Uncertainty and Forest Land Use Allocation in British Columbia: Fuzzy Decisions and Imprecise Coefficients

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by

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ABSTRACT

Recently, increasing weight has been placed on non-timber values in forest management. Both the multiple objectives and the parameters that support decision making in forestry are often imprecise and vague. In this paper, the concepts of fuzzy set theory are explained and then applied to the problem of allocating public forest land on Vancouver Island among competing land uses. Two principal sources of fuzziness are identified—those related to uncertainty in classification (specification of management objectives) and those related to uncertainty concerning how actions affect objectives (imprecise technical coefficients). By comparing the results of classical and fuzzy decision models, we conclude that the latter approach can be judged an improvement over the former. The fuzzy land-use allocation appears to be more consistent with the political decision making process that has evolved in British Columbia, a process which relies on consultation and consensus seeking among various interest groups,. The analysis also yields insights into the robustness of outcomes and suggests priority areas for further research.

Key Words: Multiple-objective management of public forest lands; fuzzy set theory; imprecision and vagueness in technical coefficients and objectives

that emerged during the CORE process was to treat each of the multiple objectives of land use as equally important, so that economic efficiency was not given preeminent status. Under this condition, traditional multiple objective decision making (MODM) is the most appropriate tool of analysis. The usefulness of classical MODM models is limited, however, because of the following characteristics inherent in the land-use decision making process:

- the objectives of society are ill-defined;
- the values that society attaches to various forest activities (such as recreation or preservation of biodiversity) are imprecise at best, or simply unknown;
- the effects of silviculture and other forest management decisions are uncertain, both from a biological and socioeconomic perspective;
- land-use and silvicultural decisions often pertain to an uncertain and distant future; and
- there is uncertainty about forest tenures, the macro economy, future product prices, and the ability of, or need for, governments to reduce deficits/debts.

While some uncertainty is related to the *randomness* of events (e.g., price movements), not all uncertainty is related to randomness (e.g., the objectives of society, the ranking of those objectives, and the value of a bequest of natural forests to future generations are uncertain "event", but they are not random). The distinction between complexity, ambiguity, vagueness, imprecision and uncertainty is frequently blurred, and probabilistic description and stochastic modeling are often inadequate tools for dealing with uncertainty. Therefore, in this paper, we employ fuzzy logic to deal with uncertainty related to vagueness, ambiguity and imprecision.

Although the literature on fuzzy set theory has expanded significantly in recent years, applications in the field of forestry and land-use planning are scarce. Mendoza and Sprouse (1989) proposed a two-stage approach to forest planning, and developed a fuzzy model for more flexible and robust generation of alternatives. The uncertainty in their model arose from imprecise coefficients and was modeled by tolerating some constraint violations. Bare and Mendoza (1992) and Pickens and Hof (1991) compared classical (i.e., crisp) and fuzzy models for describing optimal harvest over time. They found that, by relaxing the constraint of non-declining harvest volume over time, net present value (NPV) could be significantly increased. Mendoza *et al.* (1993) developed a fuzzy multiple objective linear programming model for forest planning that accomodated uncertainty in the objective function by making coefficients interval-valued. Finally, Tecle *et al.* (1994) developed an interactive fuzzy multi-criterion decision model in which the decision maker is allowed to search the frontier of efficient solutions instead of being confronted with a uniquely preferred solution. Fuzzy set theory was used to deal with a vague objective and constraint.

This paper differs from previous ones because its scope is generally broader. In Bare and Mendoza (1992) and Pickens and Hof (1991), for instance, the focus is on timber yield only. We have a social orientation in which timber is but one of many services provided by forests.[1] Obviously, extending the analysis to allow for recreation and preservation benefits requires data that are comparatively less precise than similar data for timber, because they are unobservable in markets and are difficult to measure. While Bare and Mendoza (1992)

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[1]The papers by Mendoza *et al.* (1993) and Tecle *et al.* (1994) deal with multiple objectives, but are based on NPV maximization models.

argue that imprecise (timber) coefficients are better viewed as stochastic rather than fuzzy, in this paper we address imprecision in coefficients as fuzzy measures.

The broader scope also implies that the impact of fuzzifying a crisp model is different. If fuzzifying simply amounts to relaxing a constraint of a NPV maximization model, then the predictable result is that NPV will increase! Relaxing a binding constraint will always have this effect. In our analysis, fuzziness actually affects the allocation of land among uses, a less than trivial result of the fuzzy approach.

Finally, in this paper we combine the existence of vague objectives and constraints, as in Tecle *et al.* (1994) and Pickens and Hof (1991), and imprecise coefficients, as in Mendoza and Sprouse (1989), Bare and Mendoza (1992) and Mendoza *et al.* (1993). Moreover, imprecise coefficients are explicitly dealt with by modeling them as fuzzy numbers instead of implicitly incorporating them in the analysis as admissible violations of constraints, as in Mendoza *et al.* (1993). The approach taken in this paper gives more insight into the effect of the various sources of uncertainty on land allocation and makes better use of available information.

The purpose of the current research is to capture the uncertainty inherent in describing the socioeconomic impacts of land-use and forest management decisions on Vancouver Island. The major objectives of this study are threefold: (1) to develop a fuzzy multiple objective decision-making model that incorporates uncertainties both in objective specification and parameter values; (2) to contrast the fuzzy models to a classical multiple objective approach where uncertainty is not considered; and (3) to demonstrate the usefulness of fuzzy set theory in the context of a multiple objective decision-making model for land use on Vancouver Island. Vancouver Island was selected because it is a region where land-use conflicts are intense and the recent CORE (1994) land-use recommendations (subsequently adopted by government) have been controversial.

The rest of the paper is organised as follows. In section 2, we provide a heuristic overview of uncertainty, the contribution of fuzzy set theory and the notion of fuzzy numbers for dealing with fuzzy quantities and concepts, followed in section 3 by a more formal description of these concepts. Three decision support models are developed in section 4. The derivation of the required parameters are presented in section 5, while the empirical results are provided in section 6. Our conclusions ensue.

2. Uncertainty: Fuzzy Sets and Fuzzy Measures

Current literature concerned with modeling uncertainty provides a wide range of definitions both for the concept of uncertainty itself as well as the various types of uncertainty that may be addressed. The extent of the divergence is demonstrated by the fact that some typologies define uncertainty as a subset of ignorance, while others consider ignorance to be a subset of uncertainty (Krause and Clark 1993). In what follows, it is the latter conceptualization that is employed.

According to Kruse *et al.* (1991), uncertainty has to do with the degree of belief or faith in the validity of a particular proposition or datum. Uncertainty arises from many sources, including measurement error, lack of judgement, imprecision, unreliability, variability, vagueness, ignorance and ambiguity. The theory of fuzzy sets is most widely used in dealing with vagueness and ignorance, although one often sees fuzzy sets recommended as a means for dealing with ambiguity.

Ambiguity may be defined as the property of possessing several distinct but plausible and reasonable interpretations (Cox 1994). The term "hot" is considered ambiguous until the context for its use has been defined, be it temperature, spiciness or trendiness to name but a few. The failure to define clearly the context associated with a particular event or concept is not a source of uncertainty amenable to formalization through the use of fuzzy set concepts. The recommendation of fuzzy set theory for use in this situation arises from confusion regarding the terms ambiguity and vagueness.

Vagueness has perhaps the widest range of interpretations advanced for any of the various types of uncertainty; in fact, completely contradictory definitions can be found. However, within the fuzzy set literature there is a consensus. Vagueness is said to occur when an object is completely known but its classification is in doubt because the set to which it may belong is poorly defined (Barret and Pattanik 1989); vagueness refers to the lack of clear-cut boundaries for the set of objects to which the symbol or meaning is applied (Fedrizzi 1987). Cox (1994) refers to this form of vagueness as imprecision of content. Consider classifying a person as "young", "middle-aged" or "old". You may know their age exactly, but at what age is one to be described as old versus middle-aged. These describers of age are poorly defined, they are vague.

The concept of vagueness has direct application to decision making in identification of the "optimal" or "best" solution. The characterization of the best solution may be incomplete or unknowable, and thus the relation of any one solution to that ideal is unclear or vague. The solution is known, but the description of the best solution is not. This approach is clearly distinct from that of probability theory. Probability deals with the quantification of an uncertain event, while fuzzy set theory deals with the quantification of the uncertainty of the description of the event. Put another way, probability deals with unknown elements of fixed and well known sets, while fuzzy set theory deals with fixed and known elements of ill-defined sets (Dubois and Prade 1993; Kosko 1992, pp.264-68).

Fuzzy set theory also has application in incorporating uncertainty due to ignorance or incomplete information, a condition often encountered in the process of identifying decision alternatives, and described by the term "possibility". A view of possibility is that, although a degree of belief may be assigned to a set as a whole, the assignment of that degree of belief among the individual elements of the set requires more knowledge than is present. In this situation, a possibility measure describes the ordering of the elements within the set in terms of their relative likelihood of occurrence, a preference relation on the possible elements of the set (Dubois and Prade 1993). To say a person is "about 30 years old" is to implicitly define a set of ages that contains the number 30. Possible ages that would fit the definition of about 30 may lie in the range of 27 to 33. We consider that they are most likely to be 30, less likely to be either 29 or 31, less likely again to be 28 or 32, and so on. We can order the ages based on relative possibility, but we cannot assign a cardinal ranking to the possibilities.

3 Fuzzy Set Theory: Membership and Possibility

In this section, we provide a formal treatment of fuzzy logic by considering membership or indicator functions for fuzzy sets (objective targets) and fuzzy numbers for imprecise values of the technical coefficients in the decision model. This background constitutes the formal foundation for the fuzzy and the fuzzy possibilistic MODMs that are developed in section 4.

Fuzzy sets and membership functions

An element x of X is assigned to an ordinary (crisp) set A via the characteristic function μ_A, such that:

$$\mu_A(x) = 1 \qquad \text{if } x \in A. \tag{1}$$

$$\mu_A(x) = 0 \qquad \text{otherwise.}$$

The element has either full membership ($\mu_A(x) = 1$) or no membership ($\mu_A(x) = 0$) in the set A. The valuation set for the function is the pair of points {0,1}. A fuzzy set $\tilde{A}$ is also described by a characteristic function, the difference being that the function now maps over the closed interval [0,1].

Formally, a fuzzy set $\tilde{A}$ of the universal set X is defined by its membership function

$$\mu_{\tilde{A}} : X \rightarrow [0,1], \tag{2}$$

which assigns to each element x∈X a real number $\mu_{\tilde{A}}(x)$ in the interval [0,1], where the value of $\mu_{\tilde{A}}$ at x represents the grade or degree of membership of x in $\tilde{A}$ (Sakawa 1993). While membership functions can take on a variety of functional forms, linear specifications are often employed.

As an example of fuzzy membership, consider the set of “natural forests”. It is clear that old-growth forests belong to this set, they have a degree of membership equal to 1. As we consider progressively heavier logged forests, the descriptor "natural" becomes less apt. Is a selectively logged forest “natural”? To capture the uncertainty surrounding their membership in the set of “natural forests”, partly logged forests are assigned a partial degree of membership, something less than one. This is an example of a one-sided fuzzy set. Membership in this set approaches zero as the exploitation pressure increases.

In this regard, fuzzy set theory can be used to deal with unclear objectives. This will be illustrated with an example. An objective of the land-use decision model developed below will be to preserve wilderness by setting land aside as protected areas. The question is: how much land should be protected? According to the PAS, 12% of the land base of B.C. should be protected. Since "undershooting" of this goal will be politically sensitive, it can be argued that 12% serves to define the lower limit of acceptable objective values—any solution that provides a lower percentage of the land base as wilderness will be unacceptable and have a membership value of 0. On the other hand, there are many who would argue that more land should be set aside. Claims up to 30% have been put forward. If we adopt 30% as a perfectly satisfactory level of forest protection, then the membership function describing the fuzzy set of acceptable or satisfactory solutions, denoted by i, is:

$$\begin{aligned} \mu_i(x) &= 1, && \text{if PA} \geq 30\% \\ \mu_i(x) &= (\text{PA-12})/(30\text{-}12) && \text{if } 12\% \leq \text{PA} < 30\% \qquad (3) \\ \mu_i(x) &= 0, && \text{if PA} < 12\%. \end{aligned}$$

where PA refers to the percentage of the land base that is to be protected. If the solution to the optimization problem allocates 21% of the land base to protected areas, $\mu_i(x) = 0.50$.

The preceding definitions have employed the concept of a normalized fuzzy set. A fuzzy set A, defined over a finite interval, is said to be normal if there exists an $x \in X$ such that $\mu_A(x) = 1$, and $\mu_A(x) \leq 1 \; \forall x \in X$. A subnormal fuzzy set is normalized by dividing $\mu_A(x)$ by its height or greatest membership value.

Set theoretic operations are defined for fuzzy sets. Among these are the concepts of containment, complement, intersection and union. A fuzzy set A is contained in the fuzzy set B ($\tilde{A}$ is a subset of $\tilde{B}$), if and only if the membership function of $\tilde{A}$ is less than or equal to that of $\tilde{B}$ everywhere on X:

$$\tilde{A} \subseteq \tilde{B} \quad \Leftrightarrow \quad \mu_A(x) \leq \mu_B(x) \text{ for all } x \in X. \tag{4}$$

The complement of $\tilde{A}$ (written as $\bar{A}$) is defined as:

$$\mu_{\bar{A}}(x) = 1 - \mu_{\tilde{A}}(x) \tag{5}$$

The intersection of the fuzzy sets $\tilde{A}$ and $\tilde{B}$ is defined as:

$$\tilde{A} \cap \tilde{B} \quad \Leftrightarrow \quad \mu_{(\tilde{A} \cap \tilde{B})} = \min\{\mu_A(x), \mu_B(x)\} \text{ for all } x \in X, \tag{6}$$

and the union as:

$$\tilde{A} \cup \tilde{B} \Leftrightarrow \mu_{(\tilde{A} \cup \tilde{B})} = \max\{\mu_A(x), \mu_B(x)\} \text{ for all } x \in X. \tag{7}$$

Hence, the intersection $\tilde{A} \cap \tilde{B}$ is the largest fuzzy set contained in both $\tilde{A}$ and $\tilde{B}$, and the union $\tilde{A} \cup \tilde{B}$ is the smallest fuzzy set containing both $\tilde{A}$ and $\tilde{B}$.

While both union and intersection of fuzzy sets are commutative, associate and distributive, as is the case for ordinary or crisp sets, fuzzy logic deviates from crisp logic because, if we do not know $\tilde{A}$ with certainty, then its complement $\bar{A}$ is also not known with certainty. Thus, $\tilde{A} \cap \tilde{\tilde{A}}$ does not produce the null set as is the case for crisp sets (where $A^c \cap A = \phi$). Thus, fuzzy logic violates the "law of noncontradiction". It also violates the "law of the excluded middle" because the union of a fuzzy set and its complement does not equal the universe of discourse—the universal set X. Thus, $\tilde{A}$ is properly fuzzy iff $\tilde{A} \cap \tilde{\tilde{A}} \neq \phi$ and $\tilde{A} \cup \tilde{\tilde{A}} \neq X$ (Kosko 1992, pp.269-272).

Fuzzy numbers and alpha cuts

A fuzzy number describes the situation where a parameter value is "approximately m" or "about n." Fuzzy numbers are approximations of a central value and can be represented by "bell" curves, triangular distributions and so on (Cox 1994). A standard form of fuzzy number that allows for computational efficiency is that of the L-R (left-right) fuzzy number. A fuzzy L-R number M is fully characterized by three parameters—m is the mean value of M and σ and β are the left and right spreads, respectively. It is defined as:

$$\mu_M(x) = \begin{cases} L\left(\frac{m-x}{\sigma}\right) & x \leq m \quad \sigma > 0 \\ R\left(\frac{x-m}{\beta}\right) & x \geq m \quad \beta > o \end{cases} \tag{8}$$

and written as $(m,\sigma,\beta)_{LR}$. Operations for fuzzy numbers of the L-R type have been provided by Sakawa (1993, pp.26-30). Given the fuzzy numbers $M=(m,\sigma,\beta)_{LR}$ and $N=(n,\gamma,\delta)_{LR}$, the basic L-R fuzzy operators, modified for symmetric possibility functions (where $\beta=\sigma$ and $\gamma=\delta$), are as follows:

Addition: $(m,\beta)_{Sym} \oplus (n,\gamma)_{Sym} = (m+n, \beta+\gamma)_{Sym}$ (9)

Subtraction: $(m,\beta)_{Sym} \ominus (n,\gamma)_{Sym} = (m-n, \beta+\gamma)_{Sym}$ (10)

Multiplication: $(m,\beta)_{Sym} \otimes (n,\gamma)_{Sym} = (mn, n\beta+m\gamma)_{Sym}$, iff $m,n \geq 0$ (11)

Scalar multiplication: $k \otimes (m,\beta)_{Sym} = (km,k\beta)_{Sym}$ (12)

The fuzzy number $M=(m,\beta)_{Sym}$ resembles a fuzzy membership function in appearance and is used to describe a continuous quantity distribution about an imprecise parameter. The fuzzy membership function can also be considered a fuzzy number to describe the fuzzy class associated with the fuzzy objective.[2]

Another concept required for model building with fuzzy sets is that of the α-level set. The α-level set A_α is simply that subset of $\tilde{A}$ for which the degree of membership exceeds the level α, and is itself a crisp set (an element either meets the required level of α or it does not).

$$A_\alpha = \{ x \mid \mu_A(x) \geq \alpha \}, \alpha \in [0,1]. \tag{13}$$

A_α is an upper level set of $\tilde{A}$. The use of α-level sets provides a means of transferring information from a fuzzy set into a crisp form. Defining an α-level set is referred to as taking an α-cut, cutting off that portion of the fuzzy set whose members do not have the required membership or possibility value. It can be argued that the level of the α-cut is a measure of the faith that the decision maker has in the reliability of the imprecise

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[2]Although fuzzy numbers assume symmetry, this does not presuppose symmetry in solution sets. The response in any one parameter value to a change in the fuzzy quantity is strictly a function of the spread (β_{ij}) defined for that number. The spread completely defines the slope of the linear possibility distribution and thus the rate of change in value. It is the net result of all such independent movements that determines the ultimate solution.

coefficient. The more the decision maker's confidence, the higher the α-cut is set.

4. Fuzzy Multiple Objective Decision Making

A fuzzy model with vague preferences and imprecise coefficients can be formulated as a crisp LP, as illustrated in this section. We focus on a decision maker seeking to maximize satisfaction of a number of different vaguely defined objectives, subject to imprecise biophysical and socioeconomic constraints. Like classical goal programming, fuzzy MODM allows simultaneous consideration of all objectives and constraints without a requirement for ranking or weighing them. The specification differs because the fuzzy model centers around the concept of membership functions, and allows for uncertainty in the various model parameters. It is the philosophy of how the decision alternatives are to be measured and ranked that distinguishes fuzzy MODM from the more traditional approaches, but the measurement of the "goodness" of a solution is a matter of philosophy (Ignizio 1983).[3]

Fuzzy multiple objective linear programming

In the fuzzy MODM model, we are concerned with uncertainty surrounding the definition of satisfactory solution values for each of the objective functions. Although a precise value for each objective is provided by the model, it is unclear as to how well that

[3]We acknowledge that classical formulations exist that closely resemble the fuzzy model developed here—the class of "minimum distance models" bears some resemblance. However, the setup of the problem would be very different, and so is the conceptualisation in the context of uncertainty.

value represents the concept of a fully satisfied objective. The term satisfactory solution is vaguely defined; it is a fuzzy set. Thus, a goal G(x) or constraint C(x) may be completely satisfied by choice of the solution vector x ($\mu_G(x) = 1$ or $\mu_C(x) = 1$), completely unsatisfied ($\mu_G(x) = 0$ or $\mu_C(x) = 0$), or somewhat satisfied ($0 < \mu_G(x), \mu_C(x) < 1$). Crisp goals and constraints are accommodated in this framework by defining a crisp set as a specialized case of a fuzzy set.

The decision space, μ_D, is the fuzzy set defined by the intersection of the fuzzy goal and the fuzzy constraint, and is characterized as

$$\mu_D(x) = \min\,(\mu_G(x), \mu_C(x)). \tag{14}$$

The decision space defined is illustrated in Figure 2.

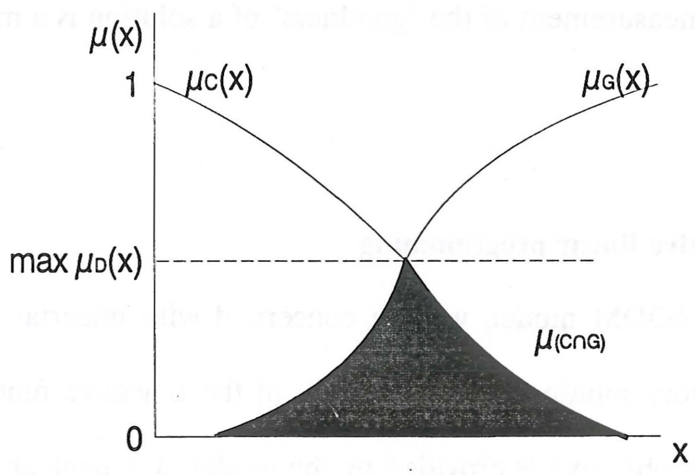

Figure 1: Illustration of Fuzzy Decision

It follows that, in order to maximize the degree of satisfaction of the goals and constraints, the objective function for the fuzzy linear programming model is:

$$\text{max. } \mu_D(x) = \text{max. min. } (\mu_G(x), \mu_C(x)), \quad x \in X \tag{15}$$

This maxmin operator is but one of several ways to represent the decision. It has the advantage that it is simple and linear, but it may fail to capture the true decision making process. There is an implicit assumption in the use of maxmin that all goals and constraints are weighted equally. This operator also fails to consider tradeoffs available between the various goals and constraints—it is non-compensatory. The solution is obtained where the minimum membership value has been maximized, or the lowest level of satisfaction has been raised as high as possible. Given its limitations, in the absence of good evidence to argue for another decision operator, the maxmin approach is favoured in the literature.

The decision model can now be written as a crisp linear program (LP). Suppose that the original fuzzy MODM is as follows:

$$\begin{array}{lll} \text{find} & x & \\ \text{s.t.} & A_i x \tilde{\geq} b_i & i = 1,2,\ldots,m \\ & x \geq 0, & \end{array} \tag{16}$$

where m is the number of goals plus constraints, A_i refers to the crisp parameter values, and $\tilde{\geq}$ refers to fuzzy objective or constraint sets. Then the crisp representation of the fuzzy MODM (16) can be written as:

$$\begin{array}{lll} \max & \lambda & \\ \text{s.t.} & \mu_i(x) - \lambda \geq 0, & i = 1,2,\ldots,m \\ & \lambda \in [0, 1], \text{ and} & \\ & x \geq 0. & \end{array} \tag{17}$$

where $\lambda = \mu_D(x)$. The interpretation of λ is that it is the degree to which the decision maker (or society) is satisfied with the simultaneous attainment of all goals. Assuming linear membership functions, fuzzy model (17) is re-written as:

$$\begin{array}{ll} \text{Max} & \lambda \\ \text{s.t.} & A_i x - b_i - p_i(\lambda - 1) \geq 0, \\ & \lambda \in [0,1], \text{ and} \\ & x \geq 0. \end{array} \tag{18}$$

Problem (18) is a maxmin formulation of the fuzzy MODM. Other formulations using various "composite" objective functions (e.g., additive specification) are available. These are reviewed by Mendoza and Sprouse (1989), Sakawa (1993), and Lai and Hwang (1994).

Fuzzy possibilistic programming

Now consider the situation where the elements of the matrix A are not precisely known. The j-th element of $\tilde{A}_i$, $\tilde{a}_{ij}$, is described by a possibility distribution. Furthermore, assume that these possibility distributions are triangular and symmetric, allowing $\tilde{a}_{ij}$ to be written as the fuzzy number (m_{ij}, β_{ij}) with the possibility distribution:

$$\begin{array}{ll} \pi(a_{ij}) = 0 & ; a_{ij} \leq m_{ij} - \beta_{ij} \\ \pi(a_{ij}) = 1 + (a_{ij} - m_{ij})/\beta_{ij} & ; m_{ij} - \beta_{ij} < a_{ij} < m_{ij} \\ \pi(a_{ij}) = 1 & ; a_{ij} = m_{ij} \\ \pi(a_{ij}) = 1 - (m_{ij} - a_{ij})/\beta_{ij} & ; m_{ij} < a_{ij} < m_{ij} + \beta_{ij} \\ \pi(a_{ij}) = 0 & ; m_{ij} + \beta_{ij} \leq a_{ij}. \end{array} \tag{19}$$

This possibility function is depicted in Figure 2.

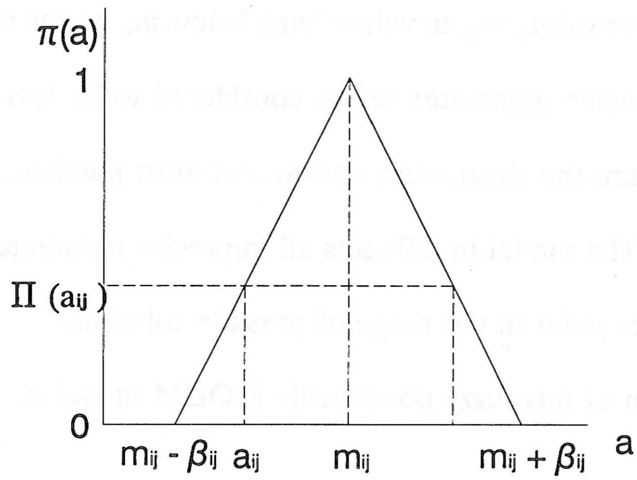

Figure 2: Possibilty Distribution for (m_{ij}, β_{ij})

To capture the effect of uncertainty in the model parameters, we employ an α-cut. This allows the definition of a crisp parameter value derived from the characteristics of the underlying possibility distribution, and permits the use of a standard LP format. The imprecise nature of the technical coefficients is incorporated into model (18) to give the following structure (Lai and Hwang 1994):

$$\begin{aligned} &\text{Max} && \lambda \\ &\text{s.t.} && [A_i - (1-\alpha)\beta_i]x - b_i - p_i(\lambda - 1) \geq 0, \\ & && \lambda \in [0,1], \text{ and} \\ & && x \geq 0. \end{aligned} \tag{20}$$

Model (20) allows for each element in the parameter matrix to be adjusted to reflect the level of possibility being considered. Each element $\tilde{a}_{ij}$ is transformed so that $\pi(a_{ij}) = \alpha$, the possibility that the value of a_{ij} is α. When $\alpha = 1$, this fuzzy possibilistic MODM formulation is identical to the fuzzy MODM discussed previously; only the most possible values are considered (i.e., α=1). As the level of possibility decreases, parameter values move away from the centre value, m_{ij}, to values lying below m_{ij} on the defined interval. The solution is now derived using parameter values considered to be less possible. This case reflects the situation where the parameters considered most possible are greater than the true parameter values. The model in (20) sets all imprecise parameters to the same level of possibility, one distinct point in the range of possible solutions.

An alternate form of this fuzzy possibilistic MODM model is:

$$\begin{array}{ll} \text{Max} & \lambda \\ \text{s.t.} & [A_i + (1-\alpha)\beta_i]x - b_i - p_i(\lambda - 1) \geq 0, \\ & \lambda \in [0,1], \text{ and} \\ & x \geq 0. \end{array} \tag{21}$$

This model uses less possible and higher values for the parameters. This represents the situation where the most possible values lie below the true values.

Considered jointly, these two models, (20) and (21), provide an upper and lower bound for possible solutions by considering the two extreme points of the possibility distributions defined by $\pi(a_{ij}) = \alpha$. As the decision maker is less confident that the central value of the possibility distribution is a correct representation of the true value, the α-cut is lowered (this can be visualised by lowering the horizontal line in Figure 2) and the length of the interval separating these bounds increases.[4] This interval identifies the range of possible solutions. Models (20) and (21) are restrictive in that they consider only the end-points of this interval, the extreme cases. By comparing the change to the solution with

[4]While λ measures the degree to which the decision maker is satisfied with the simultaneous achievement of all the objectives, the α-cut measures the confidence that the decision maker has in the central values of the uncertain parameters.

respect to a change in model possibility level, one can perform a type of sensitivity analysis with respect to uncertainty in the definition of parameters.

This maxmin approach will have a solution in which it will be impossible to increase the membership value of one objective without reducing the membership value of another. In this sense, the resulting solution is Pareto efficient. The model is now applied to land-use allocation on Vancouver Island.

5. A Decision Model for Land Use on Vancouver Island

Vancouver Island consists of nearly 3.35 million hectares (ha), of which 2.4 million ha is publicly owned and has been classified according to timber production potential. During deliberations, the Vancouver Island CORE employed the land use categories "high-intensity resource use," "integrated resource use," "low-intensity resource use," "protected areas" and "settlement" (van Kooten 1995). As public lands are the focus of this analysis, "settlement" lands and other private lands are ignored.

Goals reflecting the general public's expectations regarding forest land use in B.C. are taken from the 1989 Parksville Old-Growth Workshop (B.C. Ministry of Forests 1990). They are as follows: (1) achieve a high revenue from timber harvest; (2) create additional benefits from forest recreation activities; (3) obtain the greatest possible nonuse benefits from forests, as measured in monetary terms; (4) maintain forest employment; (5) collect substantial direct revenues from the forest industry; (6) achieve a high contribution of the forest sector to provincial GDP; and (7) expand wilderness protection.[5] Vague terminology renders each of these objectives fuzzy, and therefore the values for each cannot be precisely known. As discussed earlier, fuzziness is a measure of how well an instance or value conforms to a semantic ideal or concept. Hence, vagueness can be modeled through the specification of fuzzy objectives. Uncertainty due to ignorance about parameter values can be modeled through the use of fuzzy numbers.

Three types of MODM models for land-use decisions on Vancouver Island are compared to evaluate the usefulness of fuzzy MODM. The first is a crisp NPV maximizing formulation of the multiple goal problem.[6] The second is a fuzzy multiple objective decision model that incorporates the fuzziness of objective values. Finally, a fuzzy possibilistic multiple objective decision model with both fuzzy objectives and imprecise parameters, is considered. The models are essentially static and assume a normal forest. A planning period of 100 years is used—the assumed rotation age of the working forest. The first step in the modelling process is specification of the parameters.

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[5]An additional objective mentioned was maximizing long run sustained yield. However, since this does not seem a worthy objective in itself (rather it supports the 6 objectives listed in the text) it is not included in the analysis.

[6]This approach is standard practice in forest economics to cope with multiple objectives. Maximization of social welfare or NPV is achieved by adding the various accounts (e.g., Tecle *et al.* 1994; Mendoza *et al.* 1993).

Description of imprecise parameters

Logging benefits

Logging benefits per hectare are calculated as the difference between the price and the cost of a cubic meter (m^3) of delivered wood. Harvest volume is assumed to be a function of two harvest site attributes: site quality and management intensity.

Site quality is characterized as good, medium or poor. There is substantial variability among sites, even within the same category, in terms of their ability to provide logging benefits. Average harvest volumes by stand age, species and site class, for the B.C. coastal region, are taken from the FOREST6.0 model (Phelps *et al.* 1990a, 1990b). Uncertainty as to the realized harvest from a particular site is captured by specifying a range of possible harvest volumes based on consideration of extremes provided by species composition and consideration of a 20-year spread in harvest age.

In this paper, an area is assigned to the "high intensity management" category if intensive silviculture (spacing, pruning, pre-commercial thinning) is to be practiced. Under "integrated management," it is assumed that basic silviculture (site preparation and replanting) will be performed. Land allocated to "low intensity" management provides harvest volume from naturally regenerated stock. No harvest is available from "protected" areas.

Harvest volumes available from each of these nine land allocation categories (3 site qualities and 3 management categories) are described by a symmetrical triangular possibility distribution of the form (m_{ij}, β_{ij}). The centre value, m_{ij}, is the arithmetic mean of the extreme values.

Ranges for wood prices, based on species, age and management, are also taken from the FOREST6.0 simulation model, and possibility functions for the price parameters are calculated as for wood volume. These distributions are scaled to reflect the average 1992 wood price of \$70.71 /m^3 for the Coast region (see Price Waterhouse 1993).

Calculation of delivered wood costs follows the methodology outlined above, with the exception that costs vary with management intensity but are constant across site qualities. Two cost values are reported for the B.C. coastal region, one for low cost and another for high cost sites. The possibility distributions are based on the mean of the two cost figures and are scaled to reflect an average cost of delivered wood after stumpage fees, rents and royalties of \$65.13/m^3 (Price Waterhouse 1993). These costs do not include costs of silviculture. The Ministry of Forests (1992) provides average cost data for silvicultural activity in 1992. Basic silviculture was applied at a cost of \$21.20/ha, while incremental silviculture represented an added expense of \$20.00/ha. These costs are added in the appropriate management categories. Net logging benefits are calculated as the difference of total revenue per hectare and total costs per hectare. Using the definitions of fuzzy addition, subtraction and multiplication provided in equations (9)-(12), we obtain symmetric possibility functions. The results are summarized in Table 1.[7]

[7]The negative value associated with harvest of poor sites is consistent with the current situation: most of the current harvest is obtained from the better quality sites, and there is no margin to allow harvest of the inferior site class.

Recreation values

Recreation benefits are identified as a goal or objective for land use planning. Recreation plus recreation option value for the Vancouver forest region are estimated at $111.11 million per year (B.C. Ministry of Forests 1991), for an average recreational value of $33/ha/yr. This value was obtained under the current management, which is denoted as integrated management in this study. Land under low intensity management is assumed to offer little in increased recreational opportunities compared to integrated management, with the same recreational activities being pursued and logging ongoing. Land under intensive management is assumed to produce only 50% of the benefits attainable under integrated management as intensive forestry practices compromise recreational opportunities. Protected areas, with potentially more stringent guidelines as to appropriate recreational activities, will provide only 40% of the benefits received from the integrated management regime. Centres for the possibility distributions are scaled to preserve the gross average of $33/ha/yr, with distribution spreads arbitrarily set at $10/ha/yr for all classes. The results are summarized in Table 1.

Preservation values

Estimation of preservation or nonuse benefits are based on a survey conducted by Vold *et al.* (1994) that determined the values that B.C. residents place on wilderness protection in the province. The mean maximum annual willingness to pay for a doubling and tripling of wilderness area from a base of 5% were $136 and $168 per household, respectively. We assume that the number of households on Vancouver Island corresponds to the size of the labour force. Each household on the Island is then prepared to pay $32/yr ($168 minus $132) to increase the amount of protected area to 495,000 ha (15% of the total) from the current 10% level, corresponding to an average annual payment of $26.69/ha of protected area.[8]

Clearly the value of nonuse attributes falls with increasing management intensity, but there is little information for quantifying this relationship. The assumption is that low intensity management provides preservation benefits at 50% of that of protected areas, integrated management areas at 25% the level of protected areas, and land under high intensity management is assumed to provide no nonuse benefits. Distribution spreads for these fuzzy numbers are set to allow the range of possible values to begin at 0 and extend to twice the hypothesized value. The results are summarized in Table 1.

Forest sector employment

Forest related employment may be generated both by the forest industry and the forest-related tourism and recreation industry. Price Waterhouse (1993) reports 1.18 jobs/1,000 m^3 of wood harvested for the coastal industry. This estimate is reduced slightly to 1.16 jobs per 1,000 m^3 to reflect the fact that some of the jobs associated with the Island harvest are located in mainland mills. The spread for this fuzzy number is set at 0.07,

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[8]Preservation values are underestimated if part of the benefits accrue to people off Vancouver Island. On the other hand, they may be over-estimated because the marginal value of nonuse attributes is assumed constant while the study by Vold *et al.* indicates that these values may well be declining.

consistent with the variation reported by Statistics Canada (see COFI 1992) for the past decade.

There is little information about the relationship between employment and other uses of the forest. Regionally-based studies yield estimates of 0.0001 to 0.0003 jobs/ha (see Matas 1993; Clayton Resources Lt. and Robinson Consulting & Associates Ltd. undated). The latter figure is used to anchor the job possibility distributions with relatively large spreads to reflect the high degree of uncertainty regarding their genesis.

Direct and indirect government revenue

In 1992, the provincial government and municipalities received $5.27/m^3 of harvest, while the province collected $9.05/m^3 in stumpage fees for a total revenue of $14.32/m^3 (Price Waterhouse 1993). It is assumed that revenues can vary by as much as $5 /m^3.[9]

Indirect revenues are examined by looking at the contribution of forestry to provincial GDP. Forestry accounts for a substantial proportion of provincial GDP, indicating a high dependence on forest operations. Each cubic metre of harvest contributes about $70 directly to GDP. An interval of $20 /m^3 is chosen. The results are summarized in Table 1.

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[9] In this analysis, *direct* revenues to the provincial government do not include employee taxes paid as a result of indirect and induced employment, and revenues accruing to the federal government are ignored.

Expansion of Wilderness Protection

There is nothing uncertain about the contribution of a hectare of land towards the objective of wilderness protection. It is a crisp parameter—one hectare of land allocated to protected area provides one hectare of protected area.

Objective target values

The objectives are all modeled as fuzzy "greater than constraints". Thus, the degree of satisfaction increases as the value of the objective function increases. The value deemed to be the lowest possible to generate any satisfaction of the objective defines the lower limit of the constraint interval (b_i - d_i). The value deemed to be the lowest value at which complete satisfaction of the objective is attained defines the upper limit of the interval (b_i). This implies that the degree of membership of variable x in the fuzzy set W is given by an equation analagous to (3).

Two approaches can be used to determine the upper and lower values. The levels may be provided by a decision-maker or an expert in the area, relying on a subjective understanding of both the limits inherent in the system as well as what would constitute a satisfactory level of achievement. In this paper, "employment" is incorporated using this approach. A second approach is to define the upper and lower bounds as the maximum and minimum levels that the system can provide when each objective is considered in isolation. This approach may be especially suitable when objectives are less politically sensitive and not restricted to a narrow range *a priori*. It is an objective means to defining the fuzzy constraints and is appropriate when there is little information available regarding the problem, preventing initial specification of unrealistic objectives. In the current analysis, the objective "logging benefits" is incorporated using this approach. In theory, it would be preferable to use the second method to set initial parameters, and then use feedback information from users to refine the objective intervals, thereby incorporating new information on values or preference structure. The specification of satisfactory levels of achievement for this model employs a combination of the two approaches, without the benefit of any interactive procedure. The initial upper and lower bounds are provided in Table 2.

For logging benefits, the level for complete satisfaction is set as the maximum available from the model if only logging benefits are considered. The minimum represents the amount generated from a working forest of 700,000 hectares, even though such a scenario was rejected by CORE as too low. Recreation and preservation benefit intervals are defined by the maxima and minima available from the system (see Table 1).

Employment is a politically sensitive issue. We assume that the current level provided by the forest industry is fully satisfactory, even though it will be difficult to maintain current employment in the future as technological developments lead to a decreasing number of jobs per unit of harvest. Jobs related to recreation are also considered satisfactory at current levels, although, in actual fact, it would probably be less than satisfactory if current levels were simply maintained. However, recreation contributes only a very small number of jobs compared to those related to timber harvest; requiring an increase in this component has little impact in the model. The lower bound for job provision is arbitrarily set at 15% below the current level, on the assumption that it would be politically unwise for government to allow employment levels to drop below this figure (Table 2).

Maximal values for both direct and indirect revenue are determined by the ability of the system to generate revenues and timber-related GDP. Lower bounds again reflect the political nature of these objectives. It is assumed that a decrease of more than 20% in direct revenues, or of 25% in indirect revenues (i.e., forestry's contribution to GDP), would be politically unsatisfactory. Values for these fuzzy objectives are also provided in Table 2.

The final objective is that of wilderness expansion. Any increase in protected areas will most likely come from Crown land. A doubling of protected area on the Island would mean that 660,000 ha would be removed from the working forest, or about 30% of total Crown land. It is assumed that protecting almost a third of the public land on Vancouver Island would allow all PAS objectives to be met; thus, the decision maker is assumed to be completely satisfied at that level of wilderness protection. The lower level for the fuzzy objective is defined as the current area under protection, a level below current legislated requirements and thus considered unacceptable. The data are summarized in Table 2.

6. Empirical Results

A crisp MODM (in the form of an NPV maximizing LP model), a fuzzy MODM and a fuzzy possibilistic MODM were constructed. First, the results of the crisp formulations are compared to the fuzzy MODM, which is then compared with the fuzzy possibilistic model where the coefficients in the model are imprecise.

Fuzzy and crisp MODM

The crisp MODM model (or CRISP) maximizes net present value (NPV) subject to employment and wilderness conservation constraints. NPV is defined as the sum of logging benefits, recreation benefits, nonuse values and direct government revenues. The fuzzy MODM (FUZZY) considers all objectives of the previous section as independent and equal in terms of priority. The land allocations resulting from these models is presented in Table 3.

The CRISP model concentrates good and medium quality sites into high intensity management regimes, and allocates all poor sites to the low intensity system. In contrast, the FUZZY model places the larger proportion of medium quality forest land under integrated management as well as a small amount of good quality area. Total area assigned to the high management regime is less, and protected area is greater. Given that the B.C. Government has recently indicated that logging should take better account of non-timber benefits, it is interesting to note that the outcomes of the fuzzy model are more in line with government policy than the outcomes of the crisp model. On the other hand, it must be noted that there is very little movement (if any) of high quality land into protected areas in both the crisp and the fuzzy models, which is at odds with the philosophy and intent of the Protected Areas Strategy.

Differences in the allocation schemes are evident in the levels attained for each of the objective functions (see Table 4). The logging benefits are greater under the crisp formulation, and as a consequence so are direct government revenues and employment, but logging benefits are of such a magnitude that they dominate other forest services when weighed equally.[10] The fuzzy model provides a higher return on the other accounts.

There is a membership function (μ_i) associated with each of the objective functions indicating the level of satisfaction attained for each objective. Focusing on the FUZZY model, we find that the minimum degree of satisfaction is attained for four of the seven objectives (Table 5). Given that the model provides a Pareto efficient solution, the interpretation is that it is impossible to increase the satisfaction level for any one of these four without compromising that of at least one of the other three. The standoff is between logging benefits and employment on the one hand, and preservation values and protected areas on the other. This situation reflects the reality of the conflicts identified in the Vancouver Island land use debate.

Perhaps the difference in performance between the crisp and fuzzy models can be explained by the fact that the former resembles a cost-benefit analysis whereas the latter is more like a true MODM. However, it is impossible to conclude to what extent the divergence between the crisp and the fuzzy models is caused by the difference between crisp and fuzzy modeling *per se*. Interpretation is blurred by the different nature of both models: maximizing NPV subject to constraints in the one and balancing objectives in the other.[11]

[10]Interestingly, both models provide an annual harvest that exceed the current LRSY of 11.0 million m^3. This is largely due to the application of intensive silvicultural practices to a substantial portion of the Crown land base, contrary to current conditions.

[11]Bare and Mendoza (1992) provide a review of the consequences of modeling random coefficients as if they are crisp.

A fuzzy possibilistic MODM

Symmetric possibility distributions are used to model the uncertainty surrounding the precision of the parameters in the model. One purpose is to gain some understanding about the sensitivity of the solution to uncertainty in parameter definition. By lowering the value of α (the fuzzy MODM has an implicit α value of 1), the effect of this uncertainty upon optimal land allocation can be explored. At any value of $\alpha<1$ there are two solutions to consider. The first is from model (20), where parameter values take on a less-likely and lower value (the LOWER results); the second is from (21), generating a solution based on parameter values of the same possibility but higher value (the UPPER model results). The results from the two models are provided in Tables 6 and 7.

The most obvious result obtained from the variation of the possibility level is in the asymmetry of the feasible solution space. While solutions may be obtained for any value of α using the UPPER model, feasible solutions do not exist below a possibility level of 0.92 for the LOWER model. Parameter values are unable to provide any level of satisfaction of at least one of the objectives; in this case, the limiting objective is timber benefits.

An unexpected result is that the LOWER model, with α = 0.95, provides for over 10% more protected area than does any of the other scenarios considered. The minimum amount is provided by the FUZZY model. The rationale for this is that the LOWER model concentrates the good and medium quality sites into the high intensity management category, a massive shift of almost 400,000 ha as compared to the integrated management allocation level of the FUZZY model. This occurs in response to the lower estimation of both wood yield and wood value. This causes a large reduction in nonuse benefits as the high intensity management category does not contribute to this objective. The shortfall is replaced by the allocation of poor quality area, with it's negative logging value, into the protected area category.[12] Harvest volume declines under this LOWER scenario and job numbers fall slightly (Table 7). Monetary benefits are also slightly lower with the largest change observed in recreation benefits; logging benefits are virtually unchanged (Table 7).

It is our opinion that the dramatic effects of small parameter adjustments provides an additional reason to model the imprecision associated with parameter estimates explicitly. From Table 6, it is clear that, if there is imprecision in the parameters, failure to model this imprecision (i.e., modeling the parameters as if they are crisp) results in distorted (less than efficient) land use allocations. The analysis conducted here enables decision makers to identify sources of imprecision when it comes to land allocation. As information gathering is costly and with limited funds available to overcome parameter imprecision, there is a definite value in knowing which areas to research first. Under the current conditions it seems that it is most crucial to address imprecision in the logging benefit parameters.

The results obtained from the UPPER model as α is decreased are as expected. All parameters of the model increase in value as α decreases, resulting in a higher provision of benefits from each hectare of land considered. Increasing yields and wood values allow less area to be allocated to the high intensity regime and more to integrated management. The result is an increase in both recreation and nonuse benefits. Harvest volume rises and job provision increases, evidence of the less possible higher per ha yield estimates and a greater number of jobs per unit of harvest.

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[12]This is likely an unacceptable result if quality of protected areas is important.

In this specific analysis, the solution provided by the FUZZY model is sensitive to overestimation of the true parameter values. If values are realized at a generally lower level than those judged most likely, a large shift in resource allocation is required to obtain the best solution as judged by the maximization of minimum objective satisfaction. Results of the UPPER model seem to indicate that we have little to fear from the general underestimation of parameter values. If true values are realized at some level above those judged most likely, the error in planning has simply been that more land than required was allocated to high intensity use, while more could have been allocated to protected area, without jeopardizing other objectives.

7. Conclusions

Increasing weight is placed on non-timber values in managing forests or making allocation plans for woodlands. The public is also becoming more involved in the planning process. Both trends are evident in British Columbia in the new forest management policies aimed at environmental concerns and a CORE process the relies on stakeholder participation. In most instances, economic efficiency is only one of many competing considerations, with cost-benefit analysis often relegated to a status below that of other concerns, such as employment. Hence, decision models need to be sensitive to the existence of multiple objectives and the fact that the objectives themselves and the parameters that characterize them are imprecise and vague. This study applied fuzzy and fuzzy possibilistic MODMs to the problem of allocating public forest land on Vancouver Island, comparing the results of the fuzzy models with a more traditional, crisp approach.

Given the nature of the process that is to be modeled, our conclusion is that the fuzzy and fuzzy-possibilistic approaches can be judged a distinct improvement over the traditional approach of constrained maximization of net present value. For example, the fuzzy MODM allocated about 25% of the land base to integrated timber management, while the traditional (crisp) model concentrated land into the extreme categories of low (natural regeneration) and intensive timber management intensity. The area assigned to the protected category was greater in the fuzzy MODM than in the crisp model, as was the number of direct jobs provided in the forest sector. We conclude that the decision by CORE not to rely on maximization of NPV is confirmed by comparing the results of the crisp and fuzzy models. The fuzzy solution was obtained without needing to specify precise values for objectives, and without an explicit ranking or weighing of the objective functions. The fuzzy MODM also clearly identified those objectives that were in direct conflict with each other, and thus the areas where compromise is required if satisfaction levels are to be increased.

The results from the fuzzy possibilistic MODM model suggest that the approach of combining fuzzy parameter specifications with fuzzy objectives constitutes an improvement over the fuzzy MODM. However, the model specified in this study was very sensitive to the possibility of lower realizations of parameter values, but this only highlights the importance of modeling imprecise parameters using fuzzy numbers instead of flexible constraints. The analysis provides an insight in what kind of additional information is especially valuable for obtaining robust land allocations—robust in the sense that small mis-specifications of parameters will not cause massive shifts from one land use option to another, which in practice may be costly to achieve.

Finally, areas for future research suggest themselves. The most important of these is that of getting stakeholders/decision makers involved in the development of both fuzzy objectives and fuzzy numbers for the technical coefficients of the decision model. Fuzzy set theory offers a means of combining information from various stakeholders. Research is required to determine how information can be updated when those involved in the decision process are presented with results and, just as importantly, how natural language can be used to develop the required fuzzy measures.

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Table 1: Fuzzy parameter values, mean and spread, in C$.

Site Quality	Logging benefits	Recreation values	Nonuse values	Employment	Direct Gov. Rev.	Indirect Gov. Rev.
High Intensity Management						
good	(260, 64)	(21.9, 10)	(0,0)	(0.0138, 0.0045)	(170, 150)	(833, 343)
medium	(98, 67)	(21.9, 10)	(0,0)	(0.0100, 0.0027)	(123, 69)	(603, 215)
poor	(39, 65)	(21.9, 10)	(0,0)	(0.0058, 0.0029)	(71, 49)	(349, 167)
Integrated Use Management						
good	(137, 75)	(43.9, 10)	(6.7,6.7)	(0.0131, 0.0040)	(162, 96)	(792, 304)
medium	(76, 67)	(43.9, 10)	(6.7,6.7)	(0.0087, 0.0026)	(107, 63)	(525, 199)
poor	(26, 49)	(43.9, 10)	(6.7,6.7)	(0.0044, 0.0023)	(54, 45)	(266, 163)
Low Intensity Management						
good	(111, 50)	(43.9, 10)	(13.4,13.4)	(0.0101, 0.0028)	(124, 71)	(608, 221)
medium	(57, 38)	(43.9, 10)	(13.4,13.4)	(0.0061, 0.0018)	(75, 44)	(368, 141)
poor	(25, 29)	(43.9, 10)	(13.4,13.4)	(0.0033, 0.0015)	(41, 30)	(202, 104)
PROTECTED	-	(26.3, 10)	(26.8,26.8)	-	-	-

Table 2: Fuzzy objective specification

Objective	Lower bound	Upper bound	Spread
Logging ($'mil.)	48.1	72.0	23.9
Recreation ($'mil.)	49.9	90.1	40.2
Passive use ($'mil.)	8.6	60.0	53.4
Employment	13.5	15,700	2,355
Direct revenue ($'mil.)	139.2	174.0	34.8
Indirect revenue ($'mil.)	640.5	854.0	213.5
Protected area ('ha)	341,000	660,000	319,000

Table 3: Simulation Results for Fuzzy and crisp MODM: Land Allocation (hectares)

Site Quality	Management Intensity	CRISP	FUZZY
	High	223845	209848
Good	Integrated	0	13994
	Low	0	0
	High	891016	363747
Medium	Integrated	0	527269
	Low	0	0
	High	0	0
Poor	Integrated	0	0
	Low	754644	658671
Protected		341000	436976

Table 4: Simulation Results for Fuzzy and Crisp MODMs

Benefits	CRISP	FUZZY
Logging	$ 68.6	$ 55.3
Recreation	$ 66.5	$ 76.7
Passive Use	$ 19.1	$ 24.0
Direct Revenue	$ 179.2	$ 166.7
Indirect Revenue	$ 875.9	$ 814.8
Employment	15066	14054

Table 5: Satisfaction of Objective Targets

	Model		
	GOAL1	**GOAL2**	**FUZZY**
Logging	0.64	0.66	0.30*
Recreation	0.65	0.62	0.67
Passive Use	0.31	0.33	0.30*
Direct Revenue	0.58	0.49	0.79
Indirect Revenue	0.65	0.57	0.82
Employment	0.06	0.00	0.30*
Protected Area	0.00	0.19	0.30*

*indicates minimum satisfaction level for model

Table 6: Simulation Results for Fuzzy Possibilistic MODM: Land Allocation (hectares)

Management Intensity	LOWER α =0.95	FUZZY α =1	α =0.95	UPPER α =0.9	α =0.80
Good Site Quality					
High	223,842	209,848	136,697	69,680	0
Integrated	0	13,994	87,145	154,162	223,842
Low	0	0	0	0	0
Medium Site Quality					
High	753,259	363,747	330,668	300,593	213,323
Integrated	137,757	527,269	560,328	590,423	677,693
Low	0	0	0	0	0
Poor Site Quality					
High	0	0	0	0	0
Integrated	0	0	0	0	0
Low	543,453	658,671	645,434	632,048	605,625
Protected Area	552,194	436,976	450,213	463,599	490,022
Total Allocated	2,210,505	2,210,505	2,210,505	2,210,505	2,210,505

Table 7: Simulation Results for Fuzzy Possibilistic MODM: Monetary and Employment Benefits

Benefits	LOWER α =0.95	FUZZY α =1	α =0.95	UPPER α =0.9	α =0.80
Logging	$54.2	$55.3	$56.3	$57.3	$66.1
Recreation	$64.7	$76.7	$79.9	$82.9	$88.1
Passive Use	$21.7	$24.0	$26.2	$28.4	$32.6
Direct Revenue	$163.2	$166.7	$170.0	$173.3	$179.7
Indirect Revenue	$806.3	$814.8	$822.7	$830.6	$846.0
Employment	13,948	14,054	14,151	14,250	14,445