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A GENERALIZED MODEL FOR DEVELOPING COMPOSITE SITE INDEX EQUATIONS

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INTRODUCTION

Characterization of site quality is important in forest management. Clutter et al. (1983) described both direct and indirect methods for quantifying site quality. Direct methods include the estimation of site quality from: (a) historical yield records, (b) stand volume data, (c) height–age relationships (site index), or (d) growth intercepts. Indirect methods include similar estimations from: (a) relationships among the dominant species, (b) lesser vegetation characteristics (or site indicators), or (c) topographic, climatic, and edaphic factors.

Site index has been the most widely used means to estimate site productivity in North America (Carmean 1975). Despite its possible shortcomings (*see* Grigal 1984, Monserud 1984a,b), site index will, in all probability, continue to be used for the foreseeable future. Indirect methods are usually adopted only when direct methods are unsuitable.

Many attempts have been made to relate site index to site factors. However, Vallee and Lawry (1970), Lawry (1972), Monserud (1984a, 1987), and Payandeh (1986) have shown that correlations between site index and soil or vegetation factors are usually poor. Consequently, site productivity functions based on these variables may be of limited use. Monserud (1984a) incorporated habitat factors into site index equations for inland Douglas-fir (*Pseudotsuga menziesii* [Mirb.]). More recently, Payandeh (1991) developed a composite site index equation for peatland black spruce (*Picea mariana* [Mill.] B.S.P.) in northern Ontario.

This paper presents a generalized model that uses both direct and indirect variables for developing composite site index equations. Such composite models are expected to improve both the accuracy and the application of site index equations. The generalized model is compared with the logistic (Monserud 1984a) and the composite site index (Payandeh 1991) models.

METHODS

Base-age specific site index models have the following form:

$$H = f(\beta, t, S) + \varepsilon \qquad [1]$$

where: H is the height of dominant and codominant trees in the stand (m);
S is site index (m);
t is the total age of dominant and codominant trees (years);
β is a parameter vector of dimension p; and
ε is random error.

Model 1 can be fitted to a data set consisting of several subsets of ecological or silvicultural categories. However, the estimated β cannot differentiate among the various categories unless each one is properly distinguished in the model. Monserud (1984a) and Payandeh (1991) employed indicator variables to differentiate several levels of one parameter of β. This procedure generates an estimate of β in which one parameter estimate varies from category to category while the rest are identical for the entire data set.

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Model 1 can be expanded into the generalized composite model:

$$H = f(\beta + I_1\beta_1 + I_2\beta_2 + ..., + I_M\beta_M, t, S) + \varepsilon \qquad [2]$$

where: β_i's are a new parameter vectors of dimension p;
I_i's are indicator scalar variables of values 1 or 0; and
M is a positive integer to be determined by the number of categories in the data set.

M is chosen as a minimum positive integer such that $2^M \geq G$ where G is the number of categories in the data. For example, if a data set contains eight categories, then M should be three; for a data set with 100 categories, M should be seven.

Two data sets were used to examine the performance of Model 2. Data Set 1 consisted of measurements (Sims et al. 1986) from 127 black spruce plots located throughout north central Ontario. On each plot, three well-formed, healthy, dominant or codominant black spruce trees were selected for stem analysis; an average height–age curve and site index were also determined for each plot. Various soil properties were measured and each plot was classified into one of 22 soil types (Sims et al. 1986). The plot summary data included: site index based on total age, site index based on breast height age, soil type, total height, total age, and age at breast height.

Most soil types were assigned to one of seven moisture/texture classes (Table 1). Several soil types represented by only a few plots, or not conforming to the seven classes, were eliminated from the data set.

Correlation analysis was used to determine the relative contribution of the above soil types or moisture/texture classes to the variability of site index (Payandeh and Sutton 1988, Payandeh and Wood 1988).

Table 1. Assignment of forest ecosystem classification to soil types by simplified soil moisture/texture classes for black spruce in north central Ontario.

Forest Ecosystem Classification soil type	Moisture/texture class
S1N1, S1N2, S2C, S2N, S3C, S3N, S4C	1 Dry-fresh sandy loamy
S4N, S4C2, S4C3, S5C, S5N	2 Dry-fresh fine loamy/clayey
S6N, S6C, S7N, S7C,	3 Moist sandy/loamy
S8C, S8N	
S9N, S9C, SS8	4 Moist fine loamy/clayey
S10N, S10C, S11F, S11S, SS9	5 Wet organic, wet organic/rock and peaty phase
SS1, SS2, SS3, SS4	6 Shallow/rock
SS5, SS6, SS7	7 Dry-fresh, moderate deep/rock

Data Set 2 came from 145 plots located in the Clay Belt of northeastern Ontario (Payandeh 1986). Stand age, tree frequency, basal area, and total height data were collected. Also, each plot was classified into one of the 14 operational groups (OGs) (site classes developed for the Clay Belt by Jeglum et al. 1983) in the Forest Ecosystem Classification (FEC) system. The plot site index was calculated based on three to five dominant or codominant trees using the site-index formula developed for peatland black spruce (Payandeh 1978).

Of the 145 plots, 122 were classified into OGs 11, 12, and 14. The other 23 plots, scattered among the remaining OGs, were discarded because of limited observations.

SITE INDEX MODELS EXAMINED

For comparison and evaluation, the two data sets were fitted to the logistic model (Monserud 1984a), composite model (Payandeh 1991), and generalized model. The logistic model takes the following form for both data sets:

$$H = \frac{b_1 S^{b_2}}{1 + e^{(b_3 + b_4 \ln(t) + b_5 \ln(S))}} \qquad [3]$$

where main symbols are as defined for Models 1 and 2. The composite model (Payandeh 1991) for the first data set containing seven soil types is:

$$H = \frac{b_1 S^{(b_2 + I_1\alpha_1 + I_2\alpha_2 + I_3\alpha_3)}}{1 + e^{(b_3 + b_4 \ln(t) + b_5 \ln(S))}} \qquad [4]$$

where: α_i's and b_i's are parameters to be estimated;
$I_1 = 0, I_2 = 0$, and $I_3 = 0$ for Soil Group 1;
$I_1 = 0, I_2 = 0$, and $I_3 = 1$ for Soil Group 2;
$I_1 = 0, I_2 = 1$, and $I_3 = 0$ for Soil Group 3;
$I_1 = 0, I_2 = 1$, and $I_3 = 1$ for Soil Group 4;
$I_1 = 1, I_2 = 0$, and $I_3 = 0$ for Soil Group 5;
$I_1 = 1, I_2 = 0$, and $I_3 = 1$ for Soil Group 6; and
$I_1 = 1, I_2 = 1$, and $I_3 = 0$ for Soil Group 7.

The composite model for Data Set 2 takes the form:

$$H = \frac{b_1 S^{(b_2 + I_1\alpha_1 + I_2\alpha_2)}}{1 + e^{(b_3 + b_4 \ln(t) + b_5 \ln(S))}} \qquad [5]$$

where: $I_1 = 1$ and $I_2 = 0$ for OG 11;
$I_1 = 0$ and $I_2 = 1$ for OG 12; and
$I_1 = 0$ and $I_2 = 0$ for OG 14.

The generalized model for Data Set 1 has the form:

$$H = \frac{\beta_1 S^{\beta_2}}{1 + e^{(\beta_3 + \beta_4 \ln(t) + \beta_5 \ln(S))}} \qquad [6]$$

$$\begin{bmatrix}\beta_1\\ \beta_2\\ \beta_3\\ \beta_4\\ \beta_5\end{bmatrix} = \begin{bmatrix}b_{01}\\ b_{02}\\ b_{03}\\ b_{04}\\ b_{05}\end{bmatrix} + I_1\begin{bmatrix}b_{11}\\ b_{12}\\ b_{13}\\ b_{14}\\ b_{15}\end{bmatrix} + I_2\begin{bmatrix}b_{21}\\ b_{22}\\ b_{23}\\ b_{24}\\ b_{25}\end{bmatrix} + I_3\begin{bmatrix}b_{31}\\ b_{32}\\ b_{33}\\ b_{34}\\ b_{35}\end{bmatrix}$$

where: b_{ij}'s are parameters to be estimated; and
I_i's are indicator variables as in Model 4.

The generalized model for Data Set 2 has the form:

$$H = \frac{\beta_1 S^{\beta_2}}{1 + e^{(\beta_3 + \beta_4 \ln(t) + \beta_5 \ln(S))}} \qquad [7]$$

$$\begin{bmatrix}\beta_1\\ \beta_2\\ \beta_3\\ \beta_4\\ \beta_5\end{bmatrix} = \begin{bmatrix}b_{01}\\ b_{02}\\ b_{03}\\ b_{04}\\ b_{05}\end{bmatrix} + I_1\begin{bmatrix}b_{11}\\ b_{12}\\ b_{13}\\ b_{14}\\ b_{15}\end{bmatrix} + I_2\begin{bmatrix}b_{21}\\ b_{22}\\ b_{23}\\ b_{24}\\ b_{25}\end{bmatrix}$$

where: b_{ij}'s are parameters to be estimated; and
I_i's are indicator variables similar to Model 5.

RESULTS AND DISCUSSION

Table 2 summarizes mean squared errors (MSE) obtained for the two data sets using Models 3–7. The MSE for the logistic Model 3 was used as a base of comparison for calculating the percent reduction by other models in each data set. For Data Set 1, the composite model (Model 4) and the generalized model (Model 6) reduced MSE by 5 percent and 1 percent, respectively. For Data Set 2, the composite model (Model 5) and the generalized model (Model 7) reduced MSE by 21 percent and 27 percent, respectively. Families of site index curves based on the generalized model for Data Set 2 (Model 7) for Site Indices 3, 6, and 9 m, and OGs 11, 12, and 14 are shown in Figure 1. Note that the three OGs produced distinctly different curves for each site index. Figure 1 also shows polymorphism more clearly, especially at younger ages, than did those curves presented earlier by Payandeh (1991). This indicates that the generalized model described both data sets better than did either the composite site index model (Payandeh 1991) or the logistic site index model (Model 3) (Monserud 1984a).

Table 2. Regression mean squared errors (MSE) for Models 3–7 and percent reduction in MSE from Model 3 for black spruce from north central Ontario (Data Set 1) and northeastern Ontario (Data Set 2).

Model	Data Set	MSE	Reduction (%)
Logistic (Model 3)	1	2.05	0.0
Composite (Model 4)	1	1.96	4.6
Generalized (Model 6)	1	2.04	1.0
Logistic (Model 3)	2	1.03	0.0
Composite (Model 5)	2	0.85	21.2
Generalized (Model 7)	2	0.81	27.2

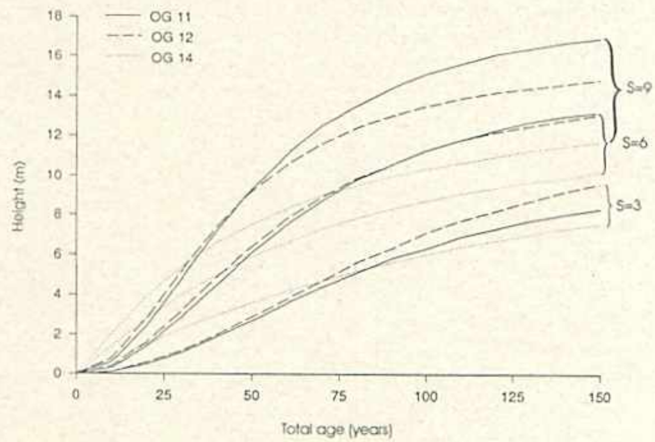

Figure 1. Graphs of the site index curves from Model 7, based on Data Set 2 for Site Indices 3, 6, and 9 m and Operational Groups 11, 12, and 14.

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