinning line within the SDMD graphic for a given species may not be stationary across all site qualities but may, in fact, vary directly with site index. Bi (2004) has also observed such a site effect on the asymptotic size–density relationship and proposed an alternative specification accordingly. Nevertheless, notwithstanding the potential site dependence of the asymptotic size–density relationship, the main conclusion drawn from this comprehensive assessment was that size–density trajectories and associated yield–density relationships generated by the structural SDMDs were consistent with the underlying ecological expectations espoused within the SDMD modelling approach.

Quantifying the empirical predictive accuracy of SDMDs is also an essential prerequisite to model acceptance and deployment, particularly when used as a crop planning tool in operational forest management. Although such assessments deploying strictly independent validation data sets are preferable, the limited availability of such data has frequently redirected efforts towards the utilization of a mixture of dependent, partially dependent, and independent validation data sets (e.g., Newton 2003b; Stankova and Diéguez-Aranda 2017, 2020). Results from these validation efforts have yielded valuable insights that have been useful in subsequent modelling efforts. For example, Newton (2003b) assessed the prediction accuracy of the principal relationships within a dynamic SDMD developed for black spruce plantations of central and eastern Canada (Newton and Weetman 1994), using partially dependent and independent data sets derived from plantation-based density control experiments. Results indicated inadequacies when projecting postthinning stand development. The magnitude of prediction errors were inferred to be related to (1) treatment-induced variation in residual stand structure (e.g., spatial pattern clustering and size-frequency distribution truncation) among the test populations and (2) not analytically accounting for the (i) growth response delay that occurs immediately following thinning as the residual crop trees rebuild their crowns and attempt to fully occupy their newly allocated growing space and (ii) growth rate increases arising from the increased availability of site resources following thinning. These latter controllable sources of error were used to inform subsequent model development when constructing the structural SDMD variants for the coniferous stand types. Specifically, crown-occupancy-based and density-repression release functions were developed to account for these effects: (1) delaying the implementation of density-dependent mortality functions following thinning until the residual crop trees were able to fully occupy their newly acquired space as inferred from the recovery of their live crown ratios (Newton 2012c) and (2) embedding a relative height growth modifier within the site-specific dominant height – age function to account for thinning-induced growth increases arising from the relaxation of density-dependent height repression effects (Newton 2015b).

Furthermore, with respect to the structural SDMDs developed for conifers, their comprehensiveness in terms of the diversity of output variables produced, the likelihood of acquiring adequate testing data sets from either inventory-based censuses (e.g., PSP systems) or density control experiments for whole-model evaluations, is minimal. Alternatively, however, focusing on the prediction accuracy of the temporal mean volume – density trajectories and associated mean tree-level metrics (e.g., dominant height and quadratic mean diameter) and stand-level variables (e.g., basal area, total volume, and relative density) can be constructive given their consequential role as predictor variables within the downstream modules (e.g., Weibull-based parameter prediction equation systems, composite height–diameter, taper, biomass and end-product volume functions, and the hierarchical fibre attribute equations, are all dependent on such predictors (Fig. 2)). To demonstrate such an approach and provide a suite of empirical performance measures for the structural SDMD variants, the predictive ability of the principal relationships within module A were examined (refer to Appendix A for a detailed summary of the data sets and analytical approach utilized).

Briefly, deploying a combination of dependent, partially dependent, and independent Ontario-centric PSPs and experimental data sets inclusive of stands that were subjected to IE, PCT, and CT treatments indicated that the mean percentage prediction errors for mean dominant height, quadratic mean diameter, basal area, total volume, density, and relative density were realizing or approaching the ±20% acceptance threshold suggested by

Huang et al. (2003) for operational growth and yield models (refer to Appendix A for stand-type-specific mean relative errors). Although these mean error metrics provide a useful measure of model prediction performance, the validation data sets were derived from repeatedly remeasured and geographically clustered sample plots that varied in their dependence with respect to the model calibration data sets (e.g., dependent, partially dependent, or independent; Appendix A). Thus, they may not be fully reflective nor independent of the populations being modelled; hence, the magnitude of the mean error metrics should be considered as an initial approximation.

Stankova and Diéguez-Aranda (2020) also evaluated the predictive ability of their dynamic structural SDMDs, developed using a state-space modelling approach, for natural-origin English oak and downy birch stand types and plantation radiata pine and Scots pine stand types in Europe. Results for quadratic mean diameter, density, total volume, and biomass projections revealed that with a 99% probability, 75% of their relative errors for all four variates would be within ±20% of their true values for three of the stand types (radiata pine, English oak, and downy birch) and within ±40% for the Scots pine plantations. A post-assessment evaluation suggested that the predictive ability of the Scot pine model was largely undermined by the magnitude of the errors arising from the mean dominant height projections.

More generally, similar to most stand-level growth and yield models used in stand-level management planning, the site-specific mean dominant height – age function is commonly deployed as the principal temporal driver of stand development within dynamic and structural SDMD models (e.g., main determinate of the rate of progression of a given size–density trajectory through the species-specific size–density space as illustrated within the traditional SDMD graphic (Fig. 1)). Furthermore, as with other growth and yield projection systems, this function is also used to incorporate genetic worth, thinning, and climate change effects on forest productivity, within the SDMD model variants (e.g., Newton 2015a, 2015b, and 2016a, respectively). Thus, departures between observed and expected height growth patterns will have consequential effects on the precision of all downstream module-based predictions and thus affect the accuracy of the derived individual tree, diameter class, and stand level performance metrics.

Although departures arising from model inadequacies or misspecification can be analytically improved through model refinement, the consequences arising from uncertainty pertaining to climate change effects on future growing environments are among the most concerning to both modellers and resource management decision-makers. For the climate-sensitive structural SDMD variants that attempt to account for climate change effects, identifying which climate change scenario is the most plausible over the long term is inherently difficult given unknowns with respect to future mitigation efforts. Consequently, given that this is a known source of variation that is largely uncontrollable, it is advisable for stand density management decision-makers to evaluate their chosen crop plan across a plausible range of climate change scenarios. This would provide a range of outcomes from which the likelihood of realizing the stated objective could be ascertained. Additionally, density management decision-makers should be prepared to adjust their crop plan in real time if mitigation policies change. Furthermore, climate change may also illicit changes in competition relationships as moisture and nutrient cycles are impacted. For example, localized increases in drought incidence could potentially lead to accelerated growth reductions and eventual mortality of trees of all sizes given the symmetrical nature of moisture deficiency effects. From a modelling perspective with respect to the climate-sensitive structural SDMD variants for black spruce and jack pine plantations (e.g., Newton 2016a), the operational adjustment factor that is used to address density-independent mortality could be adjusted accordingly to reflect this expected mortality rate increase.

3. Exemplification of the potential utility of SDMDs within the context of anthropogenic climate change

The climate-sensitive structural SDMD variant for the managed jack pine stand type is used to exemplify the potential utility of such models in localized crop planning under climate change. Briefly, this climate-sensitive SDMD variant was developed through the incorporation of the biophysical site-specific height–age models developed by Sharma et al. (2015) within module A, dynamic SDMD (sensu Fig. 2; Newton 2016a). Essentially, the species (jack pine and black spruce) and stand-type (plantation) specific site-based height–age functions, initially used in the original structural SDMD variants, were replaced by their biophysical-based equivalents. Analytically, the biophysical model incorporates precipitation (mean total precipitation (mm) during the growing season) and temperature (mean temperature (°C) during the growing season) variables into the site-specific height–age model to explicitly account for changing growing conditions on forest productivity (Sharma et al. 2015). The actual predicted future values of these climate-based variables for a given geographic location (longitude and latitude coordinates), climate change scenario (1970–2000 climate normals and Representative Concentration Pathways (RCP2.6 and RCP4.5; IPCC 2014)), and commitment period (2010–2040, 2041–2070, and 2071–2100) were derived from a set of external models: the second-generation Canadian Earth System Model (CanESM2; Environment and Climate Change Canada 2020) in combination with a georeferencing regional spatial climatic model (McKenney et al. 2006). Conceptually, this approach is similar to the response modelling methodology used to adjust various tree and stand models to quantify silvicultural treatment effects (sensu Weiskittel et al. 2011), that is, modifying existing untreated prediction models (site productivity functions) that are the principal determinates underlying temporal change (size–density trajectories) through the addition of rate parameter modifiers (precipitation and temperature) to reflect changing temporal dynamics arising from anthropogenic influences (climate change).

Procedurally, crop plans involving IE and CT treatments at two geographically distinct locations, northwestern (Thunder Bay) and northeastern (Kirkland Lake) Ontario, were simulated. Specifically, the climate-sensitive structural SDMD variant for jack pine plantations was georeferenced to these locales and subsequently used to generate rotational-based volumetric yield, log product distributions, biomass and carbon outcomes, end-product volumes, fibre quality indicators, operability efficiency, stand stability, duration of optimal site occupancy, and economic performance metrics under varying climate change scenarios (climate normals and RCPs). In general, current climate change mitigation policies (actual and pledged) suggest that the plausible range of RCPs would be between approximately 1.9 and 6.0 (sensu Hausfather and Peters 2020), that is, current expectations between shared socioeconomic pathways (SSPs) and analogous RCPs of slightly less than SSP4 and RCP6 (3.0 °C increase in global mean temperature by 2100) and slightly less than SSP2 and RCP4.5 (2.5 °C increase in global mean temperature by 2100) under weak to modest mitigation efforts, respectively. Consequently, RCP2.6 and RCP4.5 were selected for these simulations.

Firstly, the IE crop plan at the northwestern boreal Ontario location deploying an initial planting density of 2500 seedlings·ha^{-1} on a medium- to good-quality site (18 m at 50 years) over a 75-year rotation (2020–2095) was simulated for the climate normal condition and for the RCP2.6 and RCP4.5 scenarios. Secondly, at the northeastern boreal Ontario location, crop plans involving both IE and CT treatments for the same initial density, site quality, and rotation length, but growing only under the RCP4.5 scenario, were simulated. Collectively, these simulations allowed the demonstration and comparison of (1) regional-specific climate change effects for a commonly deployed IE-based crop plan growing across a range of scenarios, (2) regional-specific climate change effects for
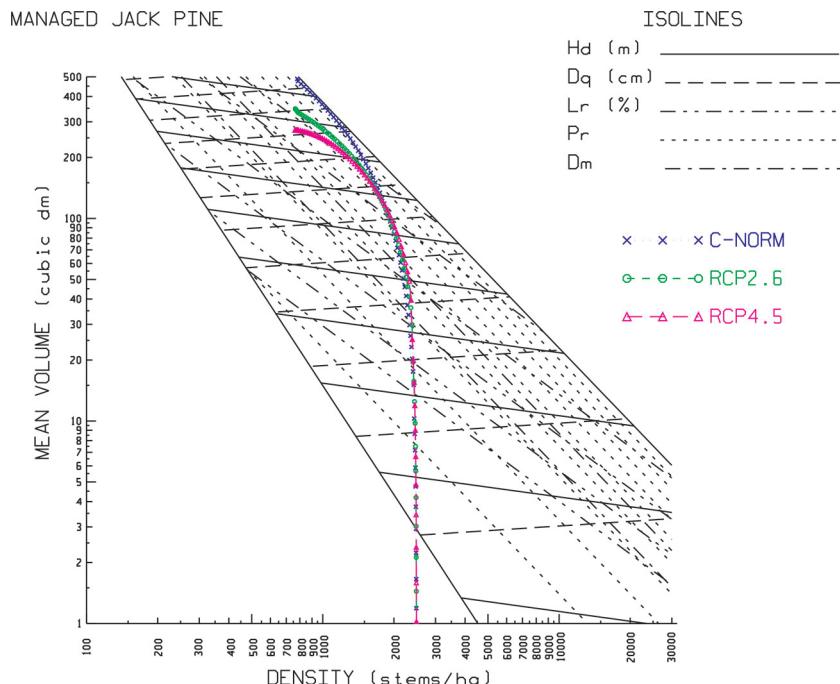
Table 1. Crop planning simulation input parameters for the climate-sensitive structural stand density management diagram (SDMD) variant developed for the managed jack pine stand type (sensu Newton 2009, 2016a, 2019) specific to plantations situated on medium- to good-quality sites in northwestern (Thunder Bay) and northeastern (Kirkland Lake) Ontario, established at a common initial espacement (IE) density level, with and without subsequent commercial thinning (CT) treatments, and growing under varying climatic conditions.

| | Crop plan set by locale | | | | | |
| | Thunder Bay: constant IE treatment/variable climate change scenario | | | Kirkland Lake: constant IE treatment + variable CT treatments/constant climate change scenario | | |
Parameter (units)[a]	Regime 1	Regime 2	Regime 3	Regime 1	Regime 2	Regime 3
Climate change scenario	Normal	RCP2.6	RCP4.5	RCP4.5	RCP4.5	RCP4.5
Mean growing season temperature (°C): 1970–2000	11.3	—	—	—	—	—
Mean growing season temperature (°C): 2010–2040	—	14.2	14.2	14.1	14.1	14.1
Mean growing season temperature (°C): 2041–2070	—	14.4	14.7	14.7	14.7	14.7
Mean growing season temperature (°C): 2071–2100	—	14.6	15.4	15.4	15.4	15.4
Growing season precipitation (mm): 1970–2000	455.4	—	—	—	—	—
Growing season precipitation (mm): 2010–2040	—	515	485	517	517	517
Growing season precipitation (mm): 2041–2070	—	550	550	579	579	579
Growing season precipitation (mm): 2071–2100	—	546	544	577	577	577
Planting year	2020	2020	2020	2020	2020	2020
Rotation length (years)	75	75	75	75	75	75
Simulation years	2020–2095	2020–2095	2020–2095	2020–2095	2020–2095	2020–2095
Initial planting density (seedlings·ha^{-1})	2500	2500	2500	2500	2500	2500
1st CT: stand age (years)/basal area removal (%)	—	—	—	—	55/35	30/35
2nd CT: stand age (years)/basal area removal (%)	—	—	—	—	—	55/35
Genetic worth (%)/selection age (years)	7/20	7/20	7/20	7/20	7/20	7/20
Operational adjustment factor (%)	0.01	0.01	0.01	0.01	0.01	0.01
Merchantable specifications						
Pulplog length (m)/minimum diameter (inside bark; cm)	2.59/10	2.59/10	2.59/10	2.59/10	2.59/10	2.59/10
Sawlog length (m)/minimum diameter (inside bark; cm)	5.03/14	5.03/14	5.03/14	5.03/14	5.03/14	5.03/14
Merchantable top diameter (inside bark; cm)	10	10	10	10	10	10
Product degrade (%)	10	10	10	10	10	10
Minimum operability targets						
Piece size (merchantable stems/merchantable volume; stems·m^{-3})	10	10	10	10	10	10
Merchantable volumetric stand yield (m^3·ha^{-1})	200	200	200	200	200	200
Economic parameters						
Interest rate (%)	2	2	2	2	2	2
Discount rate (%)	4	4	4	4	4	4
Mechanical site preparation (Can\$·ha^{-1})	500	500	500	500	500	500
Planting (Can\$·seedling^{-1})	0.8	0.8	0.8	0.8	0.8	0.8
1st CT: variable costs (Can\$·m^{-3} of merchantable volume removed)/fixed costs (Can\$·ha^{-1})	—	—	—	—	60/500	60/500
2nd CT: variable costs (Can\$·m^{-3} of merchantable volume removed)/fixed costs (Can\$·ha^{-1})	—	—	—	—	—	50/500
Rotational harvesting + stumpage + renewal + transportation + manufacturing variable costs (Can\$·m^{-3} of merchantable volume harvested)	60	60	60	60	50	40

Note: Medium to good site quality is defined as having a mean dominant height of 18 m at a breast-height age of 50 (Sharma et al. 2015). Economic rate assumptions and fixed and variable cost values are approximations.

[a]Climate change scenarios include locale-specific 1970–2000 historical-based climate normals (Normal) and Representative Concentration Pathways 2.6 (RCP2.6) and 4.5 (RCP4.5) (IPCC 2014). All forecasted climate variables were derived from the second-generation Canadian Earth System Model (CanESM2), which consists of a physical atmosphere–ocean model (CanCM4) coupled to a terrestrial carbon model (CTEM) and an oceanic carbon model (CMOC) (Environment and Climate Change Canada 2020); specific estimates for Thunder Bay and Kirkland Lake were derived from a customized spatial climatic model (McKenney et al. 2006) georeferenced to their longitude and latitude coordinate positions, in decimal degrees, of −89.2500 and 48.3833 for Thunder Bay, respectively, and −80.0333 and 48.1500 for Kirkland Lake, respectively. Genetic worth is the maximum percentage increase in dominant height growth expected to occur at the specified selection age (see Newton 2003c, 2015a for specifics). Operational adjustment factor is the annual mortality rate attributed to non-density-dependent abiotic and biotic causes (e.g., windthrow and pathogens, respectively). Product degrade is an end-user-specified allowance for correcting for the potential overestimation arising from the use of product prediction functions derived from virtual sawmill-based simulation studies (sensu Tong and Zhang 2009). Variable costs for CT treatments include all on-site equipment-related operating costs and associated stumpage payments, renewal fees, transportation expenses, and manufacturing costs, cumulatively expressed as a function of merchantable volume extracted during thinning. Fixed costs for CT include forest management and equipment-related transportation fees. Rotational variable costs for final harvesting include all on-site equipment-related operating costs and associated stumpage payments, renewal fees, transportation expenses, and manufacturing costs, cumulatively expressed as a function of merchantable volume harvested (sensu Tong et al. 2005).

Fig. 3. Rotational temporal size–density trajectories by climate change scenario for three jack pine plantations established using genetically improved stock (7% genetic worth height gain at a selection age of 20 years (Newton 2003c)) at an initial espacement (IE) planting density of 2500 seedlings·ha^{-1} on a medium- to good-quality site (site index = 18) situated in northwestern Ontario (Thunder Bay), as predicted by the climate-sensitive structural SDMD for jack pine plantations and illustrated within the context of the traditional SDMD graphic. Specifically illustrating (1) isolines for mean dominant height (H_d; 4–22 m by 2 m intervals proceeding right to left vertically upwards), quadratic mean diameter (D_q; 4–26 cm by 2 cm intervals proceeding left to right vertically upwards), mean live crown ratio (Lr; 35%, 40%, 50%, ..., 80% proceeding from left to right diagonally), and relative density index (P_r; 0.1–1.0 by 0.1 intervals proceeding left to right diagonally); (2) crown closure line (innermost left-hand-side diagonal solid line) and self-thinning rule at P_r = 1.0 (outermost right-hand-side diagonal solid line); (3) lower and upper P_r values delineating the optimal density management window (Dm; $0.32 \leq P_r \leq 0.45$); and (4) expected 75-year (2020–2095) size–density trajectories with 1 year intervals denoted for each scenario (climatic normals (C-NORM), Representative Concentration Pathway (RCP) 2.6, and RCP4.5). Refer to Table 1 for crop plan specifics and Table 2 for derived rotational predictions.
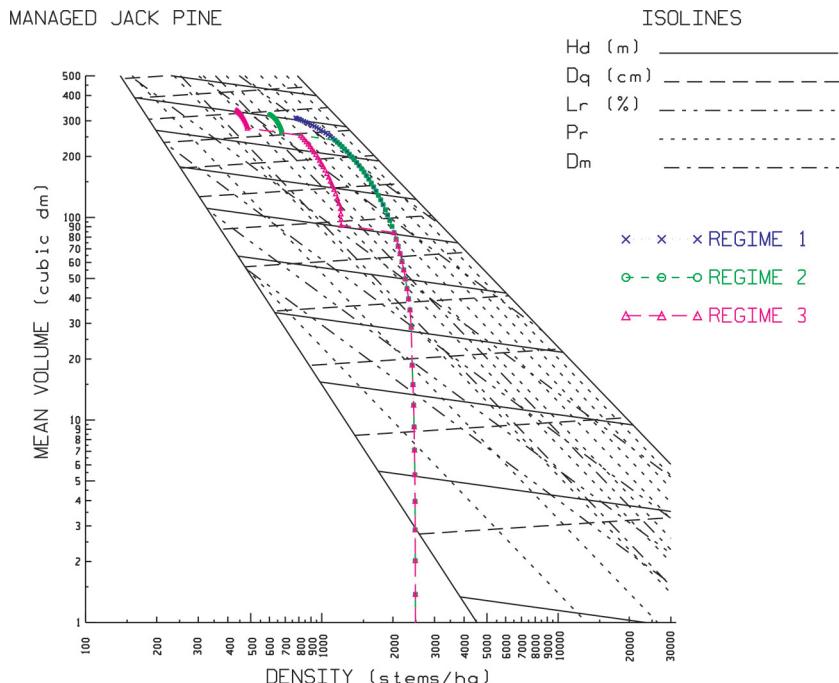
increasing silviculturally intensive crop plans growing under the RCP4.5 scenario, and (3) regional differences for a commonly deployed IE crop plan growing under the RCP4.5 scenario. Table 1 provides the input parameter settings for each set of crop plan simulations.

The resultant temporal size–density trajectories for these western and eastern sets of crop plans are graphically illustrated within the traditional SDMD graphic in Figs. 3 and 4, respectively. The rotational outcomes and associated performance metrics derived for each crop plan are presented in Table 2. For equivalently managed plantations at the western locale (Fig. 3), the rate of stand development initially increased under the RCP2.6 and RCP4.5 scenarios relative to the climate normal scenario (RCP4.5 > RCP2.6) but then rapidly declined during the last commitment period (2071–2100; RCP4.5 > RCP2.6). Although these accelerated rates of development during the first and second commitment periods led to a 14% (RCP2.6) and 19% (RCP4.5) reduction in the time to stand operability status, the decelerated rates of development during the last commitment period ultimately resulted in declines in productivity, wood quality, and economic worth at rotation (Table 2). Specifically, reduced mean tree sizes (e.g., 12% and 19% reductions in quadratic mean diameter for the RCP2.6 and RCP4.5 scenarios, respectively), duration of optimal site occupancy, volumetric yields (e.g., 31% and 46% reductions in total volume for the RCP2.6 and RCP4.5 scenarios, respectively), sawlog production (e.g., 41% and 66% reductions in number of sawlogs produced for the RCP2.6 and RCP4.5 scena-

rios, respectively), biomass and carbon yields, recoverable product volumes, wood quality indices (e.g., 13% and 20% reductions in the wood stiffness (modulus of elasticity) for the RCP2.6 and RCP4.5 scenarios, respectively), and economic worth (RCP4.5 > RCP2.6).

Comparison of the crop plans at the eastern locale, which varied in silvicultural intensity but grew under the same climate change scenario (Fig. 4), indicated that relative to the non-thinned IE regime, CT resulted in increased mean tree size, volumetric, biomass and carbon productivity, economic worth, and duration of optimal site occupancy (Table 2). Comparing the outcomes for regimes 2 and 3 indicated that the single late CT treatment resulted in larger percentage gains across most of the performance metrics, with the exception of the end-product quality measures and the duration of optimal site occupancy (regime 3 ≫ regime 2). As expected, the end-product quality indicators were largely invariant because of the underlying attribute model specifications (i.e., excluded consideration of stand-level effects such as density dependence). Contrasting the rotational metrics for the IE-only regime at both locations under the RCP4.5 scenario indicated that the western plantation would be more negatively affected by climate change than the eastern plantation (Table 2). Overall, this exemplification, although limited in scope, demonstrates the potential utility of the climate-sensitive structural SDMDs in evaluating crop plans under varying climate change assumptions while accounting for localized differences in future growing conditions.

Fig. 4. Rotational (75 years; 2020–2095) temporal size–density trajectories under an RCP4.5 climate change scenario for three jack pine plantations established using genetically improved stock (7% genetic worth height gain at a selection age of 20 years) at an IE planting density of 2500 seedlings·ha^{-1} on a medium- to good-quality site (site index = 18), with one plantation subject to a single commercial thinning (CT) treatment at 55 years (regime 2) and one plantation subject to an early (30 years) and late (55 years) CT treatment (regime 3), situated in northeastern Ontario (Kirkland Lake), as predicted by the climate-sensitive structural SDMD for jack pine plantations and illustrated within the context of the traditional SDMD graphic, as detailed in Fig. 3. Refer to Table 1 for crop plan specifics and Table 2 for derived rotational predictions.

4. Analytical opportunities for advancing SDMDs and their utility in crop planning

Analytically, the unique species-specific allometric relationships deployed in the development of SDMDs potentially provide the prerequisite quantitative foundation for innovative model expansion. For example, as demonstrated for the dynamic SDMD variant for the natural-origin black spruce stand type (Newton 2006b), the expansion of the SDMD via the inclusion of allometric equations for predicting biotic component masses (e.g., bark, stem, branch, foliage, and fine and coarse roots) and abiotic component masses (e.g., needle loss, root turnover, abscised modular components, and coarse wood debris arising from self-thinning) at any point along a given size–density trajectory yielded a stand-level forest production and carbon budgeting decision-support model. Given that a number of non-timber stand-level objectives such as enhancement of biodiversity, mitigation of fire risk, and provision of wildlife habitat are also explicitly related to the abiotic tree population arising from self-thinning, vertical stand structure, and degree of site occupancy in terms of crown cover, respectively, the SDMD platform has the potential to be readily modified to accommodate such management goals (e.g., delineation of management zones within the SDMD graphic that are conducive to achieving such a particular objective). As summarized by Newton (1997a) and more recently articulated by Stankova and Diéguez-Aranda (2020), this ability of the SDMD modelling approach to accommodate a wide array of objectives through model expansion and modification has been one of the primary determinates underlying its continued success in natural resource management.

Computationally, the provision of SDMD software analogues that can simplify the iterative crop planning decision-making process is an aspirational prerequisite for the operational deployment of SDMDs. Realization of this requirement has attained consequential importance given the increasing analytical and computational complexity of SDMDs, as they have evolved from static to dynamic to structural variants combined with their expanded inferential scope. Although computational-intensive programs have been developed for some model variants (e.g., static (Woods 1998), dynamic (Newton 1997b, 1998, 2006a), and structural (Newton 2012c)), the development of user-friendly and actively supported software suites that are compatible with current computer hardware and software technologies and formulated within a participatory framework (sensu McIntosh et al. 2011) remains outstanding. Hence, future programming efforts should be afforded in addressing this deficiency.

Consequential opportunities for continued innovation of the SDMD approach across all variants exist within this modelling domain. Potentially, these include the expansion of the structural SDMD modelling approach to accommodate more complex stand types such as uneven-aged stratified mixtures and a broader array of systematic and (or) selection thinning treatments. Similarly, development of a more unifying analytical structure for addressing the continuum of natural-origin to management stand types would be constructive. This could involve the introduction of a spatial distribution metric to account for the transition from a clustered to a uniform-like spatial pattern that commonly occurs when a natural-origin stand undergoes PCT. Furthermore, accounting for stand-level (population) effects within all of the intrinsically structured hierarchical relationships would provide

Table 2. Crop-plan-specific rotational stand structure attributes, volumetric yields, log assortments, biomass and carbon outcomes, product volumes, end-product wood quality metrics, and productivity and economic indices for jack pine plantations established on medium- to good-quality sites in northwestern (Thunder Bay) and northeastern (Kirkland Lake) Ontario, growing under specified climate change scenarios, as predicted using the climate-sensitive structural SDMD variant for the managed jack pine stand type (sensu Newton 2009, 2016a, 2019).

Index[a] (units)	Locale-specific crop plan set[b]									
	Thunder Bay locale: equal IE treatment/variable climate change scenario					Kirkland Lake locale: equal IE treatment/variable CT treatments/constant climate change scenario				
	Regime 1 (IE/CN)	Regime 2 (IE/RCP2.6)	(%Δ)	Regime 3 (IE/RCP4.5)	(%Δ)	Regime 1 (IE/RCP4.5)	Regime 2 (IE+CT/RCP4.5)	(%Δ)	Regime 3 (IE+2CTs/RCP4.5)	(%Δ)
Structural										
H_d (m)	23.4	20.9	(−11)	19.3	(−18)	20.1	20.1	(0)	20.1	(0)
D_q (cm)	25.2	22.2	(−12)	20.4	(−19)	21.3	21.8	(2)	22.4	(5)
G (m²·ha⁻¹)	38.5	29.5	(−23)	24.9	(−35)	27.6	22.23	(−19)	16.83	(−39)
N (stems·ha⁻¹)	773	765	(−1)	762	(−1)	774	567	(−27)	428	(−45)
P_r (%/100)	0.98	0.72	(−26)	0.59	(−39)	0.66	0.53	(−13)	0.40	(−26)
S_O (%)	7	5	(−2)	3	(−4)	5	5	(0)	36	(31)
S_S (m·m⁻¹)	86	86	(0)	87	(1)	84	82	(−2)	83	(−1)
Volumetric										
$\bar{v}$ (dm³)	502	348	(−31)	273	(−46)	311	324	(4)	342	(10)
V_t (m³·ha⁻¹)	388	266	(−31)	208	(−46)	240	290	(21)	279	(16)
V_m (m³·ha⁻¹)	361	245	(−32)	190	(−47)	220	265	(20)	250	(14)
Log products										
$N_{I(p)}$ (logs·ha⁻¹)	1 235	1 557	(26)	1 598	(29)	1 656	2 154	(30)	2 157	(30)
$N_{I(s)}$ (logs·ha⁻¹)	1 582	932	(−41)	540	(−66)	727	823	(13)	574	(−21)
Biomass/carbon										
M_t (t·ha⁻¹)	242	194	(−20)	153	(−37)	172	222	(29)	224	(30)
C_t (t·ha⁻¹)	121	97	(−20)	76	(−37)	86	111	(29)	112	(30)
Product volumes										
$V_{c(s)}$ (m³·ha⁻¹)	125	79	(−37)	60	(−52)	69	86	(25)	78	(13)
$V_{I(s)}$ (m³·ha⁻¹)	235	141	(−40)	96	(−59)	117	141	(21)	120	(3)
$V_{c(r)}$ (m³·ha⁻¹)	79	53	(−33)	40	(−49)	47	58	(23)	52	(11)
$V_{l(r)}$ (m³·ha⁻¹)	282	166	(−41)	116	(−59)	139	170	(22)	146	(5)
End-product quality metrics										
W_d (kg·m⁻³)	463	440	(−5)	428	(−8)	434	434	(0)	434	(0)
M_a (°)	15	14	(−7)	14	(−7)	14	14	(0)	14	(0)
M_e (GPa)	15	13	(−13)	12	(−20)	13	13	(0)	13	(0)
C_o (μg·m⁻¹)	429	399	(−7)	384	(−10)	392	392	(0)	392	(0)
W_t (μm)	2.9	2.7	(−7)	2.6	(−10)	2.7	2.7	(0)	2.7	(0)
D_r (μm)	31	31	(0)	31	(0)	31	31	(0)	31	(0)
D_t (μm)	28	27	(−4)	27	(−4)	27	27	(0)	27	(0)
S_a (m²·kg⁻¹)	292	311	(7)	321	(10)	316	316	(0)	316	(0)
Productivity										
R_{MAI} (m³·ha⁻¹·year⁻¹)	4.8	3.3	(−31)	2.5	(−48)	2.9	3.5	(21)	3.3	(14)
R_{BAI} (t·ha⁻¹·year⁻¹)	3.2	2.6	(−19)	2.0	(−38)	2.3	3.0	(30)	3.0	(30)
R_{CAI} (t·ha⁻¹·year⁻¹)	1.6	1.3	(−19)	1.0	(−38)	1.1	1.5	(36)	1.5	(36)
Economic										
E_s ($ (thousands)·ha⁻¹)	11.8	5.0	(−58)	2.7	(−77)	3.6	4.9	(36)	3.8	(6)
E_r ($ (thousands)·ha⁻¹)	13.5	6.3	(−53)	3.8	(−72)	4.9	6.3	(29)	4.9	(0)
O_T (years)	43	37	(−14)	35	(−19)	38	38	(0)	40	(5)

Note: A graphical illustration of each crop plan set within the context of the traditional SDMD is presented in Fig. 3 for the Thunder Bay locale and Fig. 4 for the Kirkland Lake locale. CN, climate normal.

[a]Predicted rotational values. H_d is mean dominant height; D_q is quadratic mean diameter; G is basal area per stand; N and P_r are total absolute and relative density, respectively; $\bar{v}$ is mean volume per tree; V_t and V_m are total and merchantable volume per stand inclusive of thinning yields, respectively; S_O is the percentage of time (rotational years) that the regime was within the optimal relative density management window ($0.32 \leq P_r \leq 0.45$); S_S is the mean height/diameter ratio; $N_{I(p)}$ and $N_{I(s)}$ are the total number of pulplogs and sawlogs per stand inclusive of thinning yields, respectively; M_t and C_t are the total oven-dry mass per stand inclusive of thinning yields, and the associated biomass-based total carbon equivalent mass per stand inclusive of thinning yields, respectively; $V_{c(s)}$ and $V_{I(s)}$ are the recoverable chip and lumber volumes per stand inclusive of thinning yields, extracted under a stud sawmill processing protocol, respectively; $V_{c(r)}$ and $V_{l(r)}$ are the recoverable chip and lumber volumes extracted under a random-length sawmill processing protocol, respectively; W_d, M_a, M_e, C_o, W_t, D_r, D_t, and S_a are breast-height basal-area-weighted mean stand values for wood density, microfibial angle, modulus of elasticity, fibre coarseness, tracheid wall thickness, tracheid radial diameter, tracheid tangential diameter, and specific surface area, respectively; R_{MAI}, R_{BAI}, and R_{CAI} are the mean annual merchantable volume, biomass, and carbon mass increments, respectively; E_s and E_r are land expectation estimates for stud mill and random-length sawmill processing protocols, respectively; O_T is the time to operability status as defined by the specified piece size and merchantability thresholds (Table 1). Refer to table 3 in Newton (2012c) for a detailed description of the computations used.

[b]Climate change scenarios and crop plan specifics are provided in Table 1. Percentage differences (Δ) are relative to the control regime (regime 1). Note that for percentage unitized metrics, the actual percentage difference is reported.

for a more complete delineation of population density effects on the derived performance indicators (e.g., currently, such deficiencies yield a lack of demarcation of wood quality attribute outcomes among diverse crop plans, as demonstrated in Table 2 for the end-product quality metrics). The lack of sufficient primary data to develop and validate plausible species-wide solutions to these analytical challenges has largely negated their resolution to date. For example, acquiring adequate spatially mapped inventory information across a species range is problematic given the magnitude of the fiscal resources required. Likewise, accounting for stand-level effects via the use of composite or hierarchical model specifications for the equations that are used to model the SilviScan-based (Evans 1994, 2006; Evans et al. 1996) fibre attribute developmental patterns within the structural SDMD variants is similarly not fiscally feasible given the large number of samples required to incorporate such population-level effects (sensu Newton 2016b, 2019). Consequently, until these issues are resolved possibly through innovations in remote-sensing technologies and non-destructive attribute estimation methodologies, fine-scale differentiation on such performance metrics among competing crop plans will be limited.

Currently, one of the most concerning forest management planning issues is the general lack of stand-level models that explicitly account for climate change effects on stand dynamic processes and associated volumetric yield, end-product, and ecosystem service outcomes. Thus, the analytical ability to readily incorporate biophysical site-specific height–age functions within the SDMD structure should prove valuable in terms of modifying existing SDMDs to partially address climate change effects. Realistically, however, given the known uncertainty with respect to the magnitude of climate change effects on future growing conditions, will largely render the derived predictions from any such climate-sensitive model as informed approximations. Furthermore, within the specific context of the climate-sensitive SDMD variants, expected climate change effects on survival probabilities due to moisture deficiencies are not explicitly accounted for and thus mortality rates are likely to be underestimated. Conversely, potential increases in productivity arising from increased carbon dioxide concentrations (CO_2 fertilization effect) are similarly not quantified. Further research on these effects and their consequences on stand dynamic processes and associated productivity outcomes would be useful in informing future SDMD modelling efforts. However, until the stand-level consequences of climate change are fully ascertained and can be accounted for within the SDMD analytical structure, it will be important to consider a range of plausible scenarios when crop planning. Acknowledgement of such prediction uncertainty when interpreting resultant volumetric yield, end-product potential, and (or) ecosystem service outcomes from SDMDs should be an essential component of the crop planning decision-making process.

In summary, consequential analytical advancements in the SDMD modelling approach have been realized since their initial introduction some 60 years ago. These innovations include the introduction of the dynamic and structure SDMD variants (e.g., Newton and Weetman 1993; Newton 2009; Stankova and Diéguez-Aranda 2020) along with unified theoretical-based modelling platforms that have analytically reconciled the reciprocal equation of the C–D effect and the asymptotic size–density relationship (self-thinning rule) during the density-dependent mortality phase of stand development (i.e., Hagihara 1999, 2000; Stankova and Shibuya 2003; Stankova and Diéguez-Aranda 2017). Similarly, novel size–density trajectory models explicitly incorporating site fertility effects that are consistent, both analytically with the self-thinning rule and conceptually with the Sukatschew effect, have been proposed (Smith and Hann 1984, 1986; Bi 2004) and presented within an SDMD context (e.g., Smith 1989). Furthermore, an array of reformulated derivations of the self-thinning rule based on plant allometry (e.g., Mohler et al. 1978; White 1981; Weller 1987a, 1987b; Newton and Smith 1990; Newton 2006a) and

fractal scaling concepts (Enquist et al. 1998) have advanced the rule's theoretical foundation and hence its continued applicability in the development of SDMDs.

Collectively, these efforts have solidified the ecological foundation, increased the analytical complexity, and expanded the application diversity of the SDMD modelling approach. The inclusion of submodels that explicitly account for genetic worth, thinning response, and climate change effects through modifications to the site-specific height–age equations (Newton 2015a, 2015b, 2016a) and deployment of more unified Y–D relationships (Stankova and Diéguez-Aranda 2020) has addressed some of the analytical challenges of previous SDMD variants. The resultant ability to localize predictions for a given crop plan for a specific geographic location by stand type, genotype, site quality, and climate change scenario and derive a multitude of rotational volumetric yield, end-product, and ecological performance metrics at the mean tree, size-class, and stand levels provides an expanded decision-space for addressing traditional forest management objectives, as well as non-timber management issues (e.g., wildlife habitat, fire risk, biological diversity, and carbon sequestration management). In summary, this documented account of the continuous evolution of the SDMD modelling approach in terms of increasing complexity, application diversity, and ongoing analytical challenges provides the basis for continued innovation within the SDMD modelling domain.

5. Conclusions

Stand density management decision-support requirements arising from the paradigm shift in forest management objectives from a singular volumetric yield maximization focus towards an aspirational trivariate one involving the collective consideration of volumetric yield, end-product value, and ecosystem service outcomes while accounting for the impacts of anthropogenic climate change has resulted in consequential developments in the SDMD modelling approach. The introduction of increased analytical complexity through modelling innovations has yielded enhanced static, dynamic, and climate-sensitive structural SDMD variants and associated algorithmic-based decision-support analogues. These stand density management models can be used to derive optimal crop plans for a wide spectrum of objectives, including those associated with maximizing volumetric productivity, enhancing end-product quality and value, increasing carbon sequestration potential, improving economic returns, increasing wildlife habitat suitability, and reducing fire risk potential. This review provided a synopsis of the analytical evolution of the SDMD modelling approach since its first introduction some 60 years ago, inclusive of an ecological-based perspective of stand density management, summary of the foundational quantitative relationships, examination of SDMD compliance with underlying ecological constructs and empirical prediction expectations, exemplification of a climate-sensitive structural SDMD variant in boreal crop planning, and identification of outstanding analytical challenges and plausible future research directions. Collectively, this resultant account solidifies the evidentiary foundation of the SDMD modelling approach and reconfirms its analytical utility in crop planning decision-making.

Acknowledgements

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Appendix A. Synopsis of validation data sets, yield variates assessed, and overall analytical approach used to evaluate the empirical prediction performance of the main model drivers of the structural SDMD variants developed for boreal coniferous (Newton 2009, 2012a, 2012b, 2012c, 2016a, 2019)

Analytically, the most important module of the structural SDMD model variant is module A, dynamic SDMD. This module consists of a key set of integrated static and dynamic yield–density relationships, which collectively comprises the volumetric yield prediction system of the SDMD. As schematically illustrated in Fig. 2, output from this module is also used to parameterize the Weibull-based parameter prediction equations from which grouped-diameter frequency distributions are recovered and subsequently combined with height–diameter, taper, biomass,

product recovery and value, and wood quality attribute functions to generate estimates for a multitude of performance indicators. These include rotational-based measures of overall productivity (mean annual volume, biomass and carbon increments), volumetric yield outcomes (total and merchantable volumes), log production (number of pulplogs and sawlogs), biomass and carbon sequestration outcomes (oven-dried masses of aboveground components and associated carbon equivalents), recoverable end-product yields and associated economic worth (sawmill-type-specific (stud and random-length processing protocols) volumes and associated monetary values of recovered chip and dimensional lumber products), wood quality (maximum branch diameter and whole-stem wood density, and SilviScan-determined end-product-based fibre measures (Evans 1994, 2006; Evans et al. 1996)), degree of optimum site occupancy, stand structural stability, operability status, and economic viability. Consequently, the scope of this assessment consisted of evaluating the predictive ability of the structural SDMD variants with respect to the key driving variables within the dynamic SDMDs module that are implicitly or explicitly responsible for the generation of these metrics. Specifically, these included mean dominant height (H_d; m), quadratic mean diameter (D_q; cm), basal area (G; $m^2 \cdot ha^{-1}$), total volume (V_t; $m^3 \cdot ha^{-1}$), density (N; stems·ha^{-1}), and relative density index (P_r; %/100).

Following an extensive assessment of potentially obtainable data sets inclusive of those from historical and operationally current PSP systems and stand density control experiments throughout central Canada, it was evident that there would be insufficient independent data sets available for model validation (i.e., data sets that were not used to calibrate the structural SDMD variants). Hence, this necessitated the employment of the original data sets (i.e., data sets that were used in initial model calibrations and henceforth denoted as dependent), partially dependent data sets (i.e., additional new remeasurements from plots previously used during initial model calibrations), and independent data sets (i.e., newly available sample and experimental plot measurements that were not utilized during the initial model calibrations). A stand-type-specific tabulation of the resultant PSPs and experimental permanent sample plots (EPSPs) selected is given in Table A1 with respect to their dependency, number of plots and measurements extracted, organizational entity involved in their establishment and remeasurement, experimental treatments implemented where applicable (i.e., initial espacement (IE), precommercial thinning (PCT), and (or) commercial thinning (CT) treatments), and geographic location. In total, 100 (58% dependent), 349 (61% dependent), 486 (30% dependent), 612 (16% dependent), 260 (100% dependent), and 415 (100% dependent) plot measurements specific to the natural-origin upland black spruce stand type, managed upland black spruce stand type (plantations), natural-origin jack pine stand type, managed jack pine stand type (plantations), natural-origin black spruce and jack pine mixed stand type, and natural-origin lowland black spruce stand type, respectively, were extracted. In terms of the area-based computations, it is noteworthy to appreciate that the PSP and EPSP measurements were obtained from plots that varied in shape (circular, square, or rectangular) and size (area). Specifically, plot sizes overall ranged from 0.010 to 0.640 ha, with a global weighted mean of 0.101 ha across all stand types. The plots within the experiments were slightly larger than those within the PSP systems (mean of 0.133 ha versus 0.073 ha, respectively). Given that area-based projections generated from the SDMDs are presented at the stand level on per unit area basis (hectare), all such plot values were accordingly scaled to the stand level.

The scope of the evaluation in regard to crop plan diversity was governed by the characteristics of the validation data set (e.g., 18% and 42% of the natural-origin upland black spruce and jack pine stand-type plots were subjected to one or more thinning treatments, respectively, whereas the remaining plots for the

Table A1. Validation data set characteristics: data set type, organizational entity, sample plot type, and geographic location by stand type.

Stand type[a]	Data set[b] Type	Data set[b] Denotation	Measurement[c] N_P	Measurement[c] N_m	Data set type; organizational entity, series, plot design, location, and applicable reference[d]
PIm$_{UL(N)}$	D	AC-PR	1	4	PSP; American-Can Inc., Pic River Series, Marathon, Ontario (AC-PR)
	D	SF-CB	6	54	PSP; Spruce Falls Inc., Clay Belt Series, Kapuskasing, Ontario (SF-CB)
	P	SF-CB	4	42	PSP; Spruce Falls Inc., Clay Belt Series, Kapuskasing, Ontario
PIm$_{UL(M)}$	D	OMNRF-BW	68	190	PSP; OMNRF, Beckwith Series, boreal Ontario (OMNRF-BW)
	D	OMNRF-ST	4	10	EPSP; OMNRF, IE experiment, Stanley, Ontario (OMNRF-ST)
	D	OMNRF-TB	4	10	EPSP; OMNRF, IE experiment, Thunder Bay, Ontario (OMNRF-TB)
	I	CFS-TL	2	9	EPSP; CFS, IE+PCT+CT experiment (Tyrol Lake Trial), Beardmore, Ontario (Robert Fleming (GLFC, CFS) 2016; per. com.; CFS-TL)
	I	CFS-ST	25	130	EPSP; CFS, IE experiment (Springer Trial), Kapuskasing, Ontario (Arthur Groot (CWFC, CFS) 2016; per. com.; CFS-ST)
PNb$_{(N)}$	D	KC-LL	29	116	PSP; Kimberly-Clark Inc., Longlac Series, Terrace Bay, Ontario (KC-LL)
	D	AC-PR	8	31	PSP; American-Can Inc., Pic River Series, Marathon, Ontario
	I	CFS-154	13	83	EPSP; CFS, PCT experiment (MS-154/Study 5 data subset), Sandilands Forest Reserve, Manitoba (Bella and De Franceschi 1974a; post-1974 remeasurements obtained from James Steward (CWFC, NoFC, CFS), 2001; per. com.; CFS-154)
	I	CFS-189	13	88	EPSP; CFS, CT experiment (MS-189/Study 6 data subset), Sandilands Forest Reserve, Manitoba (Bella and De Franceschi 1974b; post-1974 remeasurements obtained from James Steward (CWFC, NoFC, CFS), 2001; per. com.; CFS-189)
	I	OMNRF-BT	32	64	EPSP; OMNRF, PCT experiment (Bompas Trial), Burt Township, Kirkland Lake, Ontario (Stan Vasiliauskas (Timmins District Office, OMNRF) 2018; per. com.; OMNRF-BT)
	P	CFS-SE	18	54	EPSP; CFS, PCT+CT experiment (Sewell Trial) Folylet, Ontario (Newton, this study; CFS-SE)
	P	CFS-TY	5	50	EPSP; CFS, PCT+CT experiment (Tyrol Trial) Beardmore, Ontario (Newton, this study; CFS-TY)
PNb$_{(M)}$	D	OMNRF-AS	33	99	PSP; OMNRF, Anderson Series, Kirkland Lake, Ontario (OMNRF-AS)
	I	OMNRF-GT	24	96	EPSP; OMNRF, IE experiment (Garibaldi Trial), Shinning Tree Ontario (Stan Vasiliauskas (Timmins District Office, OMNRF) 2018; per. com.; OMNRF-GT)
	I	OMNRF-BL	9	24	EPSP; OMNRF, IE+CT experiment (Burt Lake Trial), Burt Township, Kirkland Lake, Ontario (Stan Vasiliauskas (Timmins District Office, OMNRF) 2018; per. com.; OMNRF-BL)
	I	CFS-236	14	84	EPSP; CFS, IE experiment (MS-236), Sandilands Forest Reserve, Manitoba (Bella and De Franceschi 1974c; post-1974 remeasurements obtained from James Steward (CWFC, NoFC, CFS), 2001; per. com.; CFS-236)
	I	OMNRF-NS	55	168	EPSP; OMNRF, IE experiment (Nelder Plot Series), Kirkland Lake, Ontario (OMNRF-NS). Notes: (1) initially established by Ken Armson and Robert Day (1968–1972; Faculty of Forestry, University of Toronto, Toronto, Ontario), and jack pine components subsequently remeasured by the OMNRF (1984, 1988; John Parton, Timmins District Office, OMNRF 2008; per. com.) and CFS (2008; Newton, this study); (2) analytically, two temporal-based yield attribute sets corresponding to initial density equivalents within the >1000–2250 and >2250–3500 stems·ha^{-1} espacement ranges were generated for each plot; and (3) refer to the methods sections of Newton (2015a, 2015b) for additional experimental design and measurement information
	I	CFS-134	4	28	EPSP; CFS, IE experiment (PNFI-134), PRF, Chalk River, Ontario (Craig Robinson (PRF, CFS) 2010; per. com.; CFS-134)
	I	FS-AT	11	37	EPSP; Forest Service, USDA, IE experiment (Aurora Trial), Superior National Forest, Hoyt Lakes, Minnesota, USA (Colin Bowling (Kenora Distinct Office, OMFRF) 2012 per. com.; FS-AT)
	I	OMNRF-RT	23	81	EPSP; OMNRF, IE+PCT+CT experiment, Barrie, Ontario (OMNRF-RT)
PImPNb$_{(N)}$	D	KC-LL	32	177	PSP; Kimberly-Clark Inc., Longlac Series, Terrace Bay, Ontario
	D	AC-PR	20	83	PSP; American-Can Inc., Pic River Series, Marathon, Ontario
PIm$_{LL(N)}$	D	AC-HE	28	89	PSP; American-Can Inc., Hearst Series, Marathon, Ontario (AC-HE)
	D	AC-PS	15	126	PSP; American-Can Inc., Pic River Series, Marathon, Ontario
	D	SF-CB	25	200	PSP; Spruce Falls Inc., Clay Belt Series, Kapuskasing, Ontario

[a]PIm$_{UL(N)}$, PIm$_{UL(M)}$, PNb$_{(N)}$, PNb$_{(M)}$, PImPNb$_{(N)}$, and PIM$_{LL(N)}$ denote natural-origin upland black spruce, managed upland black spruce, natural-origin jack pine, managed jack pine, natural-origin black spruce and jack pine mixed, and natural-origin lowland black spruce stand types, respectively (sensu Day 1967).

[b]D, P, and I denote dependent (used in model parameterization), partially dependent (additional new remeasurements for plots previously used for parameterization), and independent (measurements made newly available since model parameterization) data set types, respectively.

[c]N_p and N_m denote the total number of permanent and (or) experimental sample plots within a given uniquely denoted data set and total number of temporal-specific measurements within a given data set, respectively.

[d]PSP and EPSP denote permanent sample plot and experimental permanent sample plot types, respectively. IE, PCT, and CT denote initial espacement, precommercial thinning, and commercial thinning treatments, respectively. OMNRF, Ontario Ministry of Natural Resources and Forestry; CFS, Canadian Forest Service; GLFC, Great Lakes Forestry Centre; CWFC, Canadian Wood Fibre Centre; NoFC, Northern Forestry Centre; PRF, Petawawa Research Forest; USDA, United States Department of Agriculture; per. com., personal communication.

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Table A2. Validation data set characteristics: descriptive mensurational summary by stand type.

Variate[a]	Statistic	Stand type[b] PIm$_{UL(N)}$	PIm$_{UL(M)}$	PNb$_{(N)}$	PNb$_{(M)}$	PImPNb$_{(N)}$	PIm$_{LL(N)}$
Site index (m)	Mean	14.7	16.3	18.0	18.7	13.8	16.9
	Minimum	11.1	10.7	14.0	15.3	9.8	10.1
	Maximum	18.4	23.5	23.8	23.6	19.2	24.7
Stand age (years)	Mean	66	24	52	27	103	106
	Minimum	31	15	10	5	50	22
	Maximum	115	48	143	61	170	215
Mean dominant height (m)	Mean	16.7	8.2	17.2	11.2	19.7	17.2
	Minimum	11.0	2.4	4.5	2.9	14.3	7.2
	Maximum	21.9	17.3	25.9	24.5	23.8	26.9
Quadratic mean diameter (cm)	Mean	12.0	8.8	14.3	11.5	15.5	12.1
	Minimum	5.7	1.3	2.9	2.8	7.4	3.6
	Maximum	17.4	20.1	24.7	25.2	25.0	21.6
Basal area (m$^2 \cdot$ha^{-1})	Mean	32.6	17.9	26.6	20.9	37.2	30.8
	Minimum	12.9	0.2	2.1	0.3	17.1	0.3
	Maximum	45.7	60.1	43.8	48.5	52.8	62.6
Total volume (m$^3 \cdot$ha^{-1})	Mean	196.5	71.2	203.5	115.0	296.0	201.4
	Minimum	50.9	0.2	1.5	0.5	134.4	0.6
	Maximum	301.3	322.2	408.6	377.9	428.5	445.8
Density (stems$\cdot$ha^{-1})	Mean	3 313	2 881	2 423	2 240	2 228	3 397
	Minimum	966	603	555	435	539	222
	Maximum	9 002	6 528	27 469	6 528	8 342	13 719
Relative density index (%/100)	Mean	0.9	0.4	0.6	0.4	0.9	0.7
	Minimum	0.4	0.0	0.0	0.0	0.4	0.0
	Maximum	1.3	1.2	1.2	1.1	1.3	1.2

[a]Stand-type-specific site indices (mean dominant height at an index age of 50 years at breast height (PIm$_{UL(N)}$, PIm$_{UL(M)}$, PNb$_{(N)}$, PNb$_{(M)}$, and PImPNb$_{(N)}$) or 100 years at ground height (total age; PIM$_{LL(N)}$): (1) species-specific functions developed for natural-origin black spruce and jack pine stands (Sharma and Reid 2018) are used for PIm$_{UL(N)}$ and PNb$_{(N)}$, respectively, and in combination for PImPNb$_{(N)}$ (i.e., to generate a non-species-specific mean estimate); (2) species-specific functions developed for plantation black spruce and jack pine stands (Sharma et al. 2015) are used for PIm$_{UL(M)}$ and PNb$_{(M)}$, respectively; and (3) a function developed for natural-origin lowland black spruce (Newton 2008) is used for PIm$_{LL(N)}$.
[b]As defined in Table A1.

other stand types were not). Overall, the assessment consisted of assessing the yield prediction ability of the models for density-unregulated regimes that are consistent with an extensive or basic silvicultural management intensity (e.g., monospecific and mixed natural-origin stand types) and density-regulated regimes involving IE, PCT, or CT treatments that are consistent with an intensive or elite silvicultural management intensity (e.g., density-manipulated natural-origin and (or) planted monospecific stand types).

Information available for each plot within each PSP series or EPSP series included its geographic location, temporal disturbance and silvicultural treatment histories, ecological site type, year of plot establishment and subsequent remeasurements, dimensional plot characteristics, and measurement protocol. Briefly, the mensurational-related plot measurements included (1) individual-tree lists consisting of the measurements of diameter at breast height (DBH; breast height = 1.37 or 1.3 m; outside bark) from all biotic trees with DBH > 2.54 cm by species or a species-specific grouped-diameter frequency distribution tally (2 cm class widths) and (2) DBH and total height measurements on a subset of the trees usually selected from across the diameter range within each plot. Deploying the individual tree or diameter class values (DBHs (standardized to the 1.3 m stem height) and total height measurements or estimates derived from species-specific allometric-based height–diameter functions) in combination with plot area informa-

tion, the following species-specific variables were calculated for each temporal-based plot measurement: (1) mean dominant height defined as the mean height of the trees within the largest height quintile; (2) quadratic mean diameter; (3) stand basal area; (4) total stand volume, which was calculated as the sum of the individual tree total volumes as determined from the diameter and height measurements in combination with the applicable species-specific regionwide standardized total volume equation (Honer et al. 1983); (5) merchantable stand volume, which was calculated as the sum of the individual tree merchantable volumes as determined from the total volume estimates and merchantability specifications in combination with the applicable species-specific regionwide standardized merchantable volume equation (Honer et al. 1983); (6) total density; and (7) relative density index defined as the ratio of the observed density to the biologically maximum density attainable in a stand of equivalent mean volume.

To ensure equivalence between model simulations and the validation data sets in terms of dominant height development, the stand age and mean dominant height values from each plot measurement were assigned a site index value using the same species- and stand-type-specific function as that deployed in the models. Specifically, the (1) species-specific functions developed for natural-origin black spruce and jack pine stands by Sharma and Reid (2018) were used for natural-origin upland black spruce and jack pine stand types, respectively, and in combination for

the natural-origin upland black spruce and jack pine mixed stand type; (2) species-specific functions developed for plantation black spruce and jack pine stands by Sharma et al. (2015) were used the managed upland black spruce and jack pine stand types; and (3) function developed for natural-origin lowland black spruce by Newton (2008) was used for the natural-origin lowland black spruce stand type. The temporal coverage of the projections in terms of stand age (years) at the time of initial measurement, time of final measurement, and overall mean projection length ranged respectively as follows (mean value): (1) 31–62 (43), 68–115 (87), and 15–57 (42) for the natural-origin upland black spruce stand type; (2) 15–43 (17), 23–48 (22), and 5–20 (16) for the managed upland black spruce stand type; (3) 10–111 (36), 32–143 (57), and 5–40 (21) for the natural-origin jack pine stand type; (4) 5–61 (17), 25–61 (39), and 10–36 (22) for the managed jack pine stand type; (5) 51–139 (91), 50–170 (114), and 5–46 (26) for the natural-origin mixed black spruce and jack pine stand type; and (6) 22–195 (94), 48–215 (123), and 5–53 (29) for the natural-origin lowland black spruce stand type. A stand-type-specific descriptive summary of the resultant validation variates is given in Table A2.

The corresponding model-based simulation for each PSP or treatment-specific EPSP was initialized using the observed initial density and stand age measurement at the time of the first measurement along with the plot's mean site index value. The applicable stand-type-specific structural SDMD variant was then used to generate predictions for all seven yield variates at each temporal measurement event for each plot. The structural SDMD variants deployed were those as detailed in the stand-type-specific contributions of Newton (2009, 2012a, 2012b, 2012c) and included the analytical enhancements described by (Newton 2012c) along with the genetic worth and thinning response functions as proposed by Newton (2015a, 2015b), respectively. Overall, this yielded stand-type-specific data subsets consisting of 100, 349, 486, 612, 260, and 415 observed–predicted pairs for the natural-origin upland black spruce, managed upland black spruce, natural-origin jack pine, managed jack pine, natural-origin black spruce and jack pine mixed, and natural-origin lowland black spruce stand types, respectively.

Statistically, the mean percentage prediction error ($\bar{e}$; %) was used as the validation metric (eq. A1; sensu Huang et al. 2003).

$$(A1) \qquad \bar{e} = 100 \left[\frac{\sum_{i=1}^{n} (y_i - \hat{y}_i)/n}{\sum_{i=1}^{n} y_i/n} \right]$$

where y_i and $\hat{y}_i$ are the observed and predicted values of a given variate, respectively, and n is the total number of observed–predicted data pairs specific to a given yield variate and stand type ($i = 1, \ldots, n$). This metric is reflective of overall mean predictive performance of the model for a given variate. Although the employment of alternative interval-based error metrics (e.g., confidence, prediction and tolerance error intervals) would be likewise informative, the statistically confounding issues of temporal and spatial correlation that arise from the use of repeated and spatially clustered plot measurements negated their consideration in this assessment.

The results indicated that stand-type-specific mean relative errors for estimating H_d, D_q, G, V_t, N, and P_r were, respectively, (1) 5%, 14%, 15%, 25%, −10%, and 10% for the natural-origin upland black spruce stand type; (2) 5%, 8%, 12%, 28%, −7%, and 14% for the plantation upland black spruce stand type; (3) −2%, −13%, −21%, −16%, 26%, and −12% for the natural-origin jack pine stand

type; (4) −3%, −12%, −16%, −12%, −4%, and −10% for the plantation jack pine stand type; (5) −3%, 0%, −10%, −12%, −8%, and −16% for the natural-origin upland mixed black spruce and jack pine stand type; and (6) −3%, 3%, −9%, −9%, −15%, and −7% for the natural-origin lowland black spruce stand type. Thus, in summary, the magnitude of these prediction error measures indicated that the principal driving variables within the structural SDMD variants developed for conifers in boreal Ontario were approaching or realizing the ±20% error tolerance threshold suggested for operational growth and yield models by Huang et al. (2003).

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