APPLICATION OF BASAL AREA INDEX IN THE YIELD ESTIMATION OF BOREAL MIXEDWOOD COVER TYPES

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ABSTRACT

This paper contains a brief description of the principles of point sampling and its application in the mixedwoods of Ontario. It also describes a recently developed measure of utilized site productivity, namely "basal area index", which is used to construct variable stocking yield functions and tables for the boreal mixedwoods of north central Ontario. The basal area index employed is more appropriate for the boreal mixedwood cover type and has several advantages over the site index. For example: a) it is empirically related to yield parameters commonly used in stand level management planning, b) it produces variable stocking yield tables suitable for uneven-aged mixed species cover types, and c) it can be estimated efficiently by employing point sampling techniques. The basic improvement in mixedwood yield estimation using a basal area index may also have applications for other stand types, particularly for the disturbed hardwoods of southern Ontario and the eastern United States.

RÉSUMÉ

L'auteur décrit brièvement les principes de l'échantillonnage par points et son application aux peuplements mixtes de l'Ontario. Il décrit également une mesure de la productivité des sites utilisés, mise au point récemment et appelée «indice de surface terrière», qui est employée pour construire des fonctions et des tables de production à densité variable pour les peuplements mixtes boréaux du centre-nord de l'Ontario. Cet indice convient mieux au type de couvert des peuplements boréaux et offre plusieurs avantages par rapport à l'indice de site. Par exemple : a) il est empiriquement relié aux paramètres de la production communément employés pour la planification de l'aménagement au niveau des peuplements; b) il produit des tables de production à densité variable qui conviennent aux peuplements mixtes inéquiennes; c) il peut être estimé efficacement à l'aide des techniques d'échantillonnage par points. L'amélioration de l'estimation de la production obtenue avec cet indice de la surface terrière pour les peuplements mixtes peut également avoir des applications pour d'autres types de peuplement, en particulier les peuplements feuillus perturbés du sud de l'Ontario et de l'est des États-Unis.

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Cover photo: Point sampling in a black spruce stand in northern Ontario.

INTRODUCTION

Payandeh and Wang[1] recently developed basal area indices for major timber species in Ontario. As well, these authors constructed variable stocking yield functions and tables for the boreal mixedwoods of north central Ontario. Unlike the site index, the basal area index may be estimated quickly, accurately, and inexpensively via point sampling or prism cruising. This paper briefly describes the application of point sampling in forest inventories of the boreal mixedwoods of Ontario. Similar procedures may be used for other stand types, particularly the disturbed hardwoods of southern Ontario and the eastern United States.

PRINCIPLES OF POINT SAMPLING

Point sampling or variable plot cruising is one of the most widely used methods employed in forest sampling within North America (Husch et al. 1972). It was originally developed by Bitterlich in Austria in 1948 and subsequently introduced into North America by Grosenbaugh (1952a, b; Afanasiev 1957).

Bruce (1955) introduced the wedge prism as a simple device (relascope) for sample tree selection when using point sampling in the field. Later, an even simpler device called a Cruzall was manufactured by Forestry Suppliers Inc.[2]

In conventional plot sampling, the probability of selecting any given tree is proportional to tree frequency; in point sampling it is proportional to tree size or basal area. Therefore, point sampling employs the idea that if an observer looks around a stand of trees with a fixed visual angle, the number of trees that appear larger than the angle is proportional to the stand basal area.

Using a relascope with a constant (critical) angle, the cruiser rotates 360 degrees around a fixed spatial position (sampling point) and counts the number of trees whose diameters at breast height (DBH) appear to exceed the critical angle (Fig. 1). By multiplying the number of tallied trees by a constant, the basal area factor, the basal area per unit area is immediately known. The theory of point sampling has been described in many articles and texts (*cf.* Grosenbaugh 1952a, Dilworth 1964, Husch et al. 1972) and thus only a brief summary is included here.

In Figure 1, the circles represent the cross-section of trees at breast height, and the lines indicate the angle projected from the sampling point. Any variables associated with the selected trees may be measured, just as in the case of fixed-area plots.

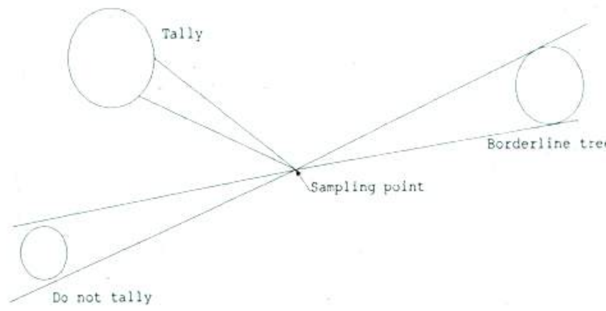

Figure 1. Selection of trees in point sampling (from *Husch et al. 1972).*

However, the unique feature of horizontal point sampling is that tree measurements are not needed to obtain an unbiased estimate of basal area per unit area from that sampling point. Regardless of its size, each qualifying tree, i.e., each tree larger than the projected angle, represents the same basal area per unit area (Fig. 2).

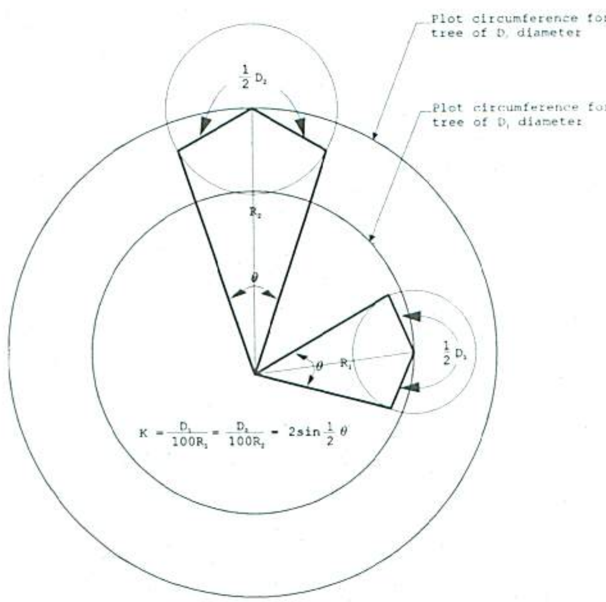

Figure 2. A plan diagram of two trees is shown where the gauge angle (θ) is precisely tangential to the breast height cross-section: Gauge constant K = D/100 R, where D is the tree diameter in centimeters and R is the plot radius in meters (from *Husch et al. 1972).*

[1] Payandeh, B.; Wang, Y. Variable stocking yield functions and tables for the boreal mixedwoods of north central Ontario. For. Chron. (In press)

[2] McCool, J.J. Forestry Suppliers Inc., Jackson, MS. Personal communication.

The essential properties to note are:

1) The angle gauge projects a fixed horizontal angle, θ.

2) At any sampling point a series of concentric circular plots, with varying plot radii for each tree diameter, is conceptually established.

3) The radius of each concentric plot is determined by each tree diameter and is not influenced by the actual spatial location of the tree. Therefore, for the purpose of development of the theory, all trees can be considered in the "borderline" condition as shown in Figure 2.

4) For the borderline condition, the ratio of tree diameter to plot radius is a constant. Thus, for a given gauge angle, θ; tree diameter in centimeters, D; and plot radius in meters, R; the angle gauge constant K may be defined as:

$$[1] \quad K = D/100R = 2 \sin \theta/2$$

5) The basal area per hectare represented by each qualifying tree, F, is independent of tree diameter, since: F = (basal area for the tree) (factor to convert to per hectare basis).

$$[2] \quad F = BA_i \frac{\textit{Area of one hectare}}{\textit{Plot area for the } i_{th} \textit{ tree}} = \frac{\Pi D_i^2}{4(10\,000)} * \frac{10\,000}{\Pi R_i^2}$$

$$\text{since } K^2 = \frac{D^2}{10\,000 R^2}, \textit{ then } \frac{D_i^2}{4R_i^2} = 2\,500 K^2$$

It should be clear that one can choose a gauge constant, K, such that 2 500 K^2 is some convenient number. In practice it is necessary to first choose a specific value of F, and so fix the ratio of $D_i/100\ R_i$ which, as a consequence, implies a different plot radius for every individual tree diameter. Thus, all trees of the same diameter that are located less than their "plot radius" distance from a given sampling point will be "in the plot" and their basal area can be converted to a per hectare basis. For example, if a gauge angle is used for which K = 0.04472, then for each tree greater than the projected angle: Basal area factor = F = (tree BA) (tree factor).

$$[3] \quad F = BA_i \left(\frac{2\,500 K^2}{BA_i} \right)$$

$$F = 2\,500\ K^2$$
$$F = 2\,500\ (0.04472)^2$$
$$F = 5.0\ m^2/ha.$$

Dixon (1958), Dilworth (1964), and Husch et al. (1972) have described several devices for horizontal point sampling. These typically use angle gauges that "project" small horizontal angles, generally under 5 degrees. A tree that appears larger than the projected angle is considered "in", and a tree that appears smaller than the projected angle is considered "out". Most devices provide a choice of angles, and some have an automatic slope correction, such as the Spiegal Relascope and the recent metric version of the Cruzall (Husch et al. 1972, Forestry Suppliers Inc. 1995). The latter provides basal area factors of 2, 4, 6, or 10 m^2/ha (Fig. 3).

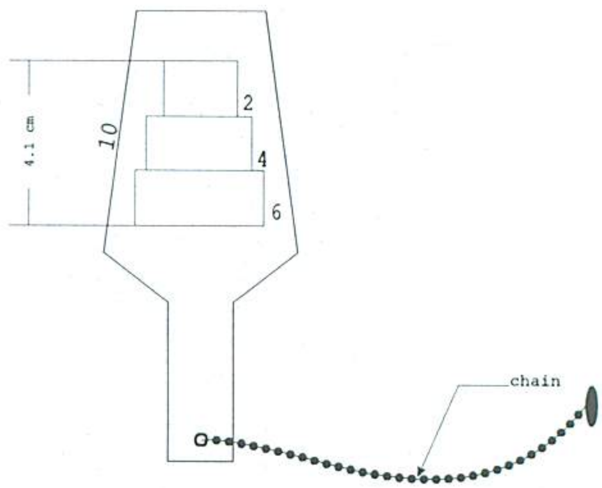

Figure 3. A metric Cruzall with basal area factors, of F = 2, 4, 6, or 10 m^2/ha.

Kullow (1966), Meyer and Beers (1968, 1971), and others have indicated that basal area factors should be chosen so as to provide an optimum number of "in" trees per sampling point. These authors have recommended 10–15 trees/point as satisfactory. However, mixing several basal area factors in the same inventory should be avoided because it complicates variance estimation.

BASAL AREA INDEX

Site index (i.e., mean dominant stand height at a specified base age, e.g., 50 years for most Canadian tree species) is widely used to measure stand productivity in North America (*cf.* Carmean 1975, Monserud 1984a). However, by definition, site index is not really applicable to uneven-aged mixedwoods stands (*cf.* Clutter et al. 1983; Monserud 1984b, 1987). To overcome such a difficulty Payandeh and Wang (1996) recently developed a basal area index (BI), based on the following constrained biological growth model (Ek 1971, Newnham 1988, Payandeh and Wang 1993), to express the total basal area growth patterns over age for major timber species in Ontario:

$$[4] \quad BAT = b_1 BI^{b_2}\left(1 - e^{-b_3 t}\right)^{p}$$

$$with\ p = \frac{\frac{BI}{b_1 BI^{b_2}}}{\ln\left(1 - e^{-b_3 t_1}\right)}$$

where: BAT is the stand total basal area (m^2/ha) at age t,
- BI is the basal area index or total stand basal area (m^2/ha) at a specified base age (e.g., 50 years),
- t is the stand age (years),
- t_1 is the index age (years),
- b_i's are model parameters,
- p is the conditional constraint (Newnham 1988, Payandeh and Wang 1993) necessary to satisfy the basal area index definition where: BAT = BI at index age or 50 years, and
- e is the base of natural logarithm.

The above model may be used either to estimate BAT from BI and t, or to estimate BI from a given BAT and t via numerical methods (Wang and Payandeh 1993). The above model was fitted to Plonski's (1974) normal yield tables for the major tree species in Ontario.[4] Such indices (summarized in Table 1) should be better correlated with the main stand attributes, such as stocking and volume, than would site index, because BI is based on all trees in the stand rather than on only a few site trees.

MATERIALS AND METHODS

The data set used here for demonstration consists of the tallies of 197 circular permanent sample plots (0.05 to 0.08 ha) maintained by the James River/Marathon Company Ltd. Each plot had been measured two to six times, mostly at 5-year intervals. Plot data included: cover type, initial stand age (average age of dominant trees of the main cover-type species), plot number, number of trees, species code, and diameter of all trees with a DBH > 1.5 cm. In the last remeasurement (1978), the height and age of five to seven dominant trees/plot were also measured. The data were described in detail by Payandeh and Field (1986). For each plot, the basal area index was calculated as described above.

ANALYSIS AND RESULTS

The following nonlinear regression models (*cf.* Clutter et al. 1983, Monserud 1984a, Payandeh 1988) were used to express various stand yield attributes as a function of basal area index and stand age:

$$[5] \quad H = b_1 BI^{b_2}\left(1 - e^{b_3 t}\right)^{b_4 BI^{b_5}}$$

$$[6] \quad N = b_1\, BI^{b_2} t^{b_3}$$

$$[7] \quad BAT = \frac{b_1 BI^{b_2}}{1 + \exp\left(b_3 + b_4 \ln(t) + b_5 \ln(BI)\right)}$$

$$[8] \quad BAM = \frac{b_1 BAT^{b_2}}{1 + \exp\left(b_3 + b_4 \ln(t) + b_5 \ln(BI)\right)}$$

$$[9] \quad VT = \frac{b_1 BI^{b_2}}{1 + \exp\left(b_3 + b_4 \ln(t) + b_5 \ln(BI)\right)}$$

$$[10] \quad VM = \frac{b_1 V_t^{b_2}}{1 + \exp\left(b_3 + b_4 \ln(t) + b_5 \ln(BI)\right)}$$

Table 1. Estimated parameters of Equation [4] for major Ontario timber species.

Species	Estimated parameters			Regression fit statistics		
	b_1	b_2	b_3	R^2	RMSE	SEE
Black spruce	10.1416	0.4256	0.0433	0.99	0.32	0.566
Jack pine	2.8210	0.6947	0.0627	0.99	0.39	0.625
Aspen	1.9608	0.8650	0.0420	0.99	0.41	0.640
White birch	1.2194	0.9796	0.0679	0.99	0.46	0.678
White spruce	2.8339	0.7686	0.1044	0.99	1.92	1.386
Tolerant hardwood	5.5366	0.6626	0.0142	0.99	0.38	0.616
Red pine	8.6018	0.4461	0.0548	0.99	0.86	0.927
White pine	4.8313	0.5963	0.0502	0.99	0.53	0.728

[4] Ibid.

Where: H is the mean stand height (m),

N is the stand density (trees/ha),

BAT is the total basal area (m^2/ha),

BAM is the merchantable basal area (m^2/ha),

VT is the total volume (m^3/ha), and

VM is the merchantable volume (m^3/ha) and other variables as defined earlier.

These models were fitted to the initial measurement (only) of the 197 permanent sample plots.

Table 2 summarizes the estimated parameters for the yield models for various stand attributes along with their respective fit statistics. With the exception of stand height and density, the resulting regression equations for the other stand attributes produced good to excellent fits, as indicated by their respective R^2 and Residual Mean Square Errors (RMSE) values. Yield tables were generated from the solution of these regression equations (*see* Tables 3–5). These tables provide various yield attributes for the boreal mixedwoods of north central Ontario and for basal area indices of 20, 30, and 40 m^2/ha, respectively.

APPLICATION

The application of basal area indices to pure even-aged, well stocked stands of major Ontario timber species (Table 1) should be straightforward. But for uneven-aged, multispecies stands (e.g., boreal mixedwoods), estimates

Table 2. Estimated parameters for various yield attributes for the boreal mixedwoods of north central Ontario.

	Parameter estimate					Regression fit statistics		
Yield characteristic	b_1	b_2	b_3	b_4	b_5	R^2	RMSE	SEE
Stand height	28.2786	-0.0221	-0.0176	1.9514	-0.0741	0.48	3.50	1.87
Density	364 661.8924	0.0101	-1.1852			0.52	1569	39.61
Total basal area	12.9024	-1.9774	-1.8540			0.83	3.90	1.97
Merchantable basal area	0.9711	1.0014	16.4496	-3.1089	-1.7072	0.92	3.36	1.83
Total volume	15.0185	1.0114	4.5222	-1.2009	0.1416	0.74	60.16	7.76
Merchantable volume	1.6028	1.1866	0.8770	-0.0172	0.1617	0.98	17.03	4.13

Table 3. Various yield attributes for the boreal mixedwoods of north central Ontario and for a basal area index = 20 m^2/ha.

Stand age (yr)	Number of trees/ha (>1.5 cm)	Average DBH[a] (>1.5 cm)	Average height (m)	Basal area (m^2/ha) (>1.5 cm)	(>10 cm)	Volume (m^3/ha) (>1.5 cm)	(>10 cm)	PAI[b] (m^3/ha)	MAI[c] (m^3/ha)
20	10 791	2.8	4.0	6.5	0.7	64.0	47.4	0.00	2.37
30	6 673	4.7	6.6	11.7	3.6	92.3	73.5	2.61	2.45
40	4 745	6.6	9.1	16.3	8.5	116.1	97.0	2.35	2.43
50	3 643	8.4	11.5	20.0	13.6	136.2	117.5	2.05	2.35
60	2 935	9.9	13.6	22.8	17.8	153.1	135.3	1.78	2.26
70	2 445	11.4	15.4	24.9	21.0	167.5	150.8	1.55	2.15
80	2 087	12.7	17.1	26.5	23.5	179.8	164.4	1.36	2.01
90	1 815	13.9	18.5	27.7	25.2	190.3	176.2	1.18	1.96
100	1 602	15.1	19.7	28.6	26.6	199.5	186.6	1.04	1.87
110	1 431	16.2	20.7	29.4	27.6	207.6	195.8	0.92	1.78
120	1 291	17.2	21.6	30.0	28.4	214.6	204.0	0.82	1.70

[a] Average DBH = stand quadratic mean DBH derived from the estimated total basal area and the number of trees/ha to maintain full compatibility.

[b] PAI = Periodic mean annual increment.

[c] MAI = Mean annual increment.

Table 4. Various yield attributes for the boreal mixedwoods of north central Ontario and for a basal area index = 30 m²/ha.

Stand age (yr)	Number of trees/ha (>1.5cm)	Average DBH[a] (>1.5 cm)	Average height (m)	Basal area (m²/ha) (>1.5 cm)	Basal area (m²/ha) (>10 cm)	Volume (m³/ha) (>1.5 cm)	Volume (m³/ha) (>10 cm)	PAI[b] (m³/ha)	MAI[c] (m³/ha)
20	10 835	4.0	4.2	13.4	2.7	92.2	69.3	0.00	3.47
30	6 701	6.3	6.8	21.1	9.9	133.5	108.2	3.89	3.61
40	4 765	8.4	9.3	26.4	17.9	168.8	143.5	3.53	3.59
50	3 658	10.2	11.6	30.0	24.0	198.6	174.6	3.11	3.49
60	2 947	11.8	13.7	32.4	28.1	224.0	201.9	2.73	3.37
70	2 455	13.3	15.5	34.0	30.8	245.7	225.7	2.38	3.22
80	2 095	14.6	17.1	35.2	32.7	264.3	246.6	2.09	3.08
90	1 822	15.9	18.5	36.0	33.9	280.4	265.0	1.84	2.94
100	1 609	17.0	19.7	36.6	34.9	294.5	281.2	1.62	2.81
110	1 437	18.1	20.7	37.1	35.5	306.8	295.6	1.44	2.69
120	1 296	19.2	21.6	37.5	36.1	317.6	308.4	1.28	2.57

[a] Average DBH = stand quadratic mean DBH derived from the estimated total basal area and the number of trees/ha to maintain full compatibility.
[b] PAI = Periodic mean annual increment.
[c] MAI = Mean annual increment.

Table 5. Various yield attributes for the boreal mixedwoods of north central Ontario and for a basal area index = 40 m²/ha.

Stand age (yr)	Number of trees/ha (>1.5cm)	Average DBH[a] (>1.5 cm)	Average height (m)	Basal area (m²/ha) (>1.5 cm)	Basal area (m²/ha) (>10 cm)	Volume (m³/ha) (>1.5 cm)	Volume (m³/ha) (>10 cm)	PAI[b] (m³/ha)	MAI[c] (m³/ha)
20	10 867	5.1	4.3	22.1	6.5	119.3	90.7	0.00	4.54
30	6 720	7.7	6.9	31.3	18.5	173.5	142.2	5.15	4.74
40	4 779	9.9	9.5	36.8	28.3	219.9	189.2	4.70	4.73
50	3 668	11.8	11.8	40.0	34.4	259.5	231.0	4.18	4.62
60	2 955	13.5	13.8	42.0	38.1	293.3	267.8	3.68	4.46
70	2 462	15.0	15.6	43.3	40.4	322.3	300.1	3.23	4.29
80	2 102	16.4	17.2	44.2	41.9	347.3	328.5	2.84	4.11
90	1 828	17.7	18.5	44.9	42.9	368.9	353.6	2.51	3.93
100	1 613	18.9	19.7	45.3	43.6	387.9	375.8	2.22	3.75
110	1 441	20.1	20.7	45.7	44.1	404.6	395.5	1.97	3.60
120	1 300	21.2	21.5	46.0	44.5	419.3	413.2	1.77	3.44

[a] Average DBH = stand quadratic mean DBH derived from the estimated total basal area and the number of trees/ha to maintain full compatibility.
[b] PAI = Periodic mean annual increment.
[c] MAI = Mean annual increment.

are needed of stand total basal area, preferably using horizontal point sampling (or a Cruzall count), and stand age for the main cover type. Equation [4] and its estimated coefficients (from Table 1) may then be used to calculate the basal area index of the stand (or the main cover type). If several species are equally represented, separate basal area index/species components should be calculated and summed to obtain the stand BI. Having the basal area index and stand age estimated in this way, one may then solve the remaining equations [Equations 5–10] for the species/cover type of interest.

The yield equations presented here have an accuracy similar to that of the earlier ones (e.g., Evert 1975, 1976a, 1976b; Payandeh and Field 1986; Payandeh 1988), and are more applicable for the boreal mixedwoods. The application of basal area index in developing yield equations has several advantages: namely, a) it serves as a valid measure of utilized site productivity and is empirically related to stand attributes as compared to site index;[5] mainly because BI is based on all trees within the stand rather than on only a few site trees; b) it produces variable stocking yield tables suitable for uneven-aged mixed species cover types; and c) unlike the site index, the basal area index may be estimated quickly, accurately, and inexpensively using horizontal point sampling.

Additional comparison of basal area and site indices may be required prior to developing basal area index and variable stocking yield functions for other tree species or geographic regions. If such comparisons confirm the superiority of basal area index over site index, development of a new family of growth and yield models and tables based on basal area index will be in order. Consequently, in addition to the boreal mixedwoods, such yield tables may also prove valuable for disturbed hardwood stands in southern Ontario and the eastern United States.

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